Mesh Parameterization: Theory and Practice

X-parameterization/Inter-surface mapping

Alla Sheffer
X-Parameterization/Inter-surface mapping

- Mapping between surface models (meshes)
  - One-to-one
  - Or not..

- Often enforce feature correspondence
Applications

Properties Transfer
Shape Spaces

Morphing/Blending
Mesh Completion
Main Approaches

• **Compatible Segmentation**
  - Coarse-to-fine [Schreiner:04]
  - Base-Mesh parameterization [Praun:01; Kraevoy & Sheffer:04; Kraevoy & Sheffer:05]

• **Global Matching** [Allen:03; Anguelov:05; Bronstein:07]
  - With/without specified feature correspondences
**Coarse-to-fine** [Schreiner:04]

- Segment meshes into compatible patches
  - Use input features as corners
- Simplify each patch to triangle
- Compute map by coarse-to-fine optimization
  - Can relax feature correspondence
Compatible Segmentation \[\text{Schreiner:04; Kraevoy \& Sheffer:04}\]

- Add pairs of matching (shortest) paths between feature vertices
  - Validity checks
    - Intersection
    - Order
    - Blocking
  - Add vertices when necessary
- Extends \[\text{Praun:01; Kraevoy:03}\]
Coarse-to-fine Optimization

- Insert vertices incrementally
  - Map each vertex inserted in one mesh to the other
    - Use local 2D parameterization
    - Use stretch metric
- Relax surroundings
  - Fairly time consuming
    - Time/Quality tradeoff
Coarse-to-fine - demonstration
Examples
Common Base Mesh

• Segment meshes into compatible patches
• Compute initial mapping
  – Construct base
    • triangle per patch
  – Barycentric mapping
    • Guarantee bijectivity
• Improve mapping
Distortion & Artifacts

- Badly shaped patches = high distortion

- To reduce distortion, must move vertices between base triangles
Improvement Approaches

- **Global** [Khodakovsky:03;Aliez:06]
  - Solve (repeatedly) global linear system

- **Overlapping domains** [Guskov:00;Kraevoy & Sheffer:04;Kraevoy & Sheffer:05]
  - Group adjacent patches
    - map to convex domain
  - Iterate
    - Maintains validity
    - Fast
Application Examples

Texture transfer
80K/7K faces
56 sec

Motion transfer
[Sumner & Popovic 04]
Mapping incomplete meshes – Base mesh approach [Kraevoy & Sheffer:05]

• Application: Mesh Completion

• Challenges
  – Compatible segmentation (multiple components)
  – Parameterizing holes
Base Parameterization with Holes/gaps

Segmentation

?  Parameterization

Idea: triangulate holes (smartly)
**Initial Parameterization**

- Map each patch to base triangle
  - Use uniform embedding
    - works for n-sided polygons
    - Bijective
- Triangulate holes
Improvement

- Iterate on overlapping domains
  - Parameterization + re-triangulation
Improvement
Completion Results

Sizes: 20455/27562  Markers: 39  Time 45s
Global Approaches

- Energy Based [Allen:03]
  - Continuous setup

- Machine Learning  [Anguelov:05]
  - Correspondence as discrete optimization

- Geodesic MDS [Bronstein:07]
  - No feature specification
Energy Based Mapping [Allen:03]

- Pull one model towards the other - minimize

\[ E = \alpha E_d + \beta E_s + \gamma E_m, \]

- feature correspondence

\[ E_d = \sum_{i=1}^{n} w_i \text{dist}^2(T_i v_i, D) \]

- attraction

\[ E_m = \sum_{i=1}^{m} \|T_{\kappa_i} v_{\kappa_i} - m_i\|^2 \]

- shape preservation

\[ E_s = \sum_{\{i,j\} \in \text{edges}(T)} \|T_i - T_j\|^2 \]

+ Oblivious to topological noise

- Robustness issues

- Dense (74) markers

- Similar pose
Correlated correspondence [Anguelov:05]

- Machine learning setup
  - Correlated correspondence (Loopy belief propagation)
    - Input: 4-10 markers
    - Output: 150-200 markers
  - Compute full registration
- Few markers
- Variety of poses/proportions
  - relies on shape similarity
Generalized multidimensional scaling
[Bronstein:07]

• Use MDS to match models in alternative space
  – use geodesics as pairwise point distances
• Assumes near-isomeric transformation
Comparison

• Compatible Segmentation
  – Need “good” features
    • “bad” features cause distortion
  – Same genus
  – Validity guarantee
    • may be at expense of distortion

• Global
  – Recent methods require less (no) features
  – Topology oblivious (almost)
  – Can fail
    • small & large overlaps
    • similarity dependant
External Material