Mean Curvature Skeletons

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1) SKELETONIZATION

Skeletons as singularities of surface flows:

S — Solid Object

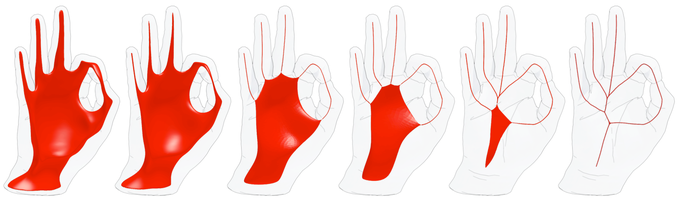
$\dot{S} = -\vec{n}$ — Medial Skeleton

$\dot{S} = -H\vec{n}$ — Curve Skeleton

2) CONTRIBUTIONS

- Anisotropy analysis of MCF
- Dynamic FEM discretization
- Medial skeletonization flow

$$\dot{S} = -H\vec{n} - w_M \vec{N}^{t=0}$$



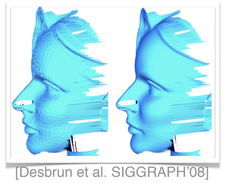
3) MEAN CURVATURE FLOW

MCF is associated with a diffusion process commonly used for surface fairing:

Each point on the surface moves... $\dot{S} = -H\vec{n}$

... with a velocity equal to its local mean curvature: H

...in a direction normal to the surface... $\vec{n}$



[Desbrun et al. SIGGRAPH'08]

More Information
Andrea Tagliasacchi (andrea.tagliasacchi@gmail.com), Ibraheem Alhashim, Matt Olson, Hao Zhang, "Mean Curvature Skeletons", *Computer Graphics Forum* (Proceedings of the Symposium on Geometry Processing), 2012. Project Homepage: https://code.google.com/p/starlab-mcfskel

4) WHY IS MCF APPROPRIATE?

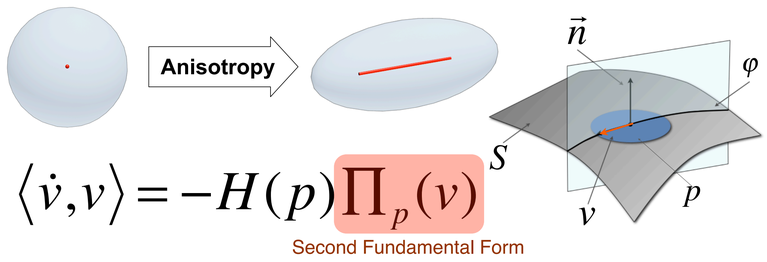
Vanishing area is a necessary condition only:

Skeleton Extraction by Mesh Contraction [Au et al. SIGGRAPH'08]

Oscar Kin-Chung Au* Chiew-Lan Tai* Hung-Kuo Chu† Daniel Cohen-Or‡ Tong-Yee Lee†
*The Hong Kong Univ. of Science and Technology †National Cheng Kung University ‡Tel Aviv University

Original shape:	$A > 0$	$V > 0$	Contraction
Curvature Flow:	$\dot{A} < 0$	$\dot{V} < 0$	
Curve-Skeleton:	$A = 0$	$V = 0$	

Instead, we highlight its anisotropic nature:



$$\langle \dot{v}, v \rangle = -H(p)\Pi_p(v)$$

Second Fundamental Form

5) MCF DISCRETIZATION (FEM)

Iteratively minimize the "contraction" energy:

$$E_{MCF} = \left\| L^t V^{t+1} \right\|^2 + w_H \sum \left\| v_i^{t+1} - v_i^t \right\|^2$$

Approximates Laplacian $L^{t+1} \approx L^t$

Bounds Approximation (by bounding velocity)

6) MEDIAL CENTERING

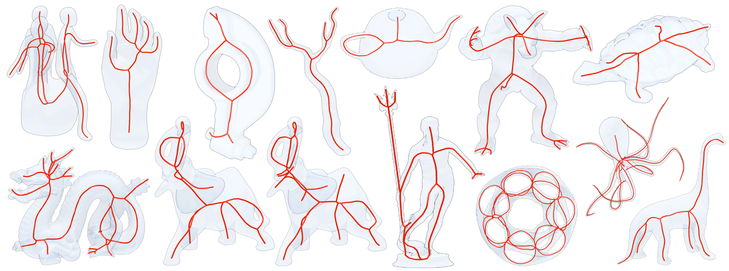
$$E = E_{MCF} + w_M \sum \left\| v_i^{t+1} - \mu(v_i^t) \right\|^2$$

Corresponding Medial Pole

- Poles approximate normals
- Maintains flow centred-ness
- (Still) converges to curve
- Boost performances
  - ➡ external vs. internal forces
  - ➡ weaker velocity control w_H

7) RESULTS (CURVE SKELETON)

We verified our results on a set of shapes:



8) RESULTS (MESO SKELETON)

The intermediate states of the medial flow are a mixture of curvilinear and surface elements: