Stable Topology Optimization: A Barycentric FEM Approach

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Acknowledgments:

NSF – National Science Foundation

SOM – Skydmore, Owings and Merrill

Pop Quiz: Is the sidewalk real or simulated ?

Modeling the Copacabana Sidewalk Pavement

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Tecgraf, Computer Science Department, PUC-Rio
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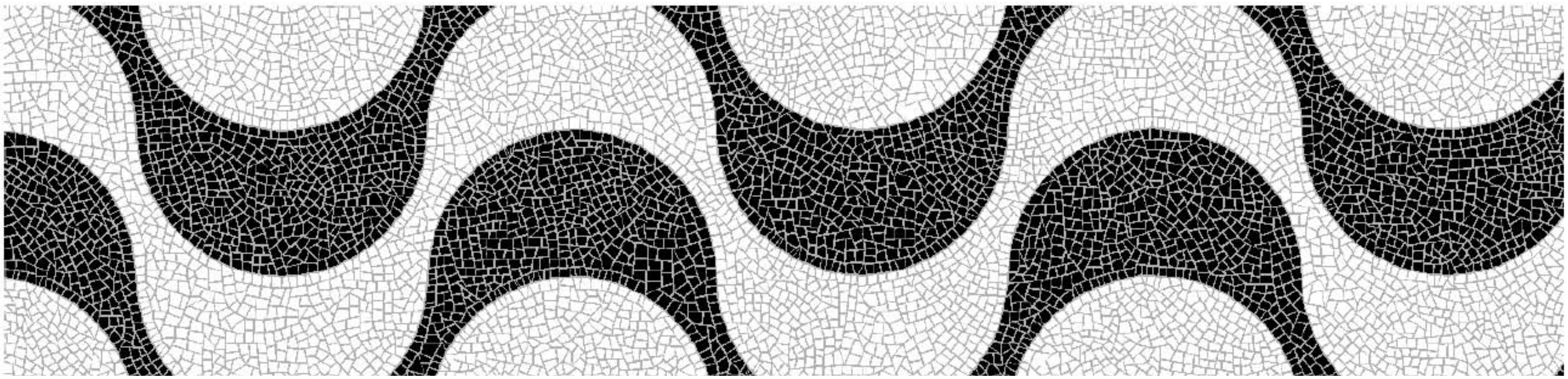
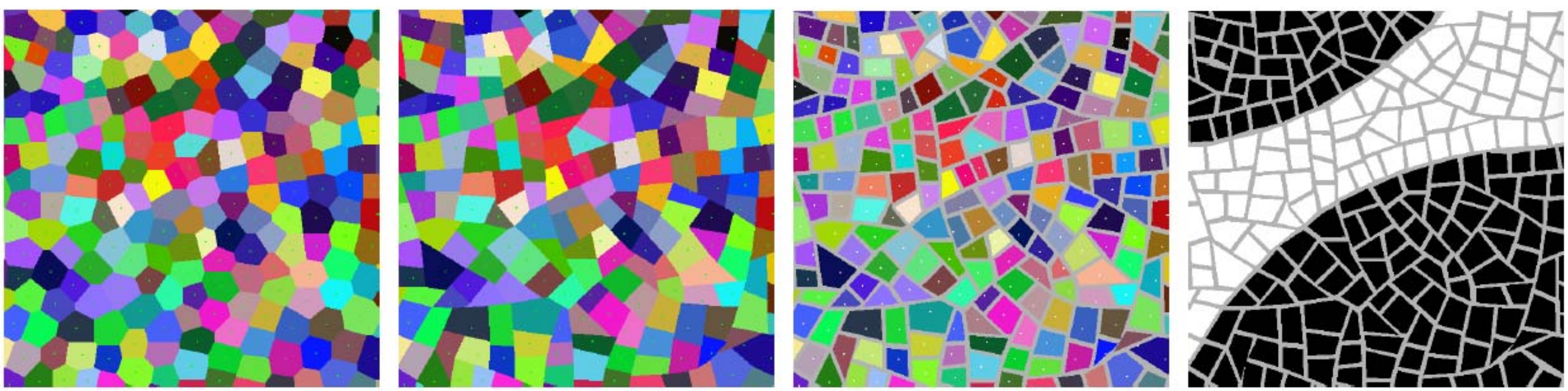




(a) Polygons from Voronoi (b) Wide angle elimination (c) Imposed separation (d) Final black and white stones

Professor Paulino's Research Group

Cam Talischi

Lauren Stromberg

Arun Gain

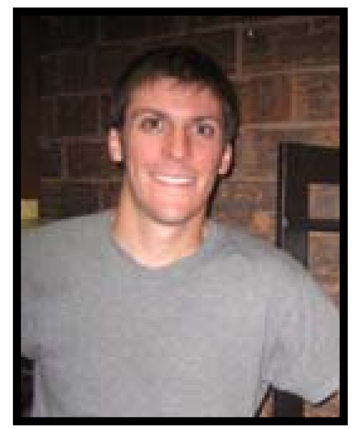
Tomas Zegard

Sofie Leon

Daniel Spring

Junho Chun

Evgueni Fillipov



Will Colletti

B

2 Posters

Engineering a new architecture through barycentric element based topology optimization

Lauren L. Beghini[1], Alessandro Beghini[2], William F. Baker[2], Neil Katz[2], Glaucio H. Paulino[1]
[1]Department of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, U.S.A.
[2]Skidmore, Owings & Merrill, LLP, Chicago, IL, U.S.A.

Structural Topology Optimization employing the Allen-Cahn Evolution Equation on Unstructured Polygonal Meshes

Arun L. Gain, Glaucio H. Paulino

Department of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, U.S.A.

Optimization of shape and topology

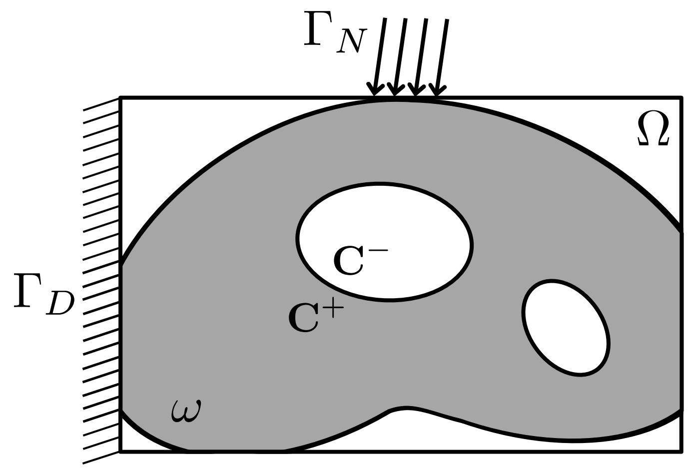
- The goal is to find the most efficient shape of a physical system

The response is captured by the solution to a boundary value problem that in turn depends on the given shape

$$\nabla \cdot [\mathbf{C}_\omega : \epsilon(\mathbf{u})] = \mathbf{0}$$

$$\mathbf{u}\big|_{\Gamma_D} = \mathbf{0}, \quad [\mathbf{C}_\omega : \epsilon(\mathbf{u})] \cdot \mathbf{n}\big|_{\Gamma_N} = \mathbf{t}$$

$$\mathbf{C}_\omega = \chi_\omega \mathbf{C}^+ + (1 - \chi_\omega)\, \mathbf{C}^-$$

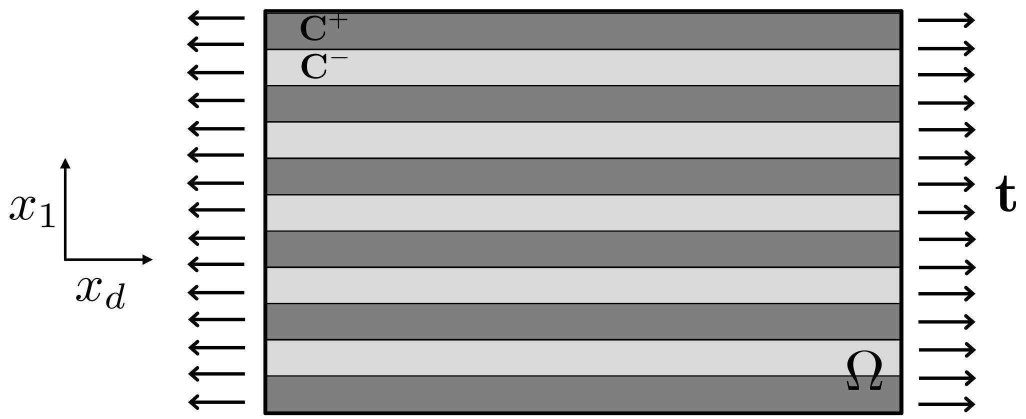


- **The cost function depends on the physical response as well as the geometric features of admissible shapes**

$$J(\omega, \mathbf{u}_\omega) = \int_{\Gamma_N} \mathbf{t} \cdot \mathbf{u}_\omega \mathrm{d}s + \lambda\, |\omega|$$

Existence of solutions

- Shapes with fine features are naturally favored, which leads to nonconvergent minimizing sequences that exhibit rapid oscillations



$$\Gamma_N = \partial\Omega, \quad \Gamma_D = \emptyset, \quad \mathbf{t} = (\mathbf{e}_d \otimes \mathbf{n}) \cdot \mathbf{t}_0$$

- **Existence of solutions can be guaranteed by introducing some suitable form of regularization in the problem**

$$\tilde{J}(\omega) = J(\omega, \mathbf{u}_\omega) + \beta \int_\Omega |\nabla \chi_\omega| \, \mathrm{d}\mathbf{x}$$

Large-scale optimization problem

- Accurate analysis of the response and capturing detailed features required fine spatial discretizations which leads to a large number of design and analysis variables

- **We can exploit the composite nature of the cost function**

$$\tilde{J}(\omega) = J(\omega) + R(\omega)$$

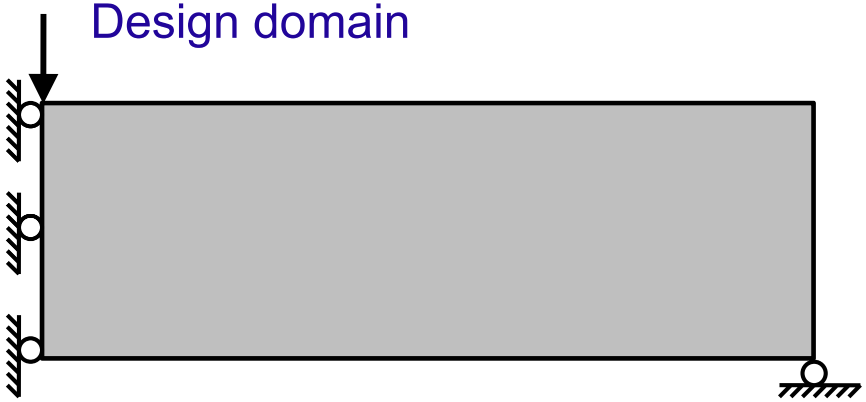
to derive convergent optimization algorithms. Forward-backward splitting, for example, leads to iterations of the form

$$\omega_{n+1} = (I + \tau_n R')^{-1}(I - \tau_n J')\omega_n$$

C. Talisch and G. H. Paulino "An operator splitting algorithm for Tikhonov-regularized topology optimization" *Computer Methods in Applied Mechanics and Engineering*, 2012 (to appear).

