Adaptive Regular Simplex Bisection Wavelets

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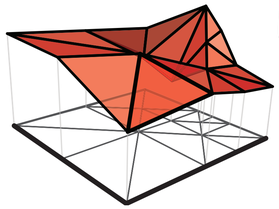
We study a family of mesh refinement schemes based on *Regular Simplex Bisection (RSB)* and lifted wavelets. We define a partial order on the vertices based on the dependencies induced by RSB and lifting which we use to reconstruct adaptive meshes.



Conforming RSB mesh with associated scalar field

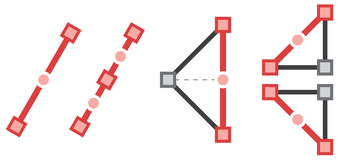
Multiresolution mesh refinement

Topological operators modify mesh discretization and ensure properties of mesh connectivity.

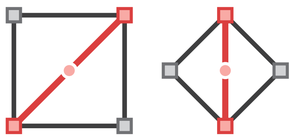
Geometric operators modify functions defined on the mesh and ensure properties of its function space.

Regular Simplex Bisection

Dimension independent affine invariant simplex bisection scheme. Enables generation of high quality adaptive simplicial complexes.



Simplex bisection in 1D and 2D



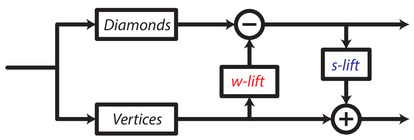
RSB diamonds in 2D

Diamonds

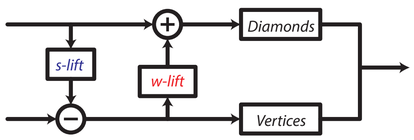
Collection of simplices sharing same bisection edge. Concurrent bisection enables extraction of conforming (*i.e.* crack-free) meshes. Refinement inserts a new vertex (●) at center of diamond and locally doubles its simplices.

Lifted wavelets

Wavelets enable localized analysis of a function at different scales.
The lifting scheme factorizes wavelets into elementary stencils with narrow support.



lifted wavelet analysis



lifted wavelet synthesis

split

splits vertices into two sets: diamond vertices (■) and diamond centers (●)

w-lift

predicts values of diamond centers from those of vertices

s-lift

updates values of vertices based on prediction error in diamond centers

Main result: Dependency relation for wavelets is a composition of diamond and lifting dependencies

$$\text{DEP}_{\text{wavelet}} := \text{DEP}_{\text{diamond}}(\text{DEP}_{\text{s-lift}}(\text{DEP}_{\text{w-lift}}))$$

diamond dependencies: *arcs from a diamond to subset of diamonds at previous depth*

s-lift dependencies: *arcs from a vertex to subset of diamond centers at same depth*

w-lift dependencies: *arcs from a diamond center to subset of vertices at same depth*

Examples

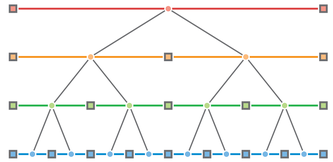
Dependencies for linear B-spline wavelets (1D)

$\text{DEP}_{\text{w-lift}}$

w-lift depends on two bounding vertices of a diamond

$\text{DEP}_{\text{s-lift}}$

s-lift depends on two diamonds incident in a vertex



$\text{DEP}_{\text{diamond}}$ *each diamond depends on a single* parent *at previous depth*

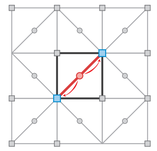
$\text{DEP}_{\text{wavelet}}$

$\approx$

$\text{DEP}_{\text{diamond}}(\text{DEP}_{\text{s-lift}}(\text{DEP}_{\text{w-lift}}))$

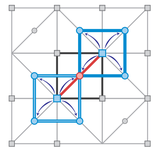
each diamond depends on its parent diamond and its neighbor incident in bounding vertex

Dependencies for linear B-spline wavelets (2D)



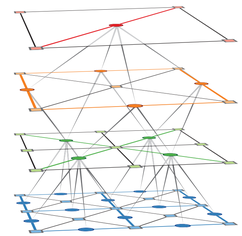
$\text{DEP}_{\text{w-lift}}$

each w-lift depends on bisection edge vertices



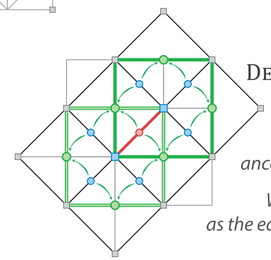
$\text{DEP}_{\text{s-lift}}$

each s-lift depends on four incident diamonds



$\text{DEP}_{\text{diamond}}$

each diamond depends on two parents *at previous depth*



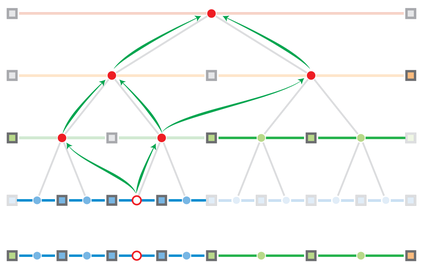
$\text{DEP}_{\text{wavelet}}$

each diamond depends on six ancestor diamonds at previous depth.

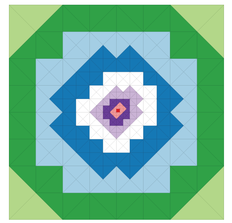
We can implicitly enumerate these as the edge centers of two squares (green).

Extracting adaptive mesh

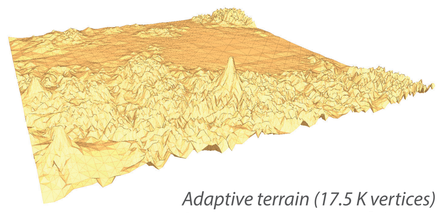
Transitive closure of the partial order defined by the dependency relations enables reconstruction of the smallest adaptive mesh satisfying application-dependent constraints.



Adaptive diamond mesh (1D) and transitive closure of dependency relation to reconstruct a given vertex



Adaptive diamond mesh (2D) supporting reconstruction of vertex (red) at depth 13



Adaptive terrain (17.5 K vertices) extracted from Puget Sound 4097² dataset reconstructed using linear B-spline wavelets

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