NSF Workshop on

Barycentric Coordinates in Geometry Processing and Finite/Boundary Element Methods

Columbia University, New York, USA

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Programme and Abstracts

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Programme

	Wednesday	Thursday	Friday
08:30–17:00	Registration	Registration	Registration
09:00–10:30	Opening *L. Moss-Salentijn* Introduction *F. L. DiMaggio* *E. Wachspress (P)*	*M. Floater (P)* *A. Gillette (P)*	*E. Grinspun (T)* *A. Yavari (P)*
10:30–11:00	Coffee break	Coffee break	Coffee break
11:00–12:30	*K. Hormann (T)*	*N. Sukumar (T)* *S. Mousavi (C)* *J. Rimoli (C)*	*P. Memari (C)* *F. de Goes (C)* *E. Vouga (C)*
12:30–14:00	Lunch	Lunch	Lunch
14:00–15:30	*C. Gotsman (P)* *O. Weber (P)*	*G. Paulino (P)* *J. Bishop (P)*	*G. Manzini (C)* *A. Rosolen (C)* *E. Wachspress (C)*
15:30–16:00	Coffee break	Coffee break	Coffee break
16:00–17:30	*A. Jacobson (C)* *G. Dasgupta (T)*	Poster session	Panel discussion
19:30–22:00		Workshop dinner	

T – Tutorials / P – Plenary talks / C – Contributed talks

Posters

A geometric discretization scheme for incompressible elasticity
Arzhang Angoshtari (Georgia Institute of Technology, Atlanta)

Blended barycentric coordinates
Dmitry Anisimov (Università della Svizzera italiana, Lugano)

Engineering a new architecture through barycentric element based topology optimization
Lauren Beghini (University of Illinois, Urbana-Champaign)

Upscaling for the Laplace problem using a discontinuous Galerkin Method
Theophile Chaumont-Frelet (INSA Rouen)

On the shape-preserving property of barycentric coordinates
Qi Chen (Karlsruhe Institute of Technology)

GPU-based algorithms for cutting deformable objects in implicit simulations
Hadrien Courtecuisse (Cardiff University)

A nonlocal damage mechanics formulation for simulating creep fracture in ice sheets and glaciers
Ravindra Duddu (Vanderbilt University, Nashville)

Structural topology optimization employing the Allen–Cahn evolution equation on unstructured polygonal meshes
Arun Lal Gain (University of Illinois, Urbana-Champaign)

Algebraic multigrid for fracture modeled with XFEM
Badri Hiriyur (Weidlinger Associates, Inc., New York)

Hybridized discontinuous Galerkin for nonlinear elasticity
Hardik Kabaria (Stanford University)

Connection between DEC and FEEC
Kaushik Kalyanaraman (University of Illinois, Urbana-Champaign)

Anisotropic Voronoi diagrams from distance graphs
Mario Kapl (Johannes Kepler University, Linz)

Discrete damage zone model for fracture initiation and propagation
Xia Liu (Columbia University, New York)

Effect of engine placement on aeroelastic trim and stability of flying wing aircraft
Pezhman Mardanpour (Georgia Institute of Technology, Atlanta)

Polygonal finite elements: cubature and application to Reissner–Mindlin plates
Sundararajan Natarajan (Indian Institute of Science, Bangalore)

Modeling honeycomb sandwich composite using polygonal finite element method & simulation of crack propagation using XFEM
Deepak Patel (Indian Institute of Science, Bangalore)

Metal fatigue in dental implants: a Markoff chain-stochastic finite element formulation
María Prados-Privado (Universidad Rey Juan Carlos, Madrid)

Biharmonic coordinates for shape deformation
Roi Poranne (Technion, Haifa)

Finite differences with high orders of approximation for irregularly discretized domains
Teseo Schneider (Università della Svizzera italiana, Lugano)

Nondestructive identification of multiple flaws in structures using a combination of XFEM and topologically adaptive enhanced artificial bee colony (EABC) optimization algorithm
Hao Sun (Columbia University, New York)

Mean curvature skeletons
Andrea Tagliasacchi (Simon Fraser University, Burnaby)

Adaptively weighted numerical integration over arbitrary domains
Vaidyanathan Thiagarajan (University of Wisconsin–Madison)

Adaptive regular simplex bisection wavelets
Kenneth Weiss (Lawrence Livermore National Laboratory)

Parallel surface and volume projection for FEM simulation on unrelated arbitrarily distributed meshes
Patrick Zulian (Università della Svizzera italiana, Lugano)

Abstracts

A polyhedral finite-element formulation using harmonic shape functions with applications to the modeling of multi-physics fracture processes

Joseph E. Bishop (Sandia National Laboratories)

Recently, several approaches for generating conformal shape functions on polyhedra have appeared, including the use of harmonic functions. However, the use of these non-polynomial based shape functions in the standard displacement-based finite-element machinery is challenging due to the apparent need for higher-order element integration. In this talk, recent research into relaxing this integration requirement is presented.