Sample result

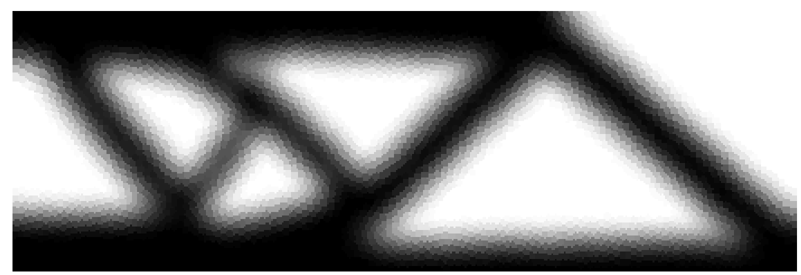
Design domain



β = 0.01

β = 0.05

Comparable “filtering” result



Stable Topology Optimization

Struct Multidisc Optim (2009) 37:569–583
DOI 10.1007/s00158-008-0261-4

RESEARCH PAPER

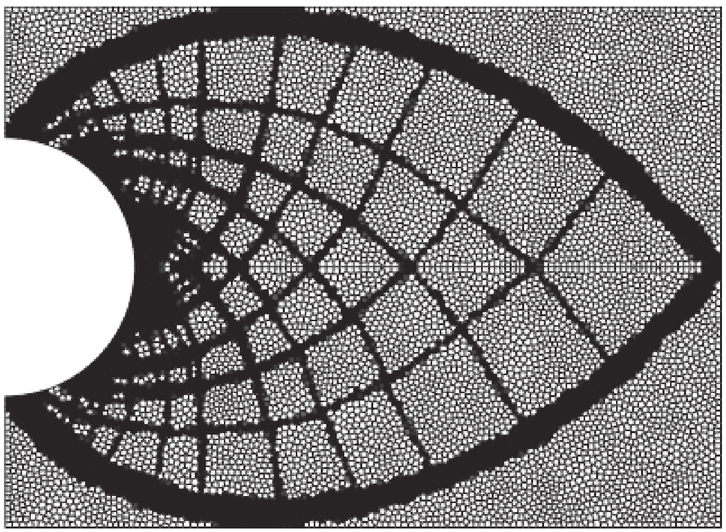
Honeycomb Wachspress finite elements for structural topology optimization

Cameron Talischi · Glaucio H. Paulino · Chau H. Le

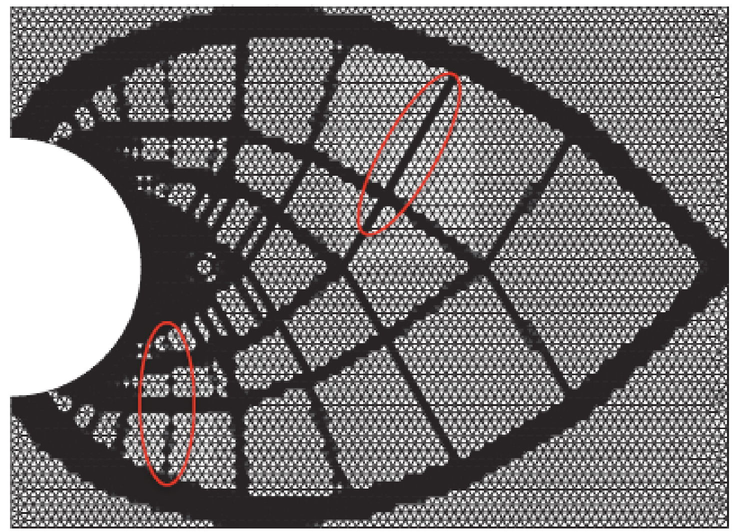
Received: 19 November 2007 / Revised: 7 February 2008 / Accepted: 9 March 2008 / Published online: 22 May 2008

Stable Topology Optimization

Baricentric FEM
Polygonal Elements



T6 Elements



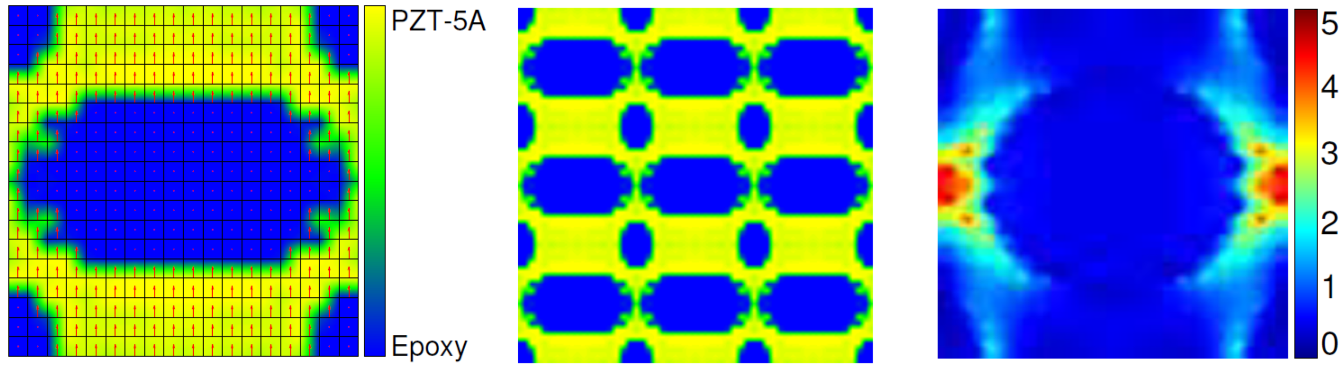
C. Talischi, G. Paulino, A. Pereira and IFM Menezes. Polygonal finite elements for topology optimization: A unifying paradigm. **IJNME**, 82(6):671-698, 2010

Design of Piezocomposite Material Unit Cell

Unit Cell | Periodic | Stress (σ/σ_0)

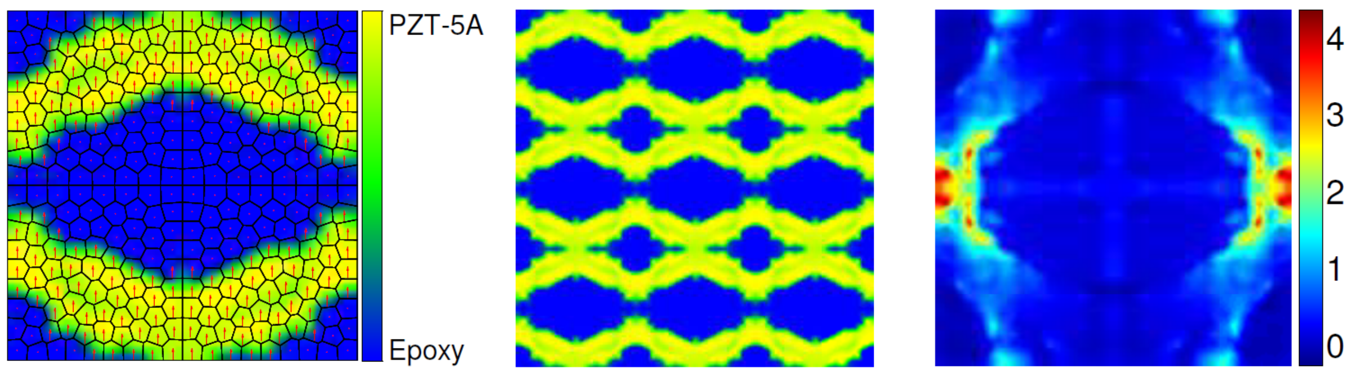
→ Quad Element
→ fixed polarization

k = 0.291
Gain[(1)] = 100.7%
σ/σ_0 = 5.1



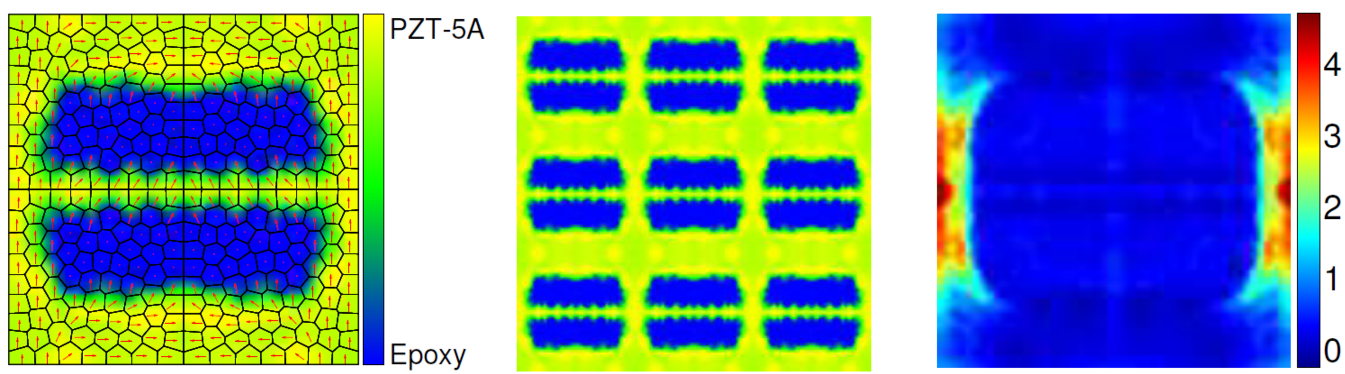
→ Polygonal Element
→ fixed polarization

k = 0.298
Gain[(1)] = 105.5%
σ/σ_0 = 4.6



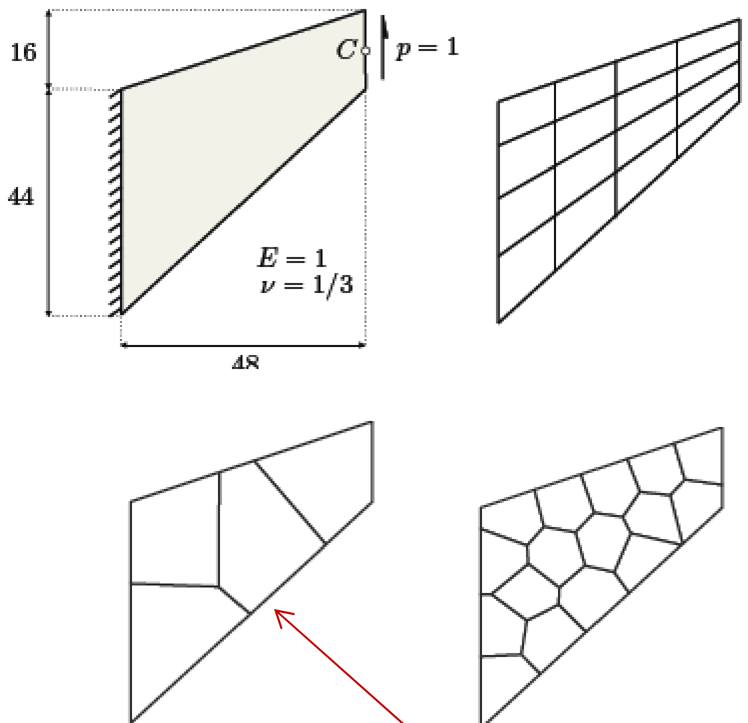
→ Polygonal Element
→ free polarization

k = 0.319 ✓
Gain[(1)] = 120.0%
σ/σ_0 = 4.9

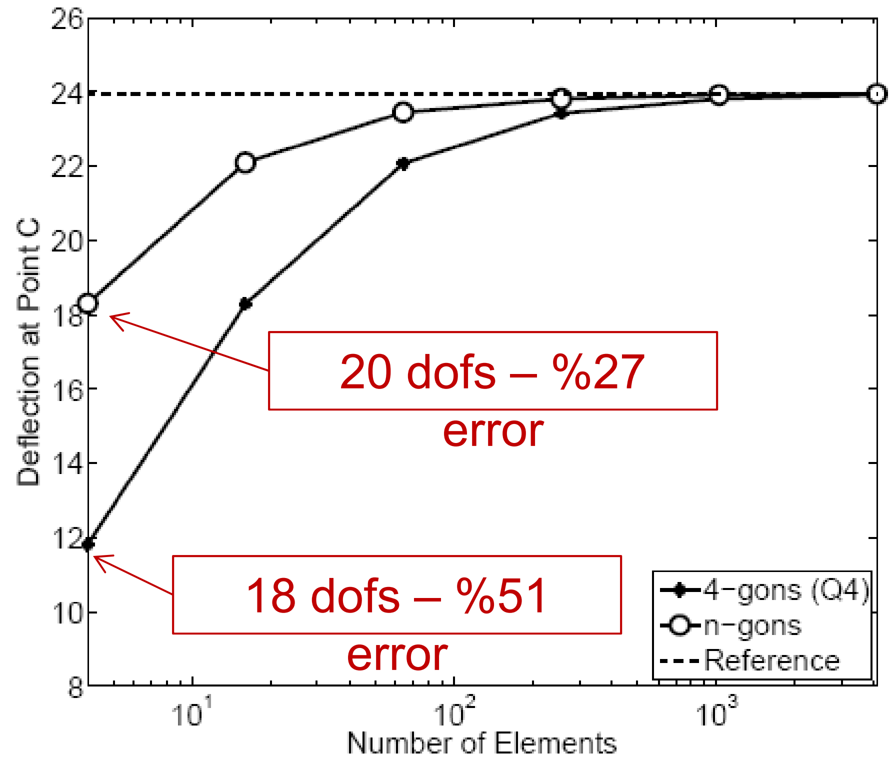


[(1)] w.r.t. PZT-5A (k = 0.145)

