Stable Topology Optimization: A Barycentric FEM Approach

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Acknowledgments:

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SOM – Skydmore, Owings and Merrill
Pop Quiz: Is the sidewalk real or simulated?

Modeling the Copacabana Sidewalk Pavement

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(a) Polygons from Voronoi  
(b) Wide angle elimination  
(c) Imposed separation  
(d) Final black and white stones
2 Posters

Engineering a new architecture through barycentric element based topology optimization

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Structural Topology Optimization employing the Allen-Cahn Evolution Equation on Unstructured Polygonal Meshes

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Optimization of shape and topology

- The goal is to find the most efficient shape of a physical system

The response is captured by the solution to a boundary value problem that in turn depends on the given shape

\[ \nabla \cdot [C_\omega : \varepsilon(u)] = 0 \]
\[ u|_{\Gamma_D} = 0, \quad [C_\omega : \varepsilon(u)] \cdot n|_{\Gamma_N} = t \]

\[ C_\omega = \chi_\omega C^+ + (1 - \chi_\omega) C^- \]

- The cost function depends on the physical response as well as the geometric features of admissible shapes

\[ J(\omega, u_\omega) = \int_{\Gamma_N} t \cdot u_\omega ds + \lambda |\omega| \]
Existence of solutions

- Shapes with fine features are naturally favored, which leads to nonconvergent minimizing sequences that exhibit rapid oscillations.

\[ \Gamma_N = \partial \Omega, \quad \Gamma_D = \emptyset, \quad t = (e_d \otimes n) \cdot t_0 \]

- Existence of solutions can be guaranteed by introducing some suitable form of regularization in the problem:

\[ \tilde{J}(\omega) = J(\omega, u_\omega) + \beta \int_\Omega |\nabla \chi_\omega| \, dx \]
Large-scale optimization problem

- Accurate analysis of the response and capturing detailed features required fine spatial discretizations which leads to a large number of design and analysis variables.

- We can exploit the composite nature of the cost function

\[ \tilde{J}(\omega) = J(\omega) + R(\omega) \]

To derive convergent optimization algorithms. Forward-backward splitting, for example, leads to iterations of the form

\[ \omega_{n+1} = (I + \tau_n R')^{-1}(I - \tau_n J')\omega_n \]

Sample result

Design domain

$\beta = 0.01$

$\beta = 0.05$

Comparable “filtering” result
Honeycomb Wachspress finite elements for structural topology optimization

Cameron Talischi · Glaucio H. Paulino · Chau H. Le
Stable Topology Optimization

Baricentric FEM
Polygonal Elements

T6 Elements

Design of Piezocomposite Material Unit Cell

→ Quad Element
→ fixed polarization

k = 0.291
Gain$^{(1)} = 100.7$
$\sigma/\sigma_0 = 5.1$

→ Polygonal Element
→ fixed polarization

k = 0.298
Gain$^{(1)} = 105.5$
$\sigma/\sigma_0 = 4.6$

→ Polygonal Element
→ free polarization

k = 0.319✓
Gain$^{(1)} = 120.0$
$\sigma/\sigma_0 = 4.9$

$^{(1)}$ w.r.t. PZT-5A (k = 0.145)
Stability

Stability

- Numerical instabilities such as the “checkerboard” problem could appear in mixed variational formulation (pressure-velocity) of the Stokes flow problems.

Paulino et al. “Polygonal finite elements for mixed variational problems”, 2012
Stability

- Numerical instabilities such as the “checkerboard” problem could appear in mixed variational formulation (pressure-velocity) of the Stokes flow problems.

Paulino et al. “Polygonal finite elements for mixed variational problems”, 2012
Stability

- Babuska-Brezzi condition (or inf-sup)
  - Required for the stability of mixed variational formulation of incompressible elasticity and Stokes flow problems
  - Polygonal discretizations satisfy the well-known Babuska-Brezzi condition

Paulino et al. “Polygonal finite elements for mixed variational problems”, 2012

G.A.M.E.S. Camp

• Makes complex structural engineering concepts accessible to middle and high school students!
Craniofacial Reconstruction

- Motivation and Background: Craniofacial reconstruction
- Load transfer mechanism
- Multi-resolution Topological Optimization
- Results
Craniofacial Reconstruction

- Head or Facial Trauma
- Cancer patients who lost part of the bony structure, soft tissue

✓ Complex and Challenging

- 3D Complex Architecture
- Serves Functional and aesthetic role
  - Facial expression, mastication, speech, and Deglutition (swallowing of food)
  - Facial Appearance
Craniofacial Reconstruction Cancer

Cancer

Tissue destruction

Deformity

Decreased Quality of Life

Treatment

• Surgery
• Radiation
• Chemotherapy
Current Approach

- Heuristic Approach
- Ad-hoc method by the surgeon during surgery

Yamamoto [2005]
Clinical Approach

Fibula Osteotomies
Clinical Problem

Fibula  Osteotomies  Surgery

Topology Optimization
Alternative …
Goals of the replacement bone

- Give support to orbital content, Avoid Changes in globe position, orbital volume, eyelid functions
- To preserve a platform for mastication, speech and dental rehabilitation
- To recreate an adequate and symmetric facial contour with other side of face

Topology Optimization
Topology Optimization Procedure

### Problem formulation

\[
\begin{align*}
\min_{\rho} & \quad C(\rho, u_d) = f^T u_d \\
\text{s.t.} & \quad K(\rho) u_d = f \\
V(\rho) = & \int_{\Omega} \rho(\psi) dV \leq V_s \\
0 < & \rho_{\text{min}} \leq \rho(\psi) \leq 1
\end{align*}
\]

### Solid and Isotropic Material with Penalization (SIMP)

- Finite Element Analysis
  - Objective Function & Constraints
  - Sensitivities Analysis
  - Filtering (Projection) Technique
  - Update Material Distribution

- Converged?
  - Yes
  - No

- Initial guess
- Result

\[
\begin{align*}
E(\psi) &= \rho(\psi)^p E^0 \\
K(\rho) &= \sum_{e=1}^{N_{el}} K_e(\rho_e) = \sum_{e=1}^{N_{el}} \int_{\Omega_e} B^T D(\rho_e) B d\Omega \\
\frac{\partial C}{\partial \rho_e} &= -u_e^T \frac{\partial K_e}{\partial \rho_e} u_e = -p \rho_e^{p-1} u_e^T K_e^0 u_e \\
\frac{\partial V}{\partial \rho_e} &= \int_{\Omega_e} dV
\end{align*}
\]
High Resolution Topology Optimization

- Large-scale (high resolution) TOP
  - Large number of finite elements
  - Computationally expensive: FEA cost

- Existing high resolution TOP
  - Parallel computing (Borrvall and Petersson, 2000)
  - Fast solvers (Wang et al. 2007)
  - Approximate reanalysis (Amir et al. 2009)
  - Adaptive mesh refinement (de Stuler et al. 2008)

Same discretization for analysis and design optimization
Multiresolution Topology Optimization (MTOP)

- Conventional element-based approach (Q4/U)
  - Same discretization for displacement and density

- MTOP approach (Q4/n25)
  - Different discretizations for displacement and density/design variables
MTOP

• B8/n125 Element

MTOP is being used as the engine for the topology optimization APP recently developed by the Danish group (2012)

Interactive topology optimization on hand-held devices, under preparation, 2012
Craniofacial Reconstruction

- MRI Data
- Select Boundary Conditions
- Select Load
2D Verification of the Concept
Craniofacial Reconstruction
Craniofacial Reconstruction
Craniofacial Reconstruction

Topological optimization for designing patient-specific large craniofacial segmental bone replacements

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Edited\textsuperscript{a} by Arner Friedman, The Ohio State University, Columbus, OH, and approved June 22, 2010 (received for review February 10, 2010)

Restoring normal function and appearance after massive facial injuries with bone loss is an important unsolved problem in surgery. An important limitation of the current methods is heuristic ad hoc design of bone replacements by the operating surgeon at the time of surgery. This problem might be addressed by incorporating special metal plates and screws. For massive defects, the bone must be transferred with a blood supply that is independent from the surrounding damaged tissues. The bone is isolated based on a single artery and vein that are surgically reattached to other uninjured blood vessels in the nearby face or neck. These vessels

- Orbit
- Buttresses
The entire process in 40 seconds …
Biological Constraints

Successful Tissue Transfer
Mechanical Variable + Biological Constraint –
Vascular Healing

Modeling oxygen transport in surgical tissue transfer

Anastasios Matzavinos, Chiu-Yen Kao, J. Edward F. Green, Alok Sutrondar, Michael Miller, and Avner Friedman

Contributed by Avner Friedman, May 14, 2009 (sent for review November 7, 2008)

Reconstructive microsurgery is a clinical technique used to transfer large amounts of a patient’s tissue from one location used to another in order to restore physical deformities caused by trauma, tumors, or congenital abnormalities. The trend in this field is to transfer tissue using increasingly smaller blood vessels, which decreases problems associated with tissue harvest but increases the possibility that blood supply to the transferred tissue may not reach areas farthest from the body. This difficulty led to introduction of a surgical flap based only on the blood vessels that pass through the muscles called perforating vessels. Flaps were designed that required the surgeon to select a single perforating vessel and follow it through the muscle without including any muscle in the flap for transfer. The perforating vessels have luminal diameters ranging from several hundred microns up to as much as 1.5 mm. The
Experimental Validation
Load-transfer mechanism

The Transmission of Masticatory Forces and Nasal Septum: Structural Comparison of the Human Skull and Gothic Cathedral

Rumy Hilliowala, D.D.S., Ph.D.; Hrishi Kanth, M.D.
Building Science Through Topology Optimization

- Topology optimization for high-rise buildings
Building Science Through Topology Optimization

- Application of pattern gradation to the conceptual design of buildings
  - Flexible tool for unique shapes
  - Increasing column sizes
  - Dominance of shear behavior at top vs. overturning moment at base

- In collaboration with Skidmore, Owings & Merrill, LLP

- Lotte Tower (Korea): optimized bracing using pattern gradation concepts
Building Science Through Topology Optimization

- Topology optimization for structural braced frames: combining continuum and beam/column elements
Building Science Through Topology Optimization

• Connecting engineering and architecture using structural topology optimization

• Goal: to create unique, innovative designs that are both aesthetically pleasing and satisfy engineering principles

• Zendai competition: optimal designs resembling nature
Concluding Remarks

- FOBOS with Tikhonov Regularized TOP leads to nearly B&W solutions
- Baricentric FEM provides a stable formulation for Topology Optimization
- Topological Optimization is a linkage between medicine & engineering – promising approach for patient-specific computer-aided design of mid-face reconstruction
- Topological Optimization is a linkage between architecture & engineering – leads bioinspired design of tall buildings
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