Siggraph Course

Mesh Parameterization – Theory and Practice

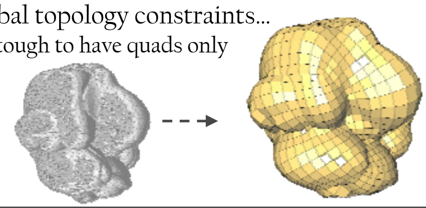
SIGGRAPH2007

Application of DEC to Global Parameterization

(and beyond)

Motivation: Quadrangulations

Needed in CAGD, Reverse Engineering
- Ubiquitous (tensor-product nature)
  - Modeling anisotropy/symmetries
  - FEM, texture atlasing
- global topology constraints…
  - tough to have quads only



Reverse Engineering

For a **local** patch of quadrangulation

- induce natural (*u*,*v*) parametrization
- **Edges: integer-valued isocurves of u/v**
- “Nice” mesh ≈ squarish mesh

$$\langle \nabla u, \nabla v \rangle = 0 \qquad \langle \nabla u, \nabla u \rangle = \langle \nabla v, \nabla v \rangle.$$

  - Cauchy-Riemann equations
  - using language of differential form, this is

one-forms → $du = \star dv$

- Thus, *u* and *v* are both *harmonic* (Laplacian=0)
  - *du* and *dv* too! Cool, DEC is perfect for that…
  - similar to [Gu and Yau ’03]

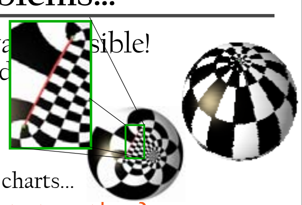
Constructing a Quad Mesh

Plan of Attack
- solve for two scalar fields (potentials)
  - *u* and *v*, stored on the input mesh
    - *parameterization!*
  - such that $du = \star dv$
- “extract” a mesh through isocontouring
  - trace isovalues (*u*=const, *v*=const) [Dong et al. 04]
  - create vertices at intersections, and connect up

Seems easy enough, right?

Problems, Problems…

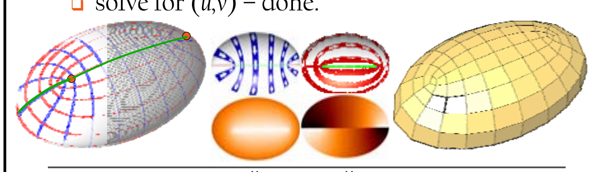
Global solve not always possible!
- *Singularities* unavoidable
  - either poles
  - or “seams”
    - T-junctions…
  - seems only ok for charts…



Can we connect charts together?
- only requirement: **continuity of 1-forms**
- so we can actually use discontinuous (u,v)!
  - [Tong et al. 2006]
- harmonicity *almost* everywhere (thru charts)

Putting it All Together

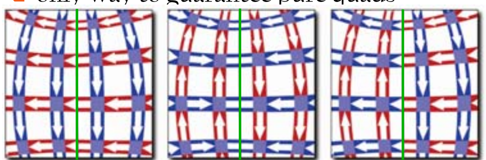
From an atlas of charts (“meta-mesh”):
- assemble a *tweaked* Laplacian matrix and rhs
  - one vertex = one line in matrix (except for metavertices)
  - away from chart boundaries: regular cot formula
  - on boundaries: make sure d*u* and d*v* aligned
- solve for (*u*,*v*) – done.



Possible Discontinuities

Only three different cases:

- only way to guarantee pure quads



$du^+ = du^-$, $dv^+ = dv^-$ | $du^+ = -du^-$, $dv^+ = -dv^-$ | $du^+ = dv^-$, $dv^+ = -du^-$

- in all 3 cases, just a tweak of the Laplacian

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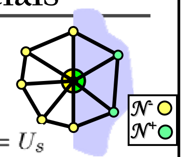
Discontinuous Potentials

Example of tweaked Laplacian

- continuity of 1-form induces:



$$du^- = du^+$$

$\Rightarrow u^- = u^+ + U_s$ *i.e.*, $u^- - u^+ = U_s$

Similarly, $v^- - v^+ = V_s$

$$\sum_{j\in\mathcal{N}(i)}(cot\alpha_{ij}+cot\beta_{ij})\begin{pmatrix}u_i-u_j\\ v_i-v_j\end{pmatrix} = \sum_{j\in\mathcal{N}^+(i)}(cot\alpha_{ij}+cot\beta_{ij})\begin{pmatrix}U_s\\ V_s\end{pmatrix}$$

Generates smooth fields *modulo* the jump!

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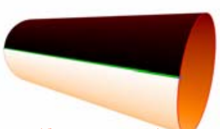
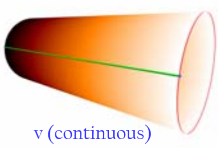
Simple Example of Tweaked Δ

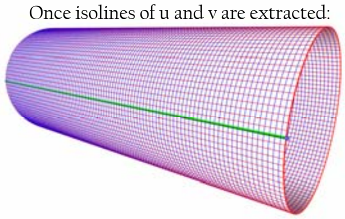


u (discontinuous)



v (continuous)

Once isolines of u and v are extracted:



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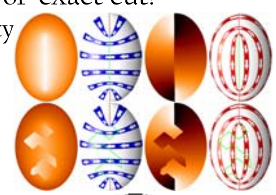
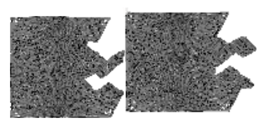
Magic Happens

Results independent of exact cut!

- only on connectivity between corners...
- parameterization





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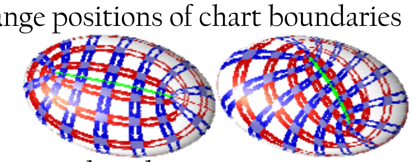
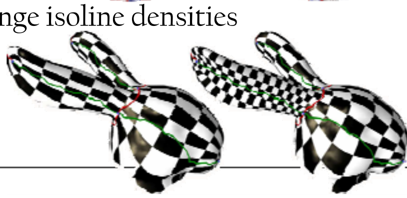
Ease of Editing

Change positions of chart boundaries



Change isoline densities



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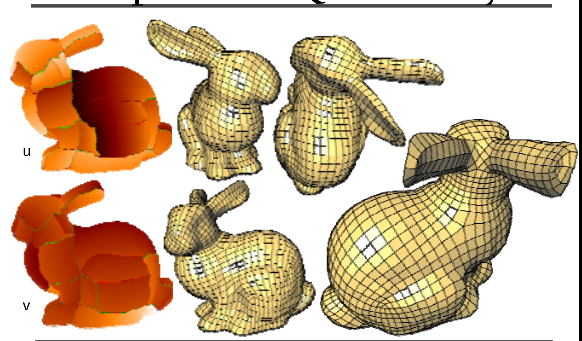
Example: Pure-Quad Bunny



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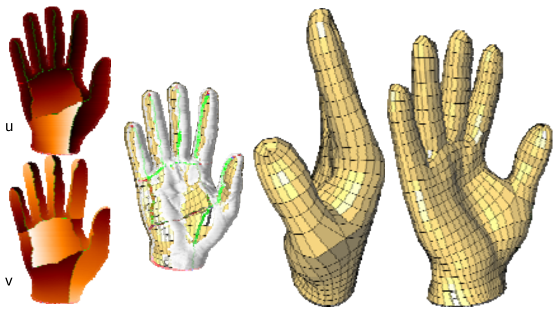
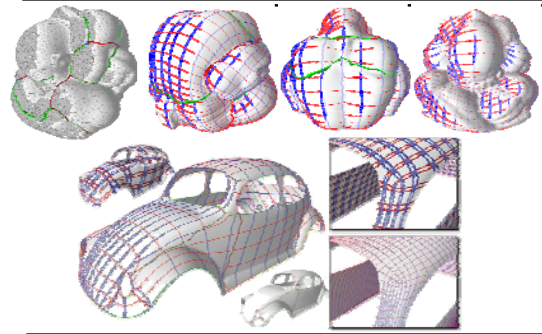
Example: Pure-Quad Hands



More Examples



Another DEC Application

Laplacian of One-Forms

- extension of Dirichlet energy to vector fields

$$E_D(\vec{u}) = \int_S |\nabla \cdot \vec{u}|^2 + |\nabla \times \vec{u}|^2 = u_e^T D u_e$$

- allows easy design of vector fields on surfaces
  - for texturing, e.g.
  - [Fisher et al. '07]

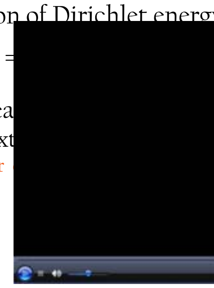
Another DEC Application

Laplacian of One-Forms

- extension of Dirichlet energy to vector fields

$E_D(\vec{u})$ = … $= u_e^T D u_e$

- allows ea… fields on surfaces
  - for text…
  - [Fisher …

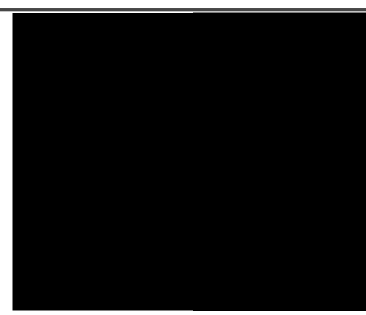


Just The Tip of The Iceberg

Wanna know more about DEC?

- "Discrete Differential Forms" by MD, Eva Kanso, Yiying Tong
  - much more details and pointers to literature:
    - homology, cohomology, Hodge decomposition
    - living document… help us improve it!
- "Build Your Own DEC at Home" by Sharif Elcott & Peter Schröder
  - for a bullet-proof implementation of DEC
- www.geometry.caltech.edu

What Else Can We Do?



Acks: Patrick Mullen, Alex MacKenzie, Yiying Tong, Eva Kanso, Sharif Elcott, Lily Kharevych, Peter Schröder, Pierre Alliez…

Submit Your Work to SGP

Symposium on Geometry Processing '08

- P. Alliez, S. Rusinkiewicz paper chairs

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