Stability



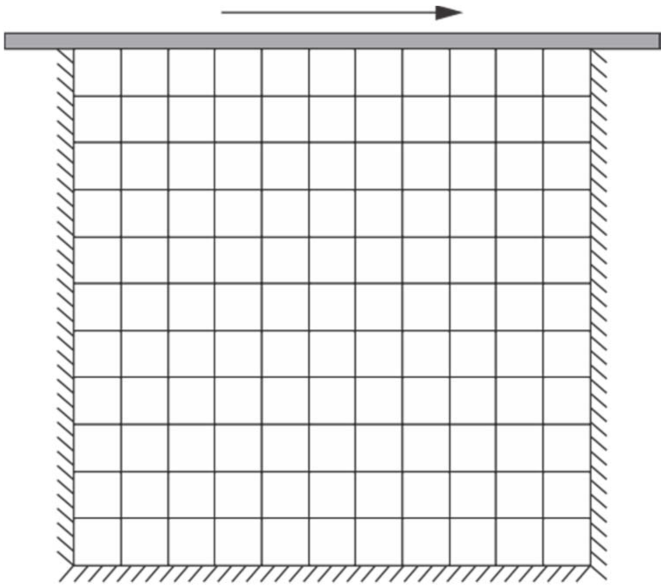
3 quads and 1 pentagon



C. Talischi, G. Paulino, A. Pereira and IFM Menezes. Polygonal finite elements for topology optimization: A unifying paradigm. **IJNME**, 82(6):671-698, 2010

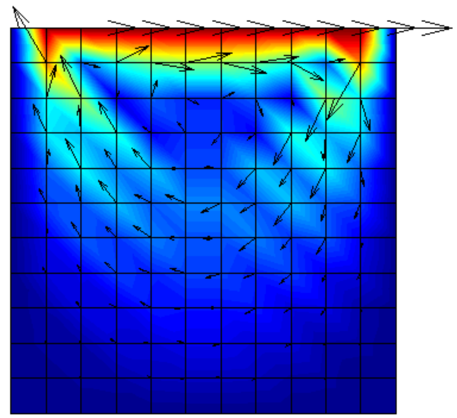
Stability

- Numerical instabilities such the "checkerboard" problem could appear in mixed variational formulation (pressure-velocity) of the Stokes flow problems.

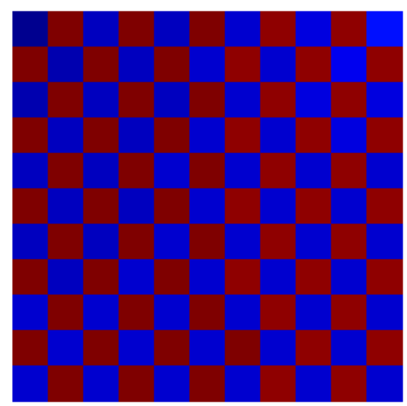


Lid-driven cavity problem

Q4 elements



velocity

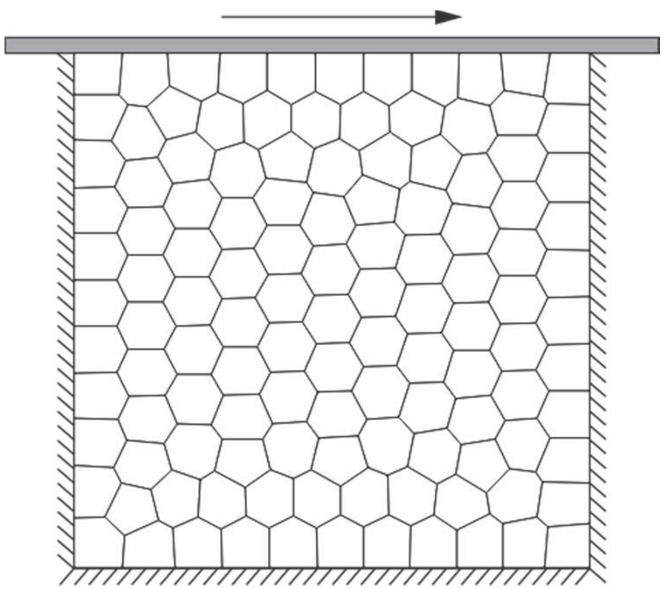


checkerboard on pressure

Paulino et al. "Polygonal finite elements for mixed variational problems" , 2012

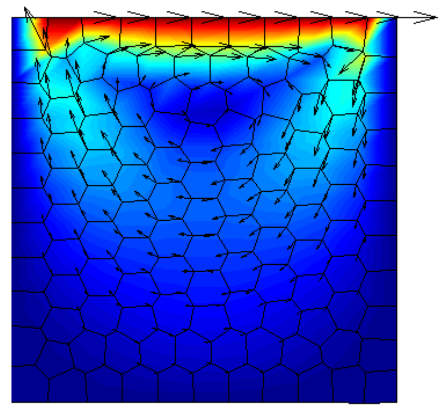
Stability

- Numerical instabilities such the “checkerboard” problem could appear in mixed variational formulation (pressure-velocity) of the Stokes flow problems.

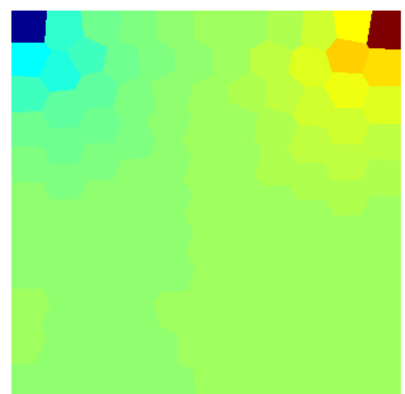


Lid-driven cavity problem

Polygonal elements



velocity

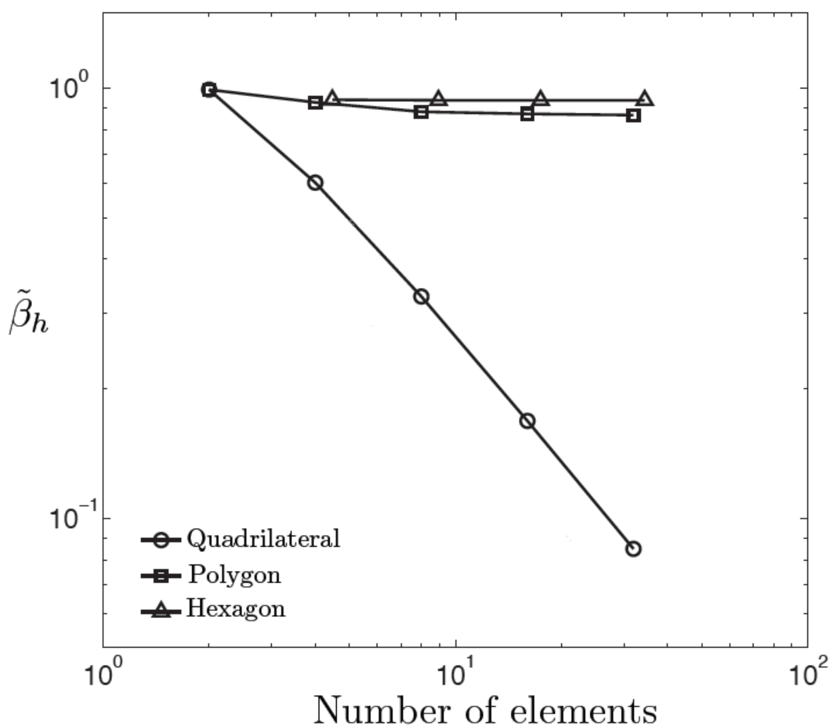


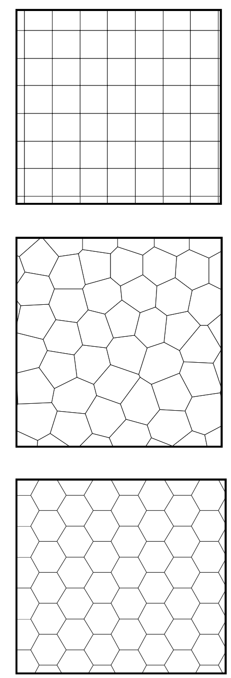
pressure

Paulino et al. “Polygonal finite elements for mixed variational problems” , 2012

Stability

- Babuska-Brezzi condition (or inf-sup)
  - Required for the stability of mixed variational formulation of incompressible elasticity and Stokes flow problems
  - Polygonal discretizations satisfy the well-known Babuska-Brezzi condition





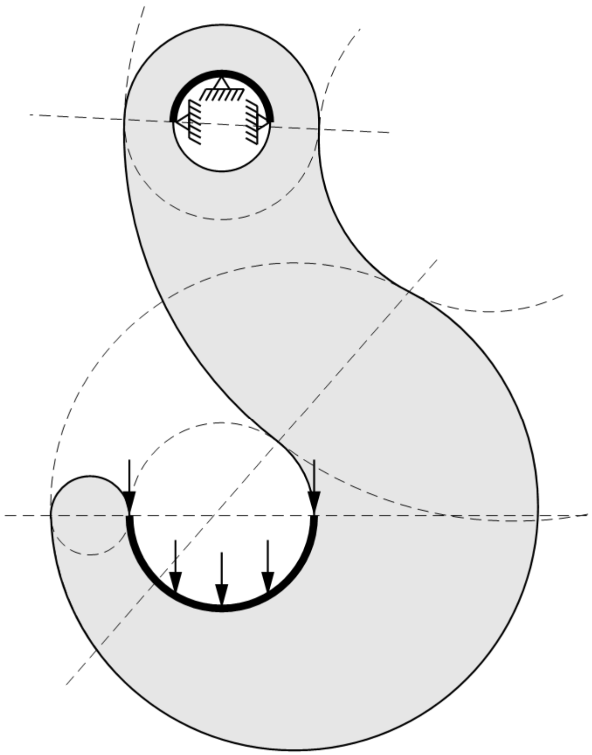
Quadrilateral

Polygon

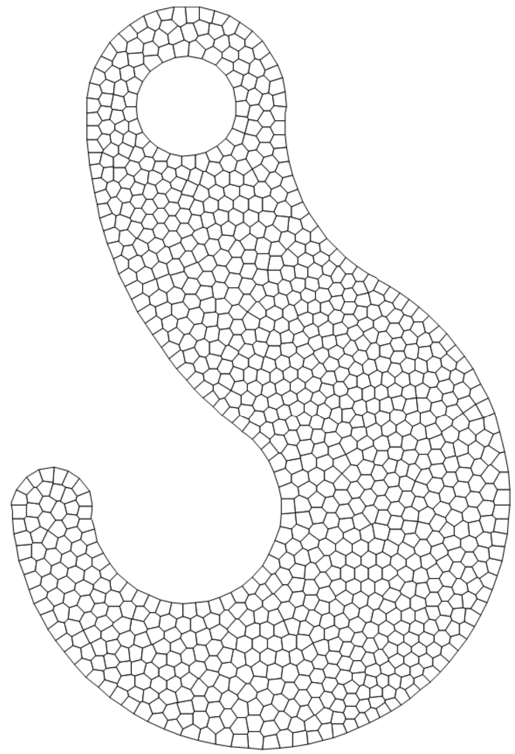
Hexagon

Paulino et al. “Polygonal finite elements for mixed variational problems” , 2012

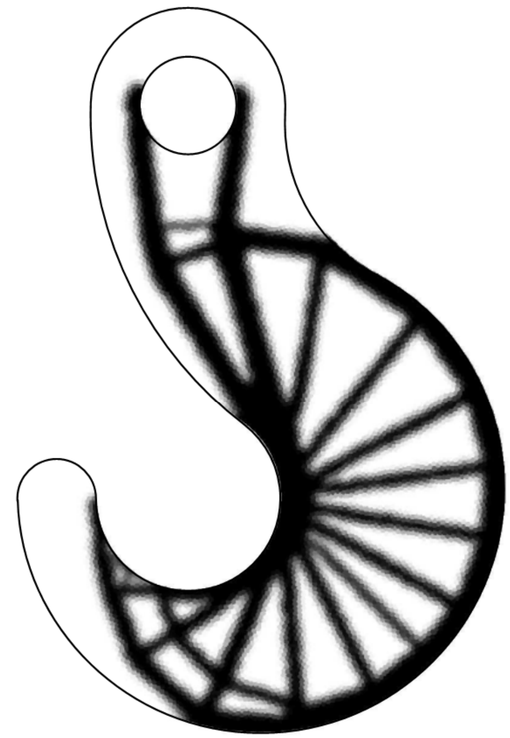
PolyMesher & PolyTop



Design domain



Voronoi mesh



Optimal shape

Talischi C, Paulino GH, Pereira A, Menezes IFM. **PolyMesher**: A general-purpose mesh generator for polygonal elements written in Matlab. Structural and Multidisciplinary Optimization, 45(3):309-328, 2012.

