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Nondestructive identification of multiple flaws in structures using a combination of XFEM and topologically adaptive Enhanced Artificial Bee Colony (EABC) optimization algorithm

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INTRODUCTION

In this poster, we propose a new computational framework based on the Extended Finite Element Method (XFEM) and a topologically adaptive Enhanced Artificial Bee Colony (EABC) algorithm, namely XFEM-EABC algorithm to detect and quantify multiple flaws in structures.

The concept is based on recent work that have shown the synergy between XFEM, used to model the forward problem, and a Genetic-type Algorithm (GA), used as the optimization scheme, to solve an inverse identification problem and converge to the "best" flaw parameters. The key idea in this scheme is that XFEM alleviates the need for re-meshing the domain in each forward analysis during the optimization process.

While previous work only considered quantification of a single flaw, we propose an adaptive algorithm that can detect multiple flaws by introducing topological variables into the search space which turn on and off flaws during run time. The identification is based on a limited number of strain sensors assumed to be attached to the structure surface boundaries. Each flaw is approximated by a circular void with three variables: center coordinates and radius. In addition the proposed EABC is improved by a guided-to-best solution updating strategy and a local search operator of the of the Nelder-Mead simplex type that shows faster convergence and superior global/local search abilities than the standard ABC and classic GA algorithms.

BACKGROUND/THEORY/METHOD

Inverse problem

The inverse problem is summarized as follows: Given Ω, Γ_t, Γ_u, $\bar{\mathbf{u}}$, $\bar{\mathbf{t}}$, $\mathbf{L}$ and some specific measured response (strains $\boldsymbol{\varepsilon}$ in this work), one's objective is to find Γ_v which can be described by a set of parameters $\boldsymbol{\theta}$. It can be formulated as an optimization process in which the objective is to minimize the difference between the measured and simulated data:

$$g(\boldsymbol{\theta}) = \sum_{\alpha=1}^{2} \frac{\|\boldsymbol{\varepsilon}_{\alpha\alpha}^{s}(\boldsymbol{\theta}) - \boldsymbol{\varepsilon}_{\alpha\alpha}^{m}\|}{\|\boldsymbol{\varepsilon}_{\alpha\alpha}^{m}\|}$$

In this work, circular voids are used to approximate the true flaws in structures: (x_k, y_k, r_k). In general, the identification problem can be summarized as:

$$\text{Find } \boldsymbol{\theta} \in S \text{ such that } g(\boldsymbol{\theta}) \to \min$$

where S is the feasible m-dimensional parameter search space which can be generally written as:

$$S = S_1 \cup S_2 \cup \cdots \cup S_k \cup \cdots \cup S_{n_t}$$

$$S_k = \left\{ \begin{array}{l|l} x_k \in \mathbb{R}^{n_t} & x_k^{\min} \le x_k \le x_k^{\max} \\ y_k \in \mathbb{R}^{n_t} & y_k^{\min} \le y_k \le y_k^{\max} \\ r_k \in \mathbb{R}^{n_t} & r_k^{\min} \le r_k \le r_k^{\max} \\ \tau_k \in \mathbb{R}^{n_t} & \tau_k = 0 \; or \; 1 \end{array} \right\}$$

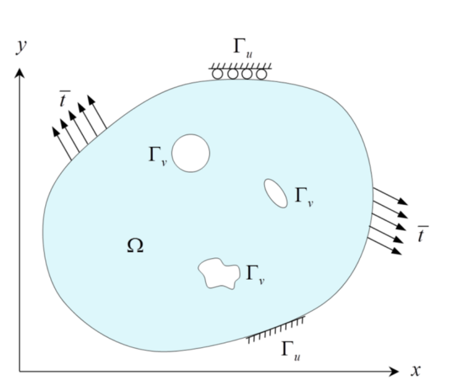


Figure 1. Generic solid with traction-free voids subjected to essential and natural boundary conditions

where n_t is the number of topological variables ($k = 1, 2, \cdots, n_t$); τ_k is the topological variable. $\boldsymbol{\theta}_k$ can be written as:

$$\boldsymbol{\theta}_k = (x_k \tau_k, y_k \tau_k, r_k \tau_k)$$

BACKGROUND/THEORY/METHOD (Continued)

Two approaches of determining the number of flaws:

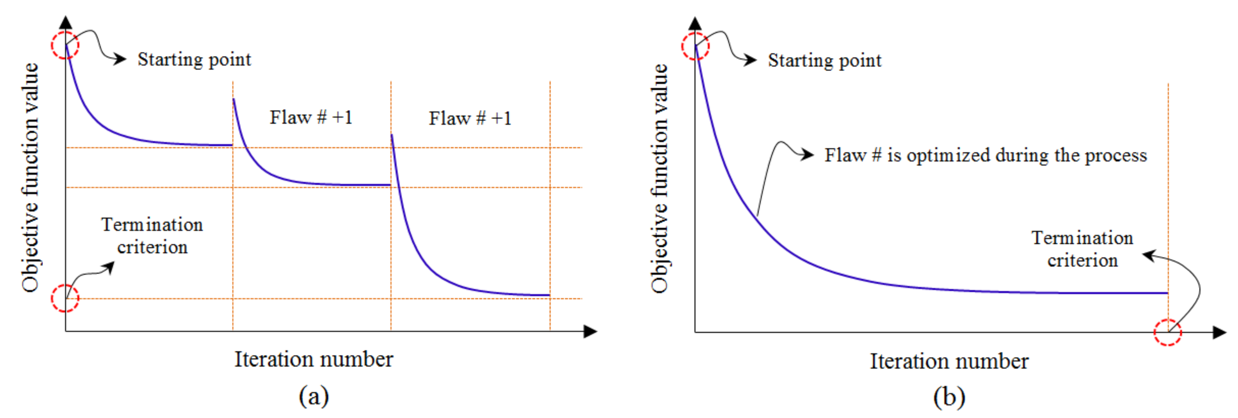


Figure 2. Convergence to the global minimum: (a) brute force sequential method: the number of flaws increases dynamically until the termination criterion is reached (global convergence is non-smooth) and (b) the topological variable approach in this paper: the number of flaws is optimized during the process (global convergence is smooth)

Extended Finite Element Methods (XFEM)

The XFEM alleviates the need for re-meshing the domain in each forward analysis during the optimization process, which can overcome the meshing limitation of standard FEMs in solving inverse problems.

In XFEM, the displacement field $u(x)$ is enriched with a weak-discontinuity function:

$$u^h(x) = \sum_{I \in \mathbb{N}} N_I(x) u_I + \boxed{\sum_{J \in \mathbb{N}^{en}} N_J(x) \psi_J(x) a_J} \qquad (u_I, a_J \in \mathbb{R}^2)$$

The discretized weak form for an enriched element with a shifted-basis is given as:

$$u_e^h(x) = \sum_{I \in \mathbb{N}_e} N_I(x) \left[u_I + \left(\psi(x) - \psi(x_I) \right) a_I \right]$$

The level set method is used for multi-circular voids modeling.

Enhanced Artificial Bee Colony Algorithm based optimization

Three search phases were presented in the standard ABC algorithm: the employed phase, the onlooker phase and the scout phase. Each phase has its own solution updating strategy.

In this work, improvements have been made such that an enhanced version of ABC (EABC) was proposed :

- *a guided-to-best solution updating strategy was proposed into the onlooker phase;*
- *a local search operator based on the Nelder-Mead simplex method (NMSM) was added to the standard ABC.*

The XFEM-EABC identification scheme

The flow of the XFEM-EABC algorithm is illustrated in Figure 3.

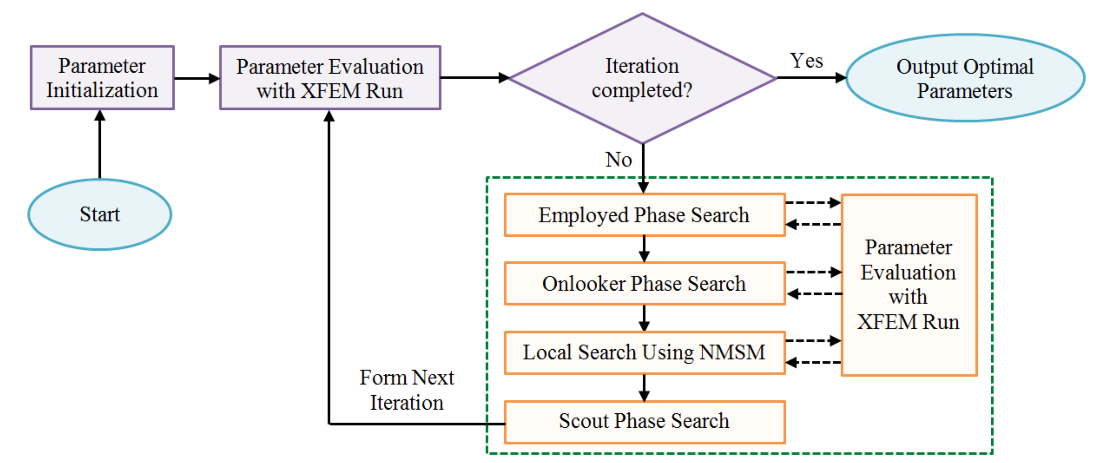


Figure 3. The XFEM-EABC based flaw detection scheme for one single independent run

RESULTS

Example 1: Detection of a single flaw within a rectangular plate

The purpose of the example is to study convergence behavior of the EABC algorithm (proposed as improvement in this work) compared to classic GAs and standard ABC algorithm.