Our research into polyhedral finite-element formulations stems from the challenge of modeling pervasive fracture processes using continuum mechanics. A pervasive fracture process is one in which a multitude of cracks are dynamically active, propagating in arbitrary directions, coalescing, and branching. Pervasive fracturing is a highly nonlinear process involving complex material constitutive behavior, post-peak material softening, localization, new surface generation, and ubiquitous contact. One approach for modeling pervasive fracture processes is by allowing new fracture surfaces to nucleate only at the inter-cell faces of a randomly close-packed (RCP) Voronoi tessellation. Each cell of the Voronoi mesh is formulated as a polyhedral finite element. The *a priori* crack paths of the RCP Voronoi mesh are viewed as instances of realizable random crack paths within a random field representation of the continuum material properties. Mesh convergence in a pervasive fracture simulation is viewed in a distributional sense rather than at the level of a single realization. In this talk, several geomechanical applications of this computational methodology are presented including the modeling of hydraulic fracturing and the assessment of caprock integrity for CO_2 sequestration.

Projective geometry and symbolic computing

Gautam Dasgupta (Columbia University, New York), José Antonio Bea (University of Zaragoza)

In nineteen eighties, Columbia Engineering and Medical Schools conducted an NIH sponsored program project to investigate biological significance of shape and size changes. A nine nodes rat skull and an eye model with 200 boundary points were analyzed as single elements. Symbolic computation made it possible to bridge the gap between the finite and boundary elements. Recent work on source terms that require side nodes will be described within the context of Wachspress coordinates.

Blue noise through optimal transport

Fernando de Goes (California Institute of Technology, Pasadena)

We present a fast, scalable algorithm to generate high-quality blue noise point distributions of arbitrary density functions. At its core is a novel formulation of the recently-introduced concept of capacity-constrained Voronoi tessellation as an optimal transport problem. This insight leads to a continuous formulation able to enforce the capacity constraints exactly, unlike previous work. We exploit the variational nature of this formulation to design an efficient optimization technique of point distributions via constrained minimization in the space of power diagrams. Our mathematical, algorithmic, and practical contributions lead to point sets with improved spectral and visual behavior, at a fraction of the computational costs of previous approaches to high-quality blue noise generation.

Barycentric coordinates and transfinite interpolation

Michael Floater (University of Oslo)

Recent generalizations of barycentric coordinates to polygons and polyhedra, such as Wachspress and mean value coordinates, have found various applications in curve and surface modelling, such as surface parameterization and curve and surface deformation. These coordinates have been further generalized to curved domains and mean value interpolants have also been generalized to Hermite interpolants: matching derivatives on the boundary. This talk summarizes some of these developments.

Error estimates for generalized barycentric coordinate finite element methods

Andrew Gillette (University of California, San Diego)

The error analysis of finite element methods employing generalized barycentric coordinates (GBCs) on meshes of polygons or polyhedra requires a subtle combination of computational geometry and functional analysis. In this talk, we will see how the derivation of a linear order error estimate can be obtained for a variety of GBCs on polygons, including Wachspress, Harmonic, Sibson, and Mean Value. The estimate is subject to certain geometric constraints depending on the type of GBCs being used. We will also see how pairwise products of GBCs can be employed to create a quadratic order finite element method with only $2n$ basis functions on a convex n-gon. On a mesh of squares, this approach recovers the well-known 'serendipity space' while providing a novel generalization to meshes of non-affinely mapped squares and polygons. We will conclude with a view toward future directions of research along these lines, both theoretical and computational.

Complex barycentric coordinates

Craig Gotsman (Technion, Haifa)

Barycentric coordinates are very popular for interpolating data values on polyhedral domains. It has been recently shown that expressing them as complex functions has various advantages when interpolating two-dimensional data in the plane, and in particular for holomorphic maps. In this talk we survey the complex representation of barycentric coordinates, especially when applied to planar domains.

We describe a construction for real-valued barycentric coordinates from a given "weight function" and show how it can be applied to generating complex-valued coordinates, thus deriving complex expressions for the classical barycentric coordinates and the more recent conformal Cauchy–Green coordinates. Furthermore, we show that a complex barycentric map admits the intuitive interpretation as a complex-weighted combination of edge-to-edge similarity transformations, allowing the design of "home-made" barycentric maps with desirable properties. Thus, using the tools of complex analysis, we provide a methodology for analyzing existing barycentric mappings, as well as designing new ones. We conclude with some open questions.

Discrete differential geometry: barycentrics, Laplacians, and mechanics

Eitan Grinspun (Columbia University, New York)

Discrete differential geometry (DDG) is a budding field of mathematics that has seen successful applications in mechanics and geometry processing. DDG seeks to expose the mathematical structures—symmetries, invariants, and theorems—of a continuous geometric picture, and then to rebuild these structures from the ground up in the discrete setting. The result is a discrete, hence immediately computable, representation of the system of interest, that exactly captures the structures of interest, independent of the resolution of the discretization. We will survey some of our recent work in this field, along the way drawing connections between barycentric coordinates, discrete Laplace operators, and computational mechanics.

Generalized barycentric coordinates

Kai Hormann (Università della Svizzera italiana, Lugano)

In 1827, August Ferdinand Möbius published his seminal work on the "barycentric calcul" which provides a novel approach to analytic geometry. One key element in his work is the idea of barycentric coordinates which allow to write any point inside a triangle as a unique convex combination of the triangle's vertices. These triangular barycentric coordinates are linear and possess the Lagrange property, and are therefore commonly used to linearly interpolate values given at the vertices of a triangle. Möbius also noticed that this construction extends nicely to linear interpolation of data given at the vertices of a d-dimensional simplex, and by giving up positivity of the coordinates, we can even extrapolate the data to every point in d dimensions.

While barycentric coordinates are unique for simplices, they can be generalized in several ways to arbitrary polygons and polytopes in higher dimensions, and over the past few years, a number of recipes for such generalized barycentric coordinates have been developed. As they are usually given in closed form and can be evaluated efficiently, they have many useful applications, e.g. in computer graphics, computer aided geometric design, and image processing.

In this tutorial, we discuss the theoretical background of generalized barycentric coordinates and present hands-on examples, ranging from colour interpolation and improved Phong shading to image warping and mesh deformations.

High-quality weight functions via constrained optimization

Alec Jacobson (ETH Zürich)

Analytically defined coordinates offer fast computation and determinable properties via closed-form expressions, but struggle to obtain all desirable properties simultaneously. We show how to construct smooth, localized, and bounded weight functions via constrained optimization of energies corresponding to polyharmonic functions. With appropriate boundary conditions these functions provide intuitive control for real-time deformation with a variety of handle types. Finally, our recent work shows how more constraints may be added to ensure against spurious local extrema, providing additional control.

From the mimetic finite difference method to the virtual element method

Gianmarco Manzini (CNR Pavia)

A numerical method is called mimetic when it mimics or imitates some properties of the continuum vector calculus, such as, for example, the Gauss–Green relation between the divergence and the gradient operators. Two new families of mimetic schemes for elliptic problems, called the node-based and the mixed Mimetic Finite Difference (MFD) methods, have been recently developed in a joint collaboration between the Los Alamos National Lab and IMATI-CNR in Pavia (Italy). These numerical formulations make it possible to perform calculations on structured and unstructured meshes of quite general shaped polygons (in 2D) and polyhedra (in 3D). A different reformulation of the family of node-based schemes as Virtual Element Methods (VEM) allows us to reconsider such mimetic schemes as finite elements. In this talk we present the mimetic formulations, some theoretical results proving the convergence of the numerical approximation, and its performance when applied to a set of academic problems.

Hodge-optimized triangulations

Pooran Memari (Télécom ParisTech)

In this talk, we will introduce Hodge-optimized triangulations, a family of well-shaped primal-dual pairs of complexes designed for fast and accurate computations in computer graphics. These triangulations are obtained via a variational optimization procedure based on a family of functionals on pairs of complexes that we derive from bounds on the errors induced by diagonal Hodge stars, commonly used in discrete computations. The minimizers of these functionals are shown to be generalizations of Centroidal Voronoi Tesselations and Optimal Delaunay Triangulations, and to provide increased accuracy and flexibility for a variety of computational purposes.

Efficient numerical integration in polygonal finite element methods

Seyed Mousavi (University of Texas, Austin)

In the standard finite element method, shape functions are polynomials and the elements in two dimensions are usually triangular or quadrilateral. As a result, Gaussian quadrature rules are accurate and efficient to compute the mass and stiffness matrices. However, in the polygonal finite element method, numerical integration of rational polynomials on irregular polygons is required. A straightforward solution is to decompose the polygons into triangles and use available rules on the triangle. This technique requires two levels of mapping—the integration points are mapped from a triangle to the parent polygonal element, and then to the physical element. Moreover, Gaussian quadratures are not designed to integrate rational functions. We show that one can use the moment fitting equations and apply the node elimination algorithm to construct efficient quadratures for the integration of polygonal shape functions and their derivatives. In this method, first a quadrature is constructed by solving the moment equations for an appropriate set of basis functions and a prescribed weight function. Then, the quadrature is optimized by eliminating one of the nodes and resolving the moment equations. Numerical experiments including the patch test will be presented to affirm the accuracy and efficiency of the weighted quadratures.

Stable topology optimization: a barycentric FEM approach

Glaucio Paulino (University of Illinois, Urbana-Champaign)

A prevalent problem in the field of topology optimization has been instabilities such as checkerboarding. The barycentric finite element approach allows to circumvent this problem by avoiding node connection between elements and emphasizing edge connections. In this talk, we will demonstrate that this approach leads naturally to stable solutions. Moreover, we recently noticed that the barycentric FEM provides stable solutions for mixed variational problems, which in turn, should lead to high quality topology optimization solutions. These features will be explored in the presentation.

Barycentric subdivision meshes in computational solid mechanics

Julian J. Rimoli (Georgia Institute of Technology, Atlanta)

We will discuss the implementation of barycentric-subdivision and barycentric-dual algorithms for simplicial meshes as well as their application to selected problems in computational solid mechanics. Specifically, we will discuss their use on polyhedral mesh generation for direct numerical simulations of polycrystals as well as their use on mesh generation for analysis of crack propagation problems. The first application, based on the Relaxed Dual Complex (RDC) method, combines a topological step, which defines an initial unrelaxed polycrystal geometry as the barycentric dual of an input triangulation of the solid, and a second relaxation step, in which the grain boundaries are relaxed by means of a gradient flow driven by grain boundary energy. The RDC method applies to arbitrary solids defined by means of a triangulation and, in this manner, it couples seamlessly to standard solid modeling engines. The second application focuses on a new type of mesh based on the barycentric subdivision of k-means meshes, termed conjugate-directions mesh. This new kind of mesh significantly reduces, for meshes of practical size, the undesired mesh-induced anisotropy and mesh-induced toughness observed when dealing with discrete cohesive models of fracture.

Blending isogeometric analysis and local maximum-entropy approximants

Adrian Rosolen (Massachusetts Institute of Technology)

We present a method to blend local maximum entropy (LME) meshfree approximants and isogeometric analysis. The blending is based on the imposition of the reproducibility conditions with a maximum entropy optimization program. The resulting schemes exploit the best characteristics and overcome the main drawbacks of each of these approximants. Indeed, it preserves the high fidelity in the representation of the boundary of the domain (exact CAD geometry) of isogeometric analysis, and it easily handles the volume discretization and unstructured grids with possibly local refinement, while maintaining the smoothness and non-negativity of the basis functions. We explain the methodology with B-Splines for the sake of simplicity, but the strategy is the same with other non-negative technologies such as NURBS or subdivision surface. The performance of the method is illustrated with different examples.

Barycentric finite element methods

N. Sukumar (University of California, Davis)

In this tutorial, I will present the development of C^0 conforming finite element methods on arbitrary planar polygons, which was first initiated by Wachspress in 1975. Linearly precise barycentric coordinates (basis functions) on planar polygons are used to construct the discrete finite element space. In the spirit of three-node triangle and four-node quadrilateral elements, Wachspress basis functions are first constructed on a regular polygon and through an isoparametric map, the basis functions are defined on any convex polygon. Wachspress, mean value coordinates (MVC) and maximum-entropy coordinates (MEC) are admissible for planar convex polygons; for meshes with nonconvex polygons, MVC and MEC are conforming.

I will present the variational/weak formulation of Galerkin methods to solve second-order (Laplace, elasticity) as well as fourth-order (biharmonic) partial differential equations. Numerical issues such as imposing essential boundary conditions and numerical integration of the weak form integrals will be discussed. Applications to the patch test and to mesh-independent simulations of elastic crack growth on polygonal and quadtree meshes will be emphasized.

Geometry of self-supporting surfaces

Etienne Vouga (Columbia University, New York)

Unreinforced masonry structures are one of the most ancient and elegant techniques for building curved shapes. Because of the very geometric nature of their failure, analyzing and modeling such structures is more a geometry processing problem than one of classical continuum mechanics. We present an iterative nonlinear optimization algorithm based on thrust network analysis to efficiently approximate freeform shapes by self-supporting ones. The rich geometry of thrust networks leads us to close connections between diverse topics in discrete differential geometry, such as a finite-element discretization of the Airy stress potential, perfect graph Laplacians, and computing admissible loads via curvatures of polyhedral surfaces. This geometric viewpoint allows us, in particular, to remesh self-supporting shapes by self-supporting quad meshes with planar faces.

Generalized barycentrics

Eugene Wachspress (Hightstown, New Jersey)

A fresh perspective on the early development of shape functions for elements with straight and curved sides in two and three dimensions will be presented. The role of algebraic geometry and algebraic surfaces of higher order will be reviewed. More recent innovations that simplify the construction will be included with applications.

Rational Basis Functions: Advanced Topics

Eugene Wachspress (Hightstown, New Jersey)

Recent work on concave elements, higher order sides, extension to four and higher dimensional spaces, and “reverse” isoparametrics will be presented in an integrated concept for the first time.

Barycentric coordinates as a fundamental tool for spatial shape deformation

Ofir Weber (New York University)

A space deformation is a mapping from a source region to a target region within Euclidean space, which best satisfies some user-specified constraints. Barycentric coordinates are widely used to perform spatial shape deformation in 2D and 3D. The conventional cage-based approach allows the user to prescribe the target position at cage vertices. The computation of the coordinate functions is done in a preprocessing step while at run time only linear combinations of the precomputed coordinates with the target vertex positions as coefficients are taken. This results in a highly efficient (and parallel) deformation method. Mappings produced this way naturally inherit the properties of the coordinate basis functions, hence achieve some of the deformation desired properties. For example, using harmonic barycentric coordinates lead to C^∞ mappings by construction.

Working with such a reduced subspace compared to the full subspace of all possible deformations, is a big advantage. However, in many scenarios, the subspace of meaningful deformations is even smaller than the one spanned by the barycentric coordinates of choice. In this talk, we will show how to further reduce this subspace in order to achieve deformations with different desirable properties that satisfy a variety of user constraints.

Focusing on harmonic barycentric coordinates, we will first show how to obtain a closed-form expression for holomorphic complex barycentric coordinates based on the classical Cauchy integral formula. We apply the Boundary Element Method to compute another set of coordinates called the Hilbert coordinates which are finally being used to construct smooth locally injective conformal mappings that satisfy user-prescribed angular boundary constraints.

Next, we construct closed-form expressions for harmonic basis functions in 3D along with their first and second derivatives and demonstrate how to use them to achieve detail-preserving shape deformation. Finally, we will show how to analyze a given deformation of one shape in order to synthesize a semantically similar deformation of a completely different shape.

A geometric structure-preserving discretization scheme for incompressible linearized elasticity

Arash Yavari (Georgia Institute of Technology, Atlanta)

In this talk we present a geometric discretization scheme for incompressible linearized elasticity. We use ideas from discrete exterior calculus to write an action for a discretized elastic body modeled by a simplicial complex. After characterizing the configuration manifold of volume-preserving discrete deformations, we use Hamilton's principle on this configuration manifold. The discrete Euler–Lagrange equations are obtained without using any Lagrange multipliers. By construction, this numerical scheme is free from any volume locking. We explicitly derive the governing equations for the two-dimensional case and demonstrate the efficiency and robustness of this geometric scheme using some numerical examples.