Talischi C, Paulino GH, Pereira A, Menezes IFM. **PolyTop**: A Matlab implementation of a general topology optimization framework using unstructured polygonal finite element meshes. Structural and Multidisciplinary Optimization, 45(3)329-357, 2012.

G.A.M.E.S. Camp

- Makes complex structural engineering concepts accessible to middle and high school students!





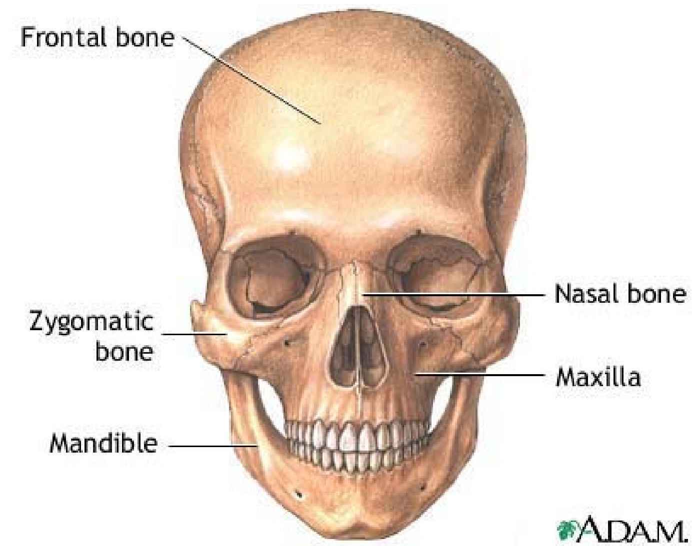



Craniofacial Reconstruction

- Motivation and Background:
  Craniofacial reconstruction
- Load transfer mechanism
- Multi-resolution Topological Optimization
- Results

Craniofacial Reconstruction

- Head or Facial Trauma
- Cancer patients who lost part of the bony structure, soft tissue



✓ Complex and Challenging

- 3D Complex Architecture
- Serves Functional and aesthetic role
  - Facial expression, mastication, speech, and Deglutition (swallowing of food)
  - Facial Appearance

Craniofacial Reconstruction Cancer

Cancer

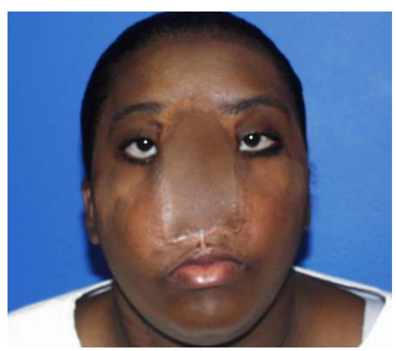
Tissue destruction

Deformity

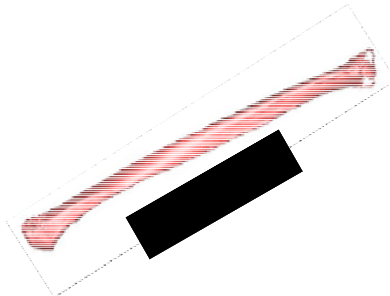
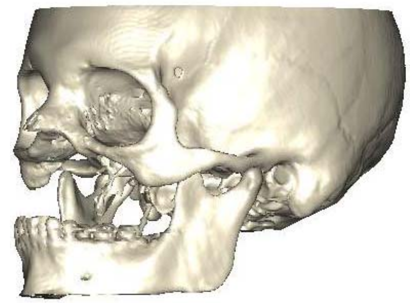
Decreased Quality of Life

Treatment

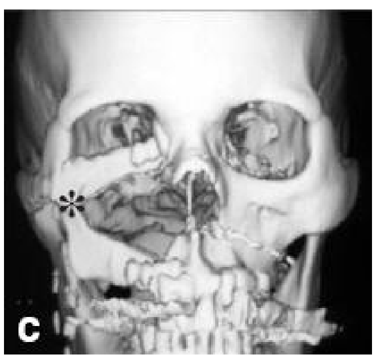
- Surgery
- Radiation
- Chemotherapy

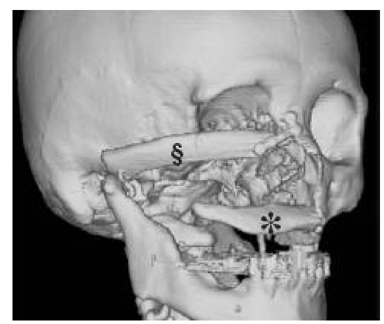


Current Approach



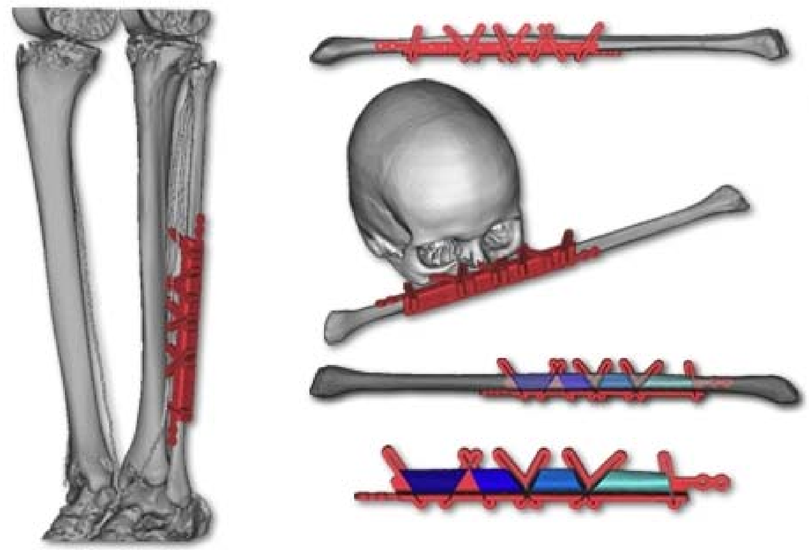





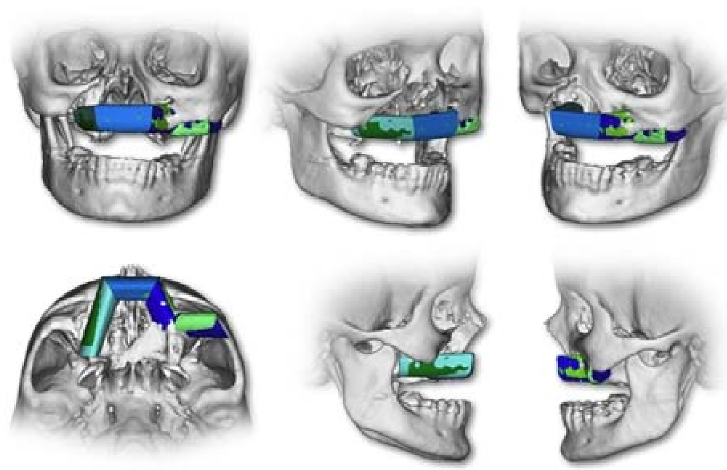
Yamamoto [2005]

- Heuristic Approach
- Ad-hoc method by the surgeon during surgery

Clinical Approach

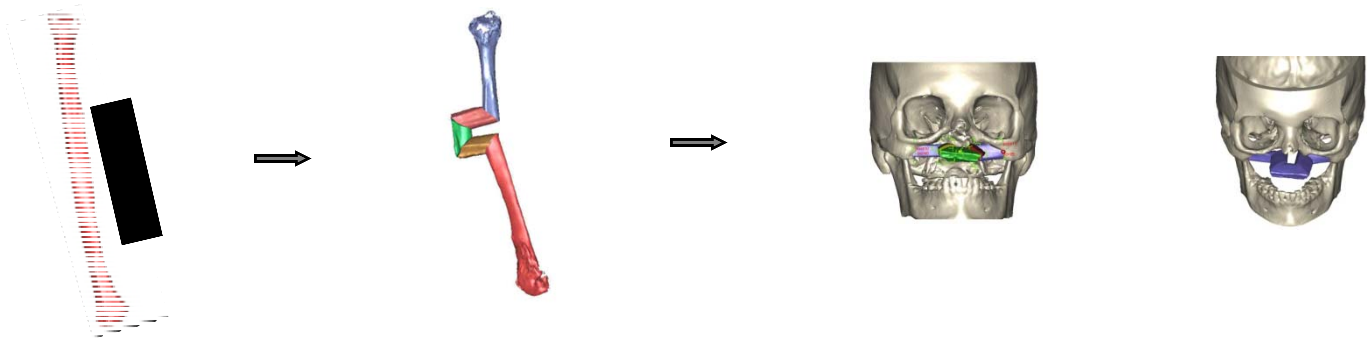


Fibula Osteotomies



Medical Modeling Inc.

Clinical Problem



Fibula Osteotomies Surgery

Topology Optimization
Alternative ...

Goals of the replacement bone

- Give support to orbital content, Avoid Changes in globe position, orbital volume, eyelid functions
- To preserve a platform for mastication, speech and dental rehabilitation
- To recreate an adequate and symmetric facial contour with other side of face

Topology Optimization

Topology Optimization Procedure

- Problem formulation

$$\min_{\rho} \quad C(\rho, \mathbf{u}_d) = \mathbf{f}^{\mathrm{T}}\mathbf{u}_d$$

$$s.t.: \quad \mathbf{K}(\rho)\mathbf{u}_d = \mathbf{f}$$

$$V(\rho) = \int_{\Omega} \rho(\boldsymbol{\psi})dV \le V_s$$

$$0 < \rho_{\min} \le \rho(\boldsymbol{\psi}) \le 1$$

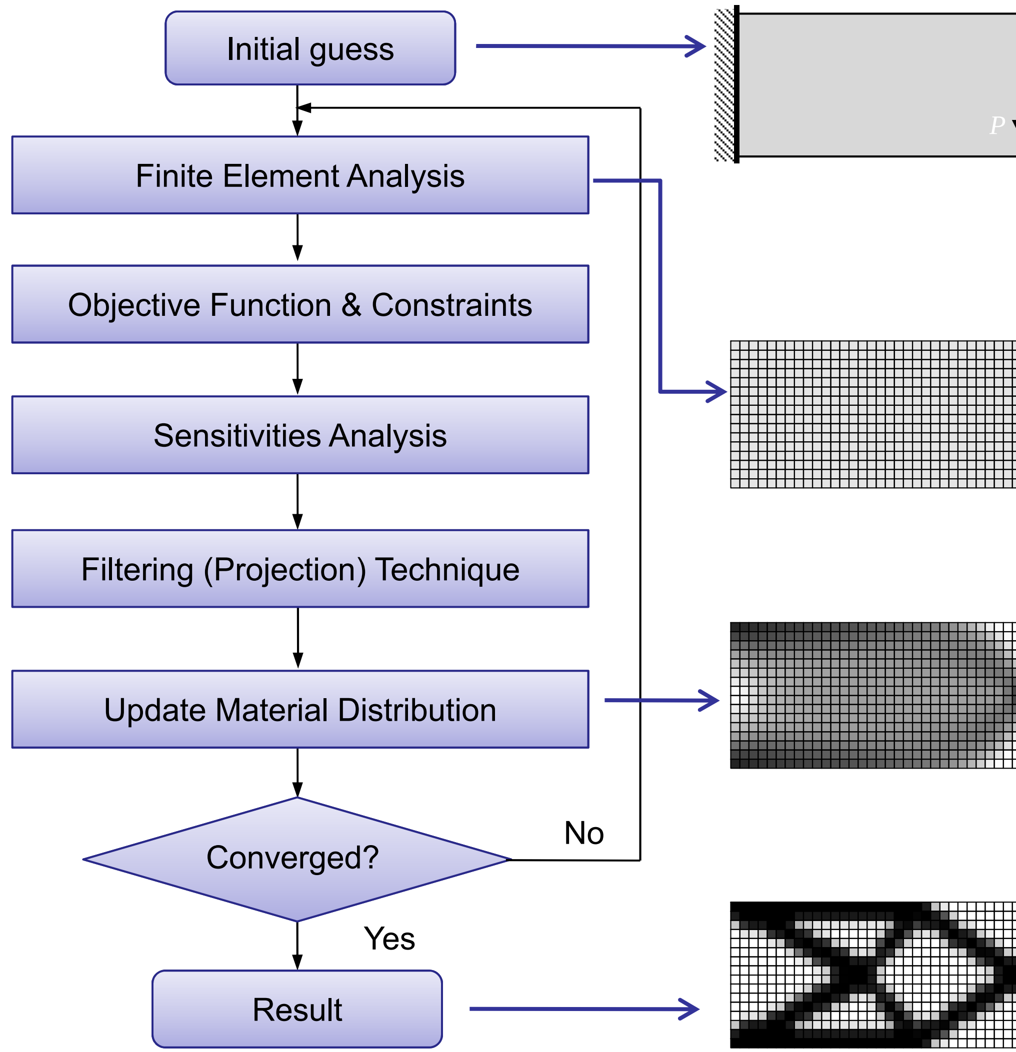
- Solid and Isotropic Material with Penalization (SIMP)