41 × 41 node structured mesh

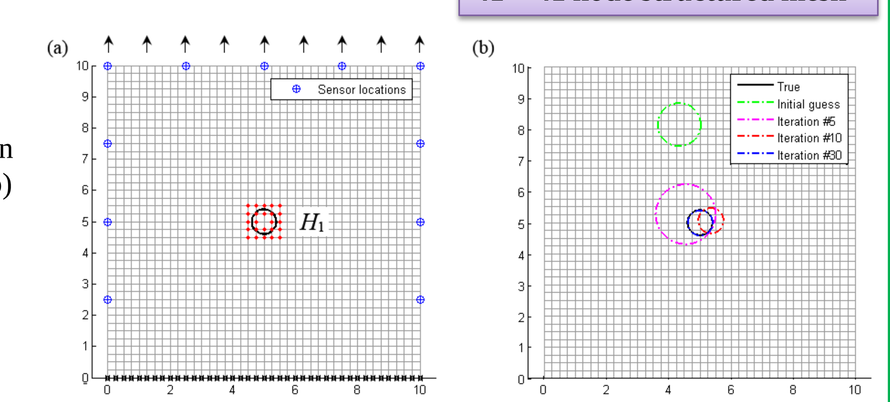


Figure 4. Detection of a single flaw within a rectangular plate: (a) mesh generation, loading condition and sensor placement and (b) snapshots of the XFEM-EABC evolutionary process for the target of a circular void of radius 0.4 (units) located at the center of a rectangular plate

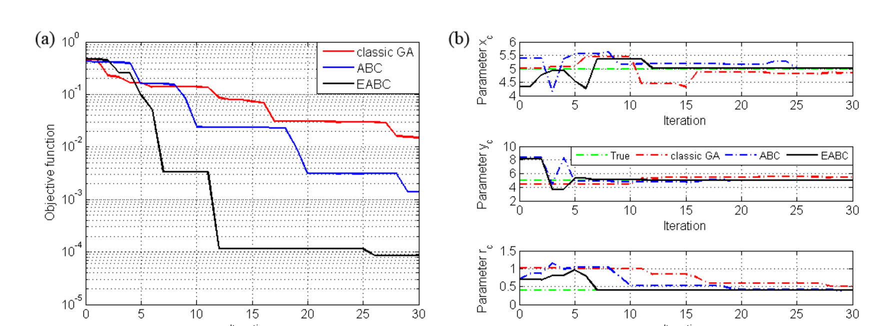


Figure 5. Comparison of convergence for alternative algorithms: (a) objective function evaluation and (b) parameter estimation

EABC shows better convergence.

Example 2: Detection of three non-regular-shaped flaws within an arch-like plate

- Three non-regular-shaped flaws;
- The reference solution is obtained from an FEM model with an unstructured mesh;
- The circular voids enrichment XFEM code is used for the forward analysis in the identification process .

Artificial Noise

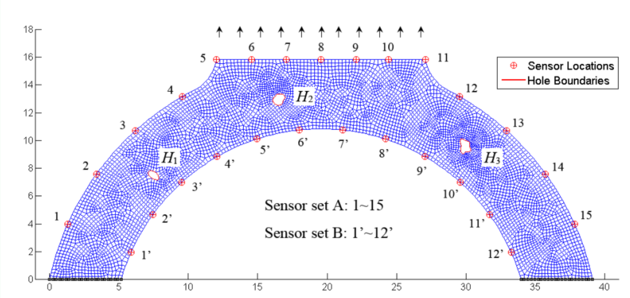


Figure 6. Unstructured FEM mesh (5827 nodes), loading conditions, sensor placement and flaw locations within an arch-like plate used as the reference solution

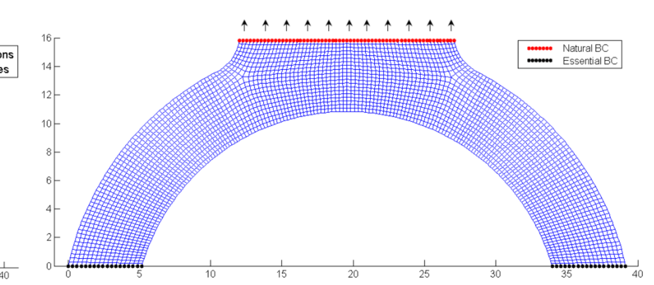