$$E(\boldsymbol{\psi}) = \rho(\boldsymbol{\psi})^p E^0$$

$$\mathbf{K}(\boldsymbol{\rho}) = \sum_{e=1}^{N_{el}} \mathbf{K}_e(\rho_e) = \sum_{e=1}^{N_{el}} \int_{\Omega_e} \mathbf{B}^{\mathrm{T}}\mathbf{D}(\rho_e)\mathbf{B}d\Omega$$

$$\frac{\partial C}{\partial \rho_e} = -\mathbf{u}_e^{\mathrm{T}} \frac{\partial \mathbf{K}_e}{\partial \rho_e} \mathbf{u}_e = -p\rho_e^{p-1}\mathbf{u}_e^{\mathrm{T}}\mathbf{K}_e^0\mathbf{u}_e$$

$$\frac{\partial V}{\partial \rho_e} = \int_{\Omega_e} dV$$



High Resolution Topology Optimization

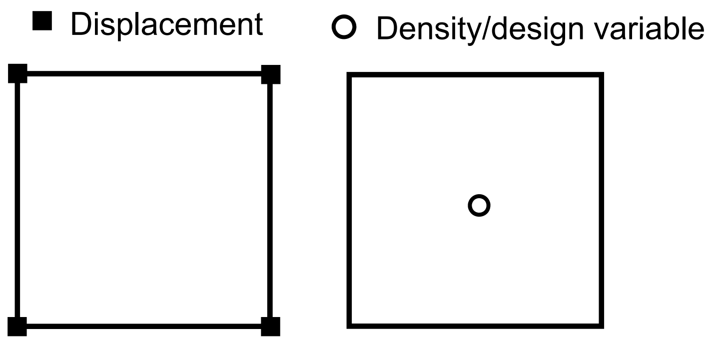
- Large-scale (high resolution) TOP
  - Large number of finite elements
  - Computationally expensive: FEA cost
- Existing high resolution TOP
  - Parallel computing (Borrvall and Petersson, 2000)
  - Fast solvers (Wang et al. 2007)
  - Approximate reanalysis (Amir et al. 2009)
  - Adaptive mesh refinement (de Stuler et al. 2008)

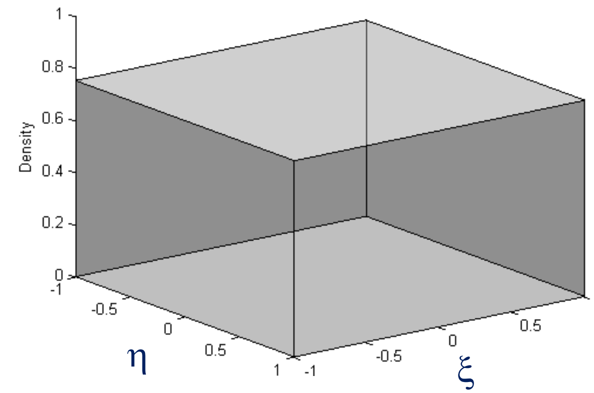
Same discretization for analysis and design optimization

Multiresolution Topology Optimization (**MTOP**)

Conventional element-based approach (Q4/U)

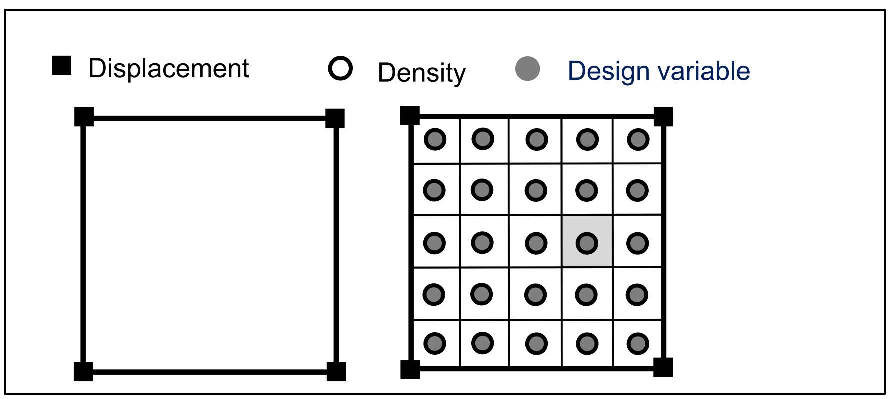
- Same discretization for displacement and density

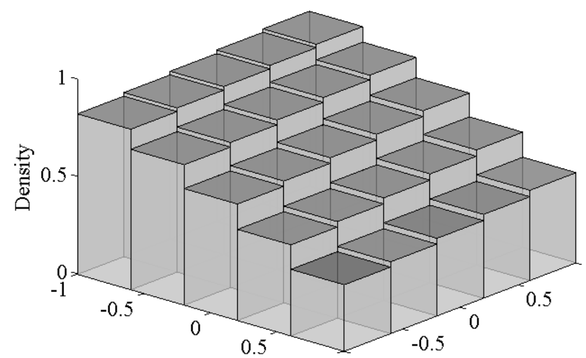




MTOP approach (Q4/n25)

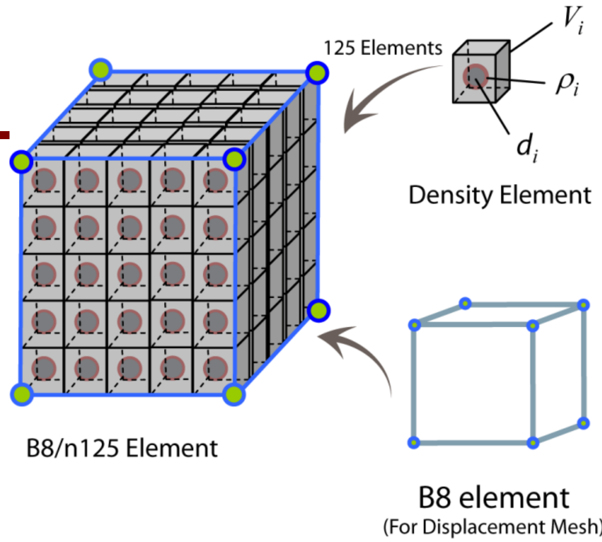
- Different discretizations for displacement and density/design variables





MTOP

- B8/n125 Element



MTOP is being used as the engine for the topology optimization APP recently developed by the Danish group (2012)

N. Aage, M.N. Jørgensen, C.S. Andreasen and O. Sigmund. Interactive topology optimization on hand-held devices, under preparation, 2012


Struct Multidisc Optim (2010) 41:525–539
DOI 10.1007/s00158-009-0443-8

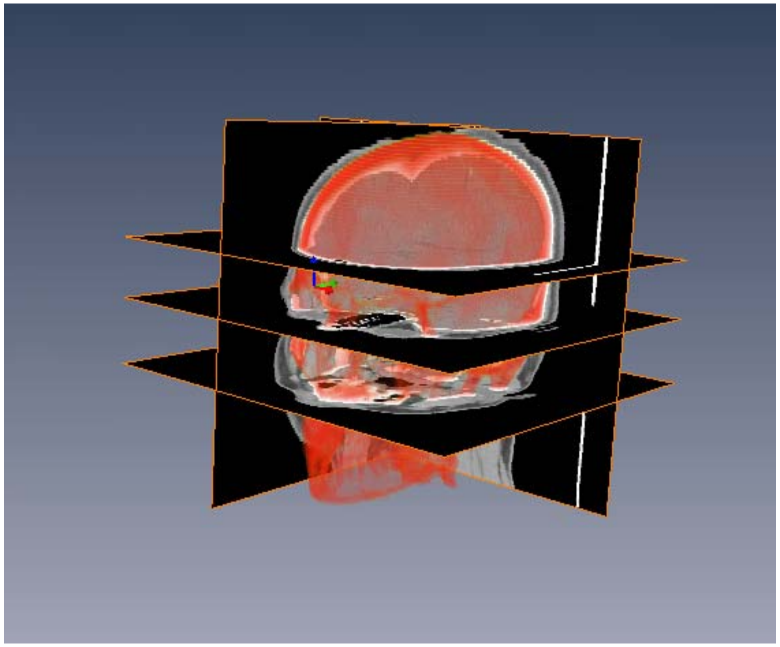
RESEARCH PAPER

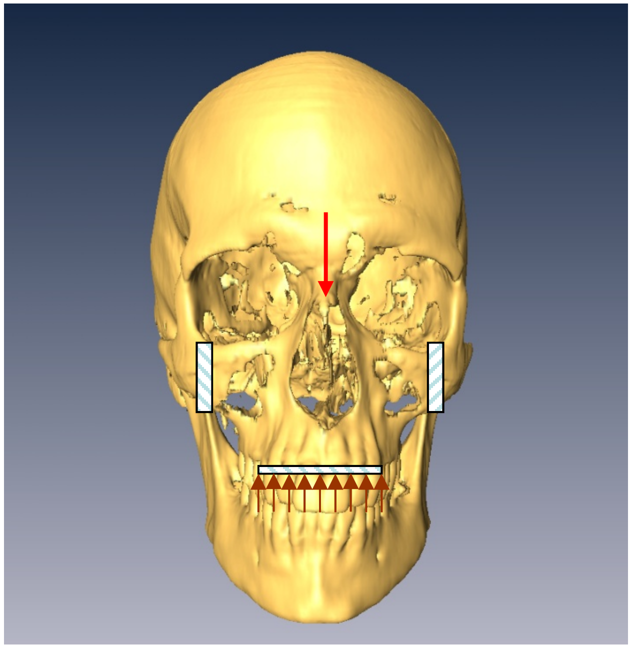
A computational paradigm for multiresolution topology optimization (MTOP)

Tam H. Nguyen · Glaucio H. Paulino ·
Junho Song · Chau H. Le

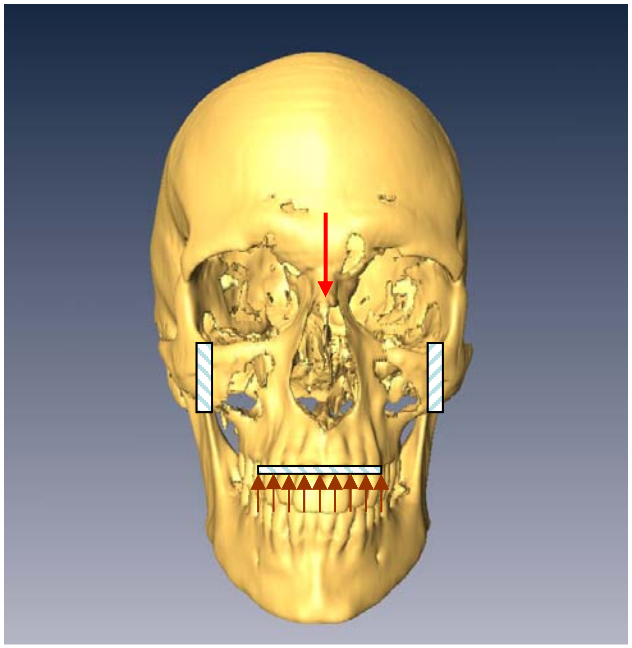
Craniofacial Reconstruction

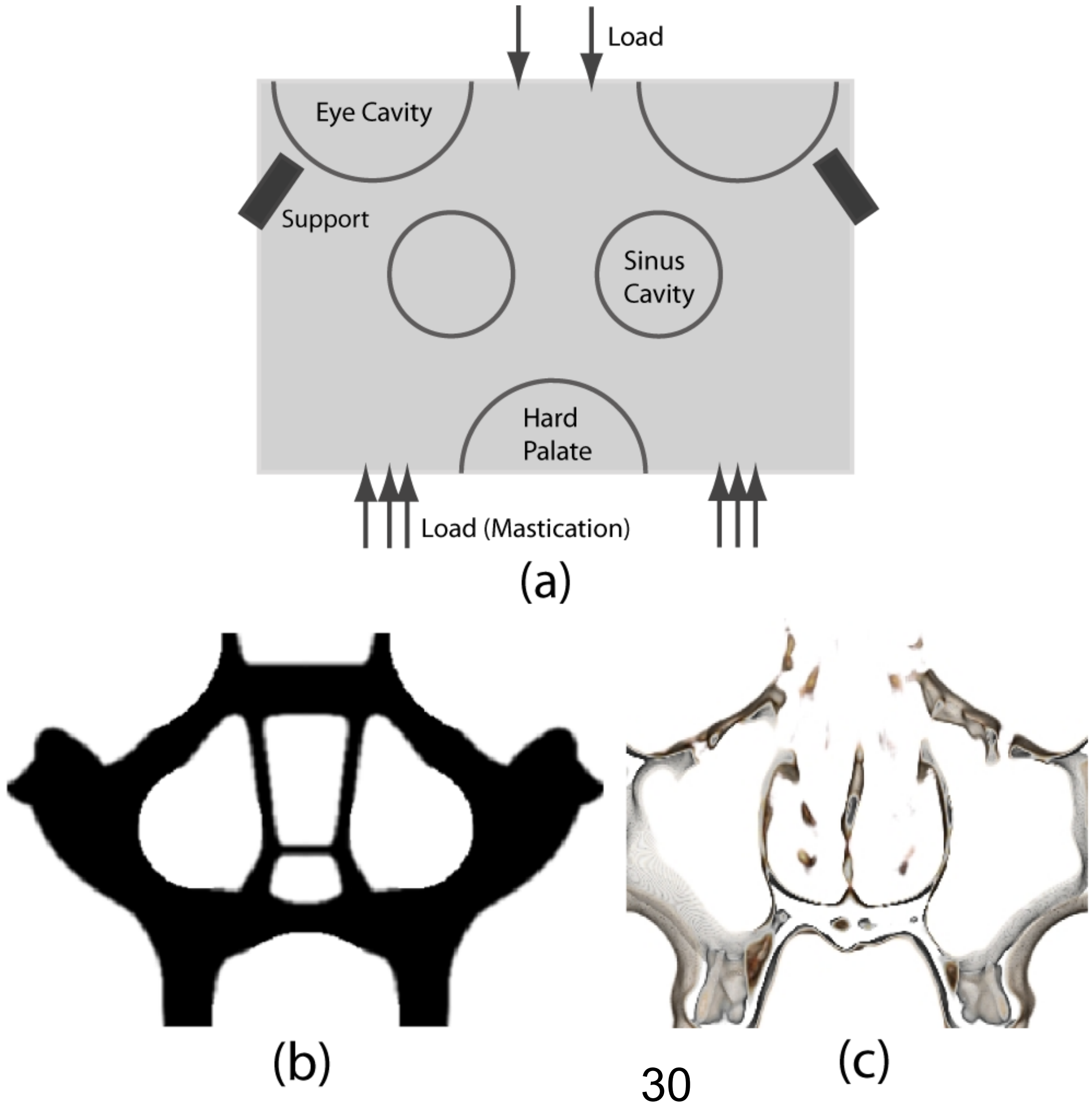
- MRI Data
- Select Boundary Conditions
- Select Load



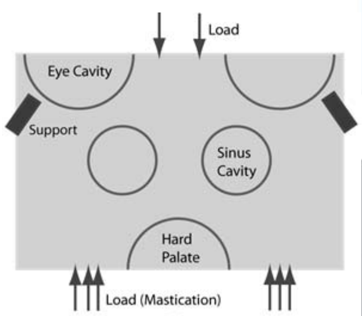


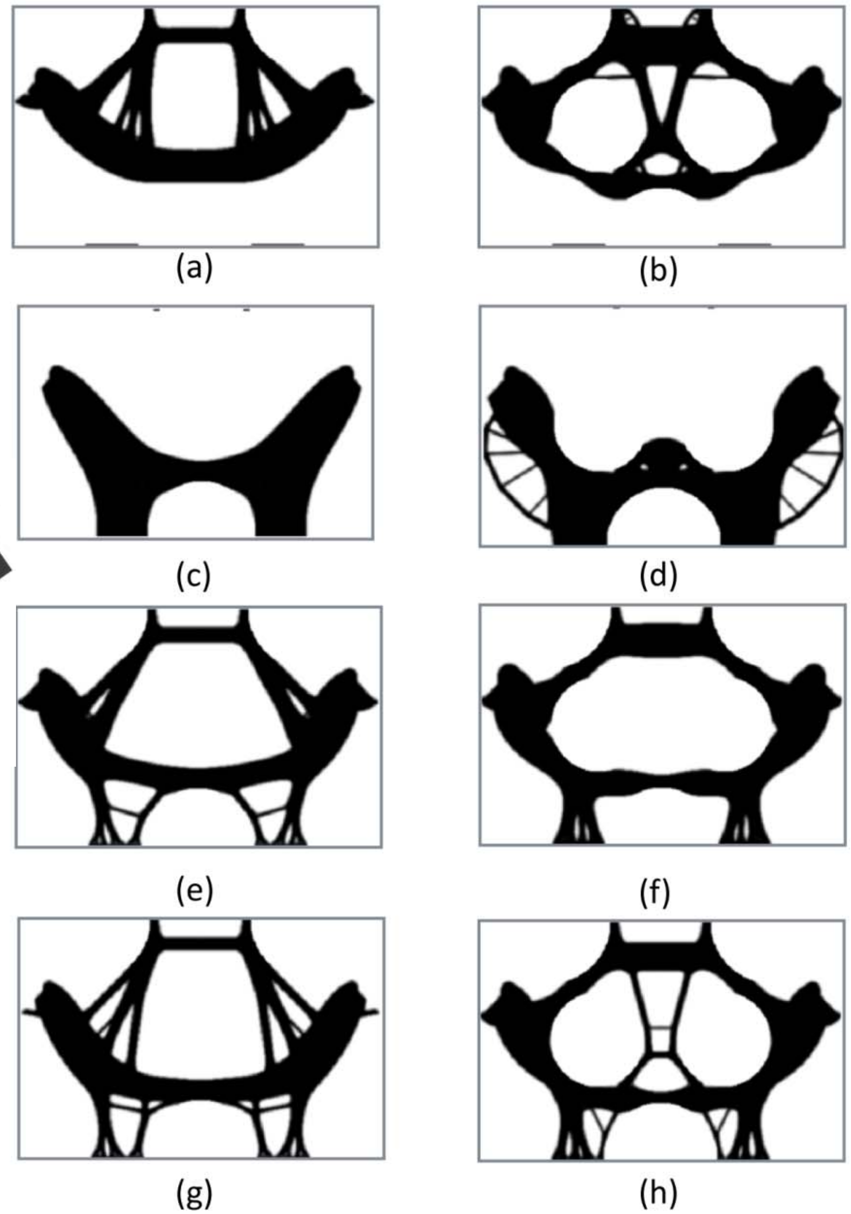
2D Verification of the Concept





Craniofacial Reconstruction





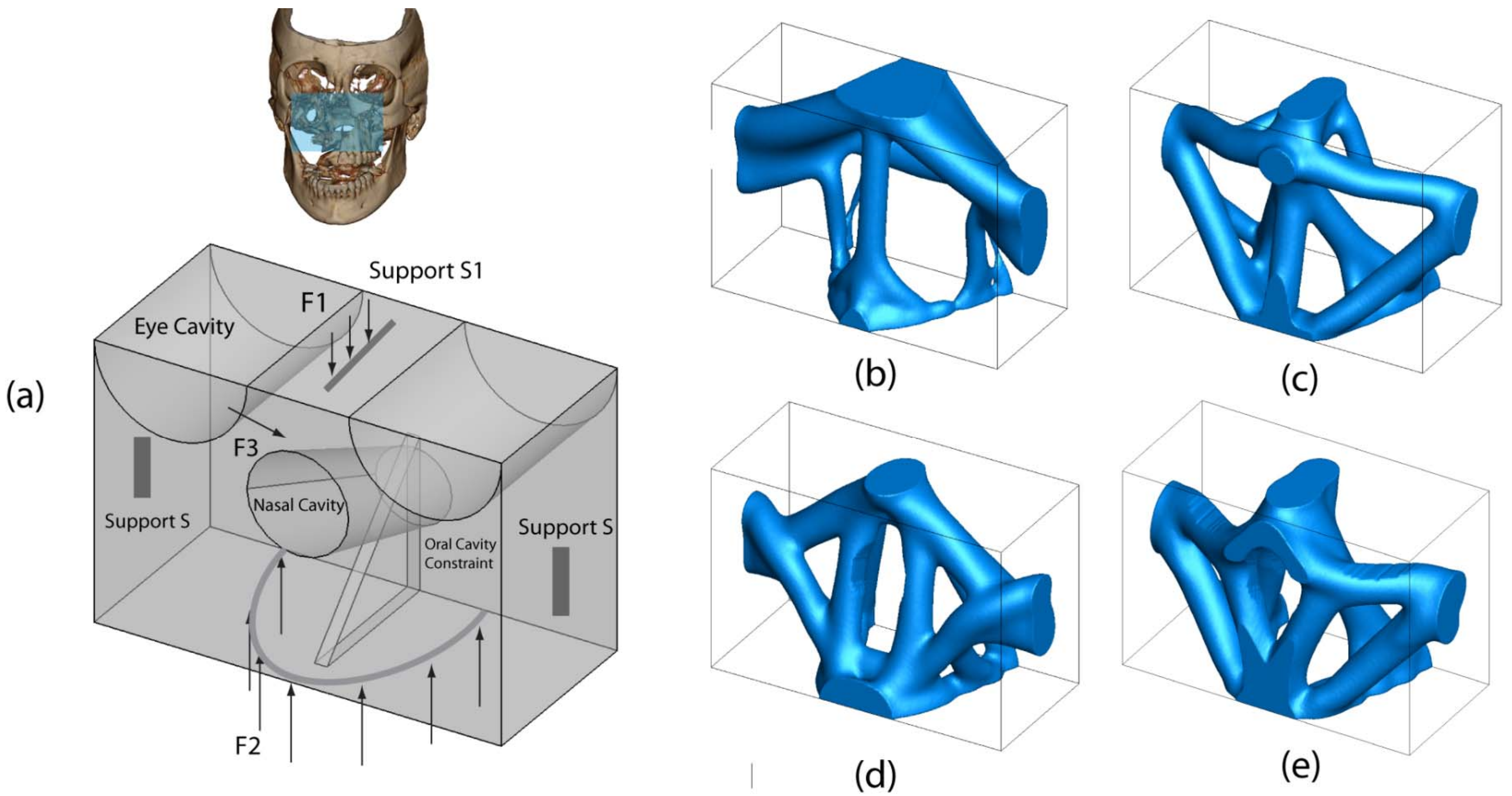
External Load F1 only

Masticatory Forces F2 Only

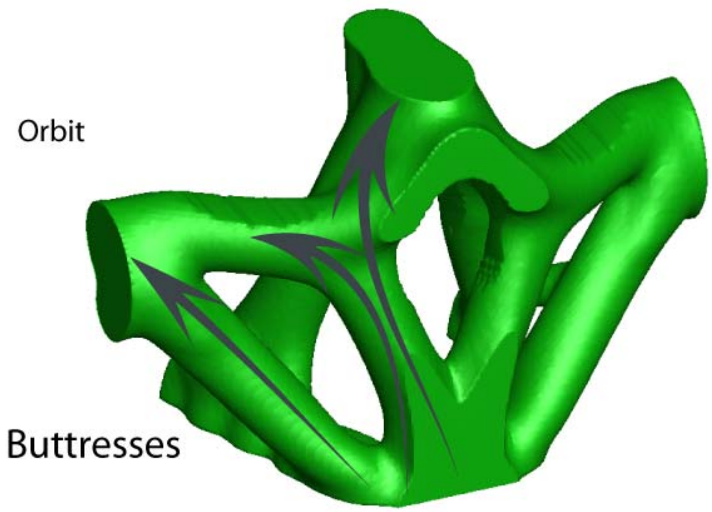
Both F1 and F2

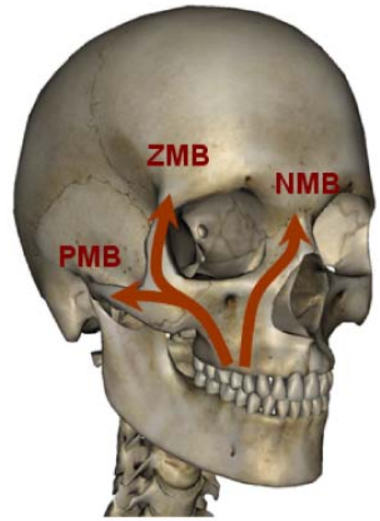
Compliance Combination using weighted method

Craniofacial Reconstruction



Craniofacial Reconstruction





PNAS

Topological optimization for designing patient-specific large craniofacial segmental bone replacements

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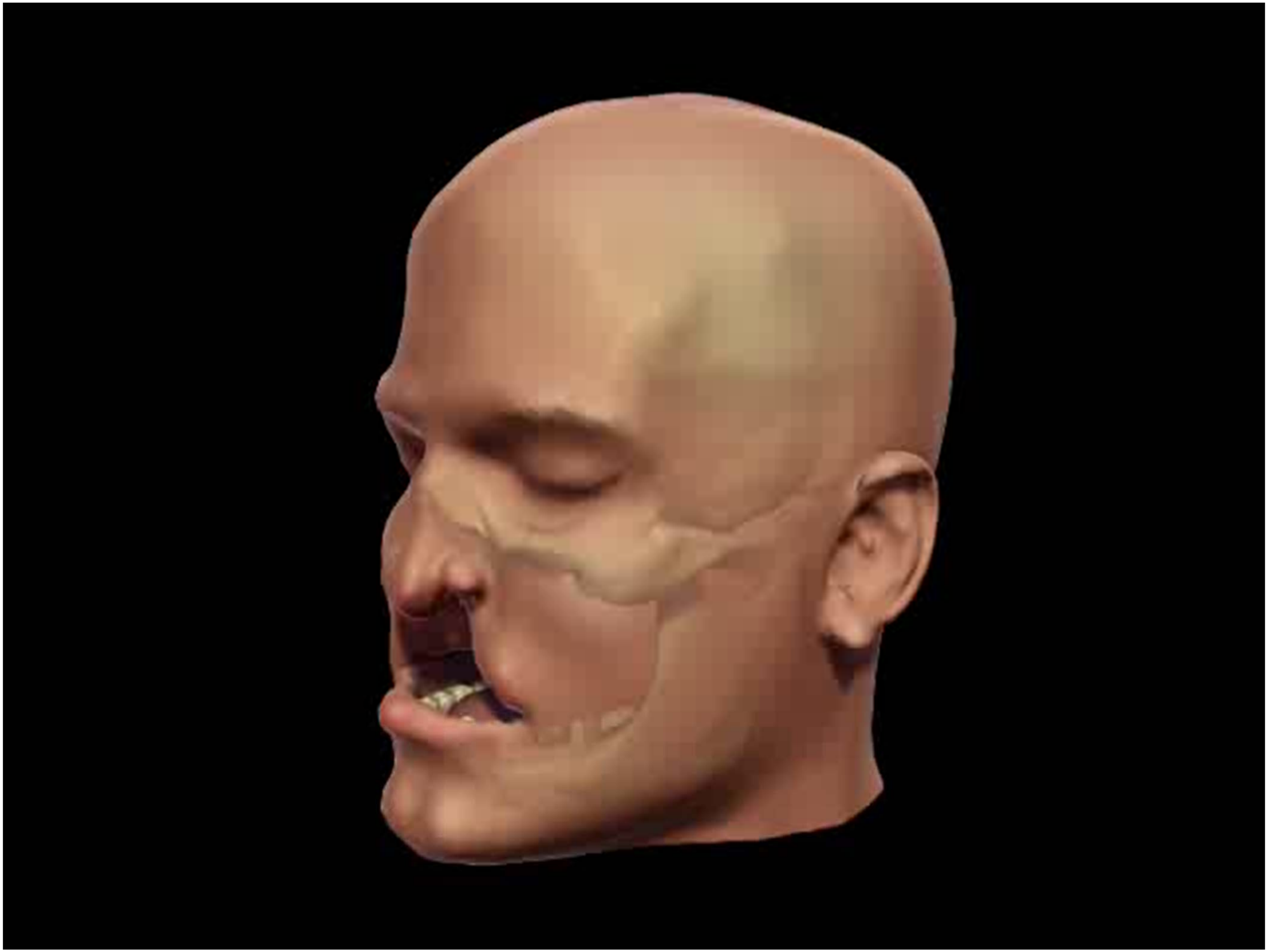
Edited* by Avner Friedman, The Ohio State University, Columbus, OH, and approved June 22, 2010 (received for review February 10, 2010)

Restoring normal function and appearance after massive facial injuries with bone loss is an important unsolved problem in surgery. An important limitation of the current methods is heuristic ad hoc design of bone replacements by the operating surgeon at the time of surgery. This problem might be addressed by incor-

metal plates and screws. For massive defects, the bone must be transferred with a blood supply that is independent from the surrounding damaged tissues. The bone is isolated based on a single artery and vein that are surgically reattached to other uninjured blood vessels in the nearby face or neck. These vessels

The entire process in 40 seconds …



Biological Constraints

Successful Tissue Transfer
Mechanical Variable + Biological Constraint –
Vascular Healing

PNAS

Modeling oxygen transport in surgical tissue transfer

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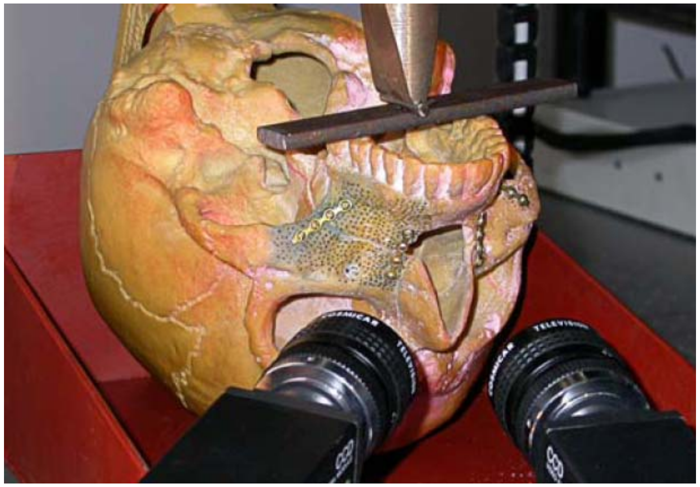
Contributed by Avner Friedman, May 14, 2009 (sent for review November 7, 2008)

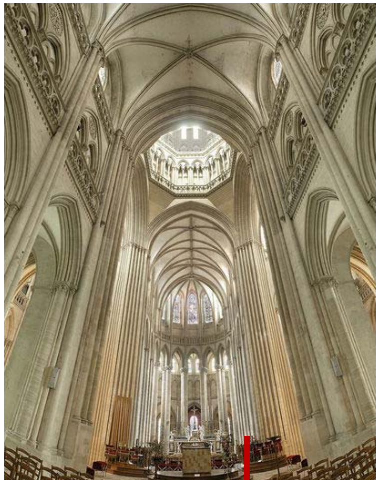
Reconstructive microsurgery is a clinical technique used to transfer large amounts of a patient's tissue from one location used to another in order to restore physical deformities caused by trauma, tumors, or congenital abnormalities. The trend in this field is to transfer tissue using increasingly smaller blood vessels, which decreases problems associated with tissue harvest but increases

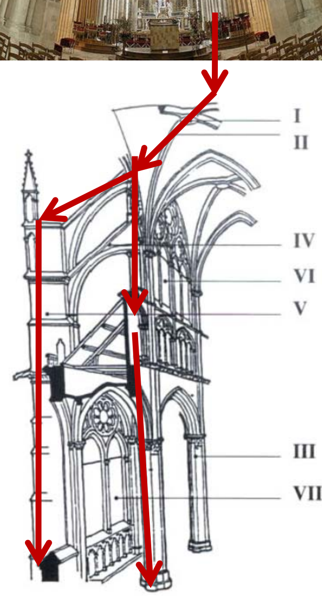
abdominal wall muscles. This difficulty led to introduction of a surgical flap based only on the blood vessels that pass through ~~the muscles calle~~d perforating vessels. Flaps were designed that required the surgeon to select a single perforating vessel and follow it through the muscle without including any muscle in the flap for transfer. The perforating vessels have luminal diameters rang-

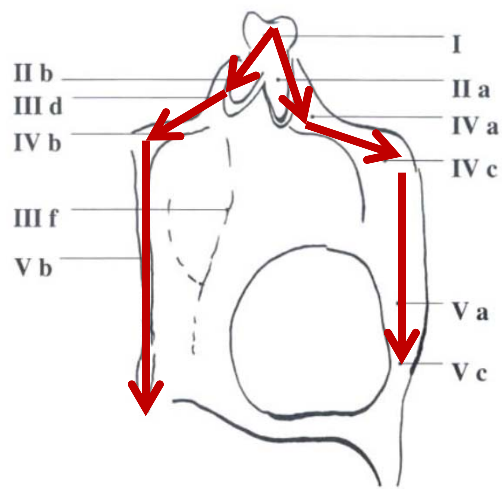
Experimental Validation



Load-transfer mechanism







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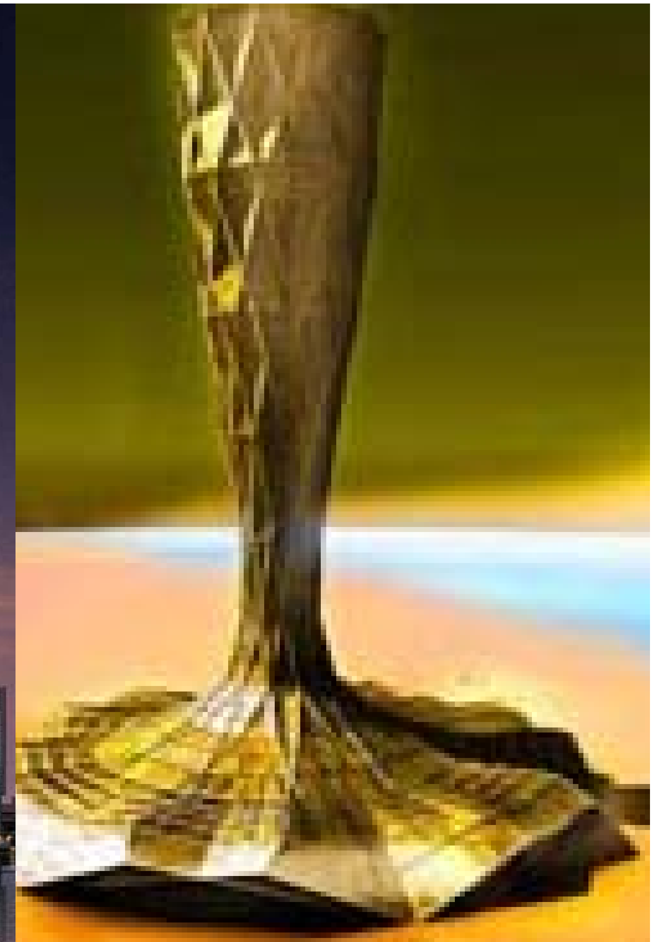
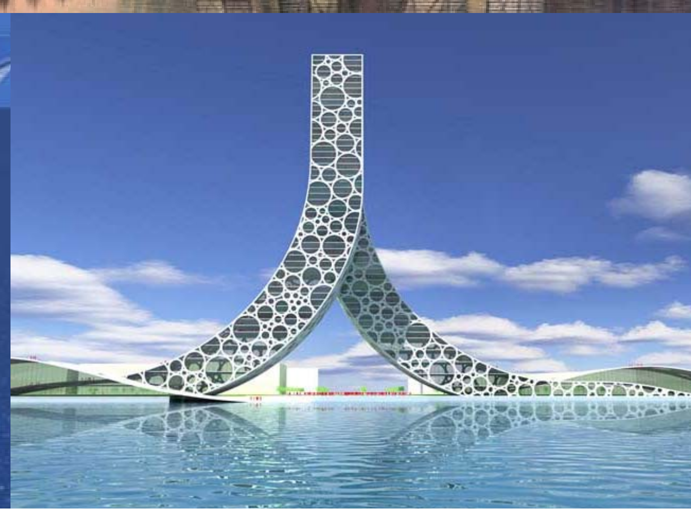
The Transmission of Masticatory Forces and Nasal Septum: Structural Comparison of the Human Skull and Gothic Cathedral

Rumy Hilloowala, D.D.S., Ph.D.; Hrishi Kanth, M.D.

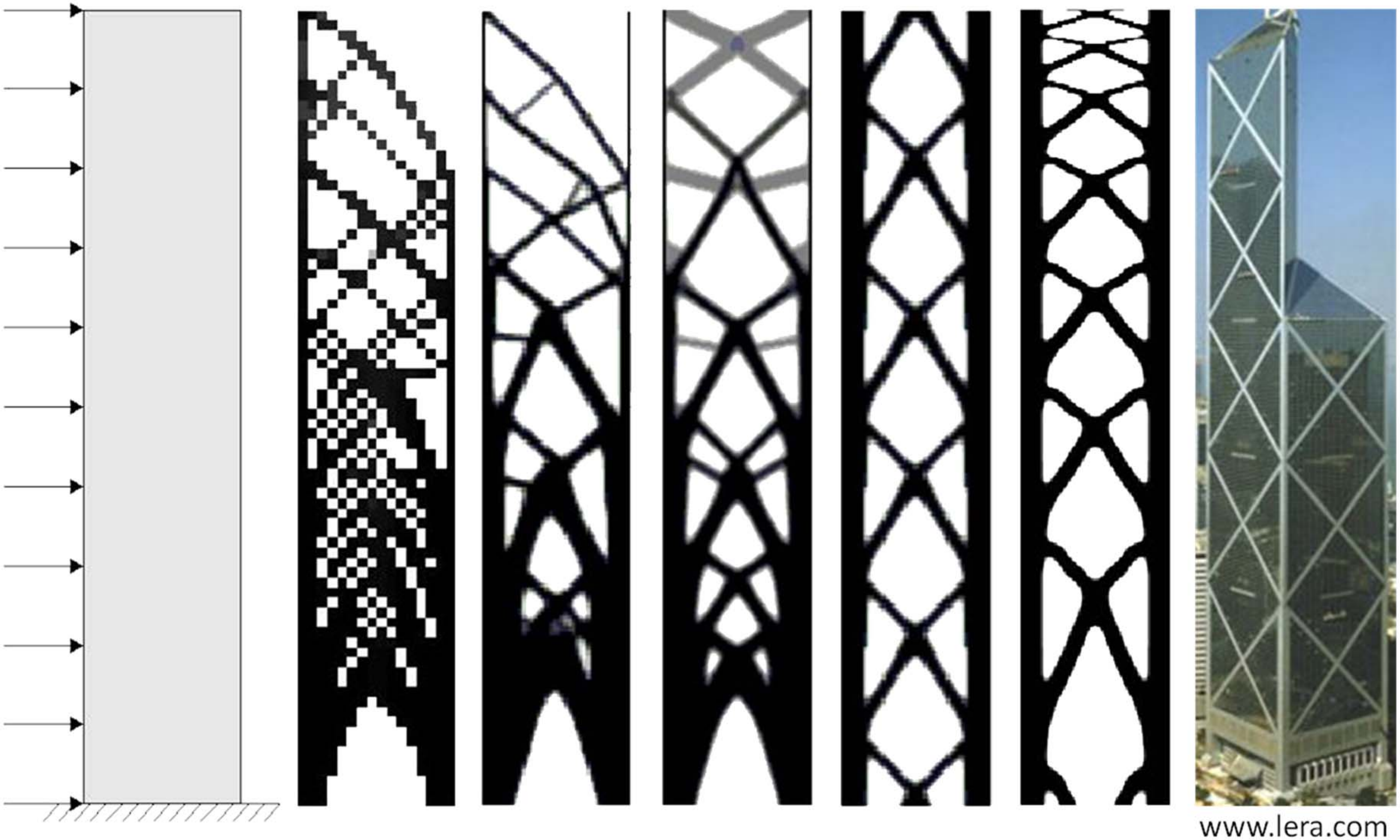







Building Science Through Topology Optimization

- *Topology optimization for high-rise buildings*

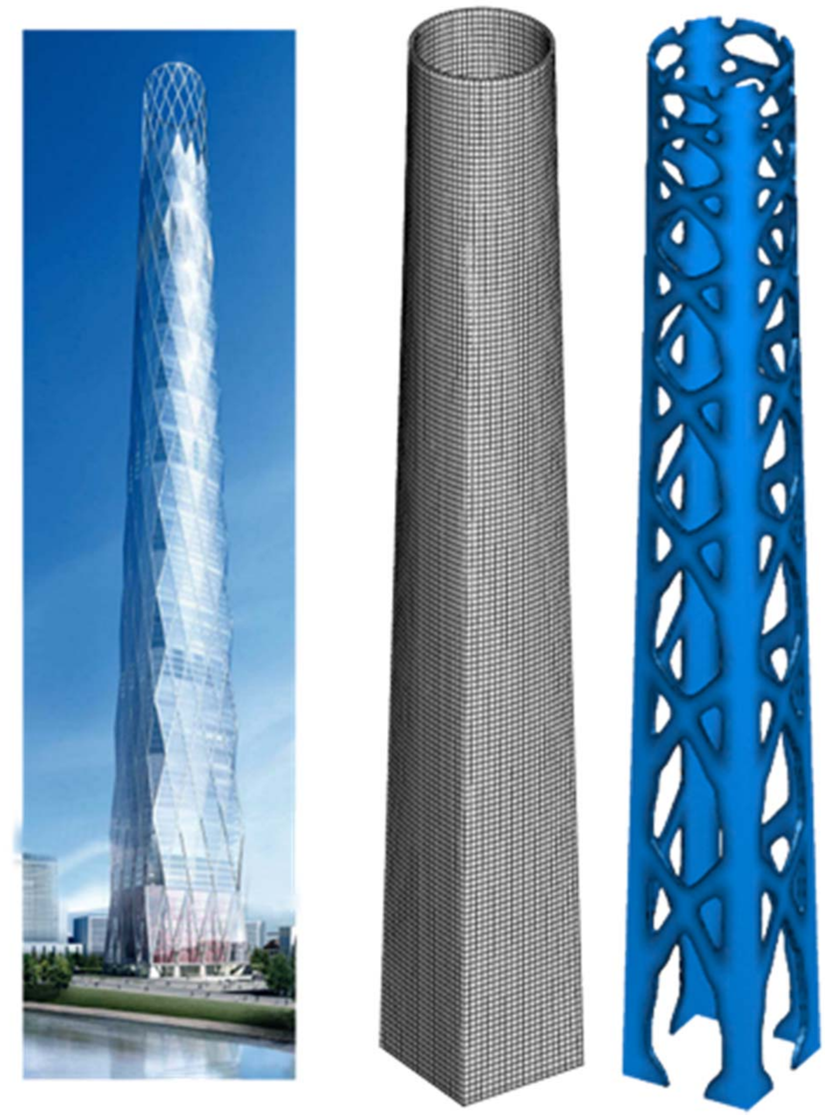


Building Science Through Topology Optimization

- *Application of pattern gradation to the conceptual design of buildings*
  - Flexible tool for unique shapes
  - Increasing column sizes
  - Dominance of shear behavior at top vs. overturning moment at base

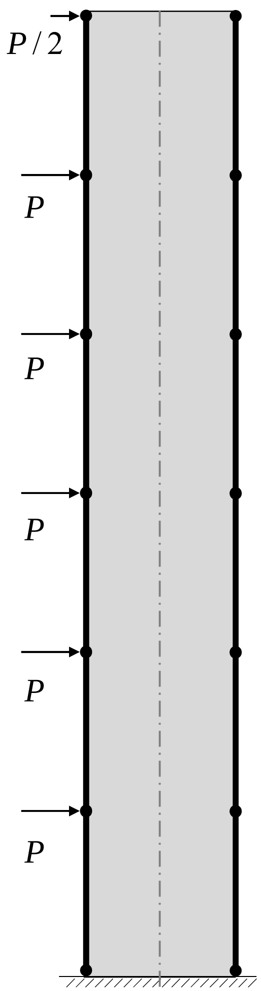
- In collaboration with Skidmore, Owings & Merrill, LLP

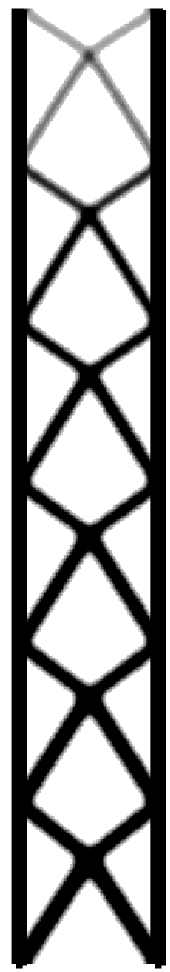
- Lotte Tower (Korea): optimized bracing using pattern gradation concepts



Building Science Through Topology Optimization

- *Topology optimization for structural braced frames: combining continuum and beam/column elements*





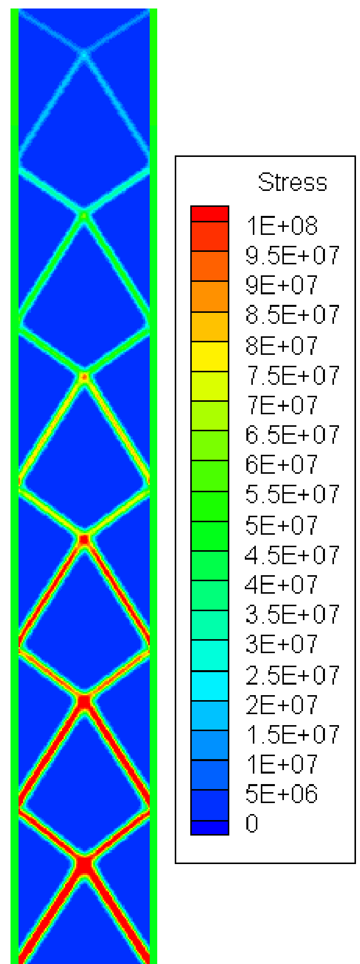


Image courtesy of SOM

Building Science Through Topology Optimization

- *Connecting engineering and architecture using structural topology optimization*
- Goal: to create unique, innovative designs that are both aesthetically pleasing and satisfy engineering principles
- Zendai competition: optimal designs resembling nature

Image courtesy of SOM

photography.nationalgeographic.com

Concluding Remarks

- FOBOS with Tikhonov Regularized TOP leads to nearly B&W solutions
- Baricentric FEM provides a stable formulation for Topology Optimization
- Topological Optimization is a linkage between medicine & engineering – promising approach for patient-specific computer-aided design of mid-face reconstruction
- Topological Optimization is a linkage between architecture & engineering – leads bioinspired design of tall buildings

Structural Topology Optimization employing the Allen-Cahn Evolution Equation on Unstructured Polygonal Meshes

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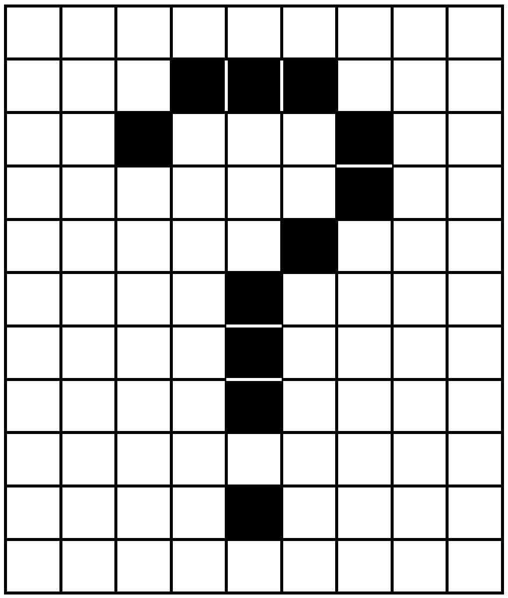

http://ghpaulino.com

Engineering a new architecture through barycentric element based topology optimization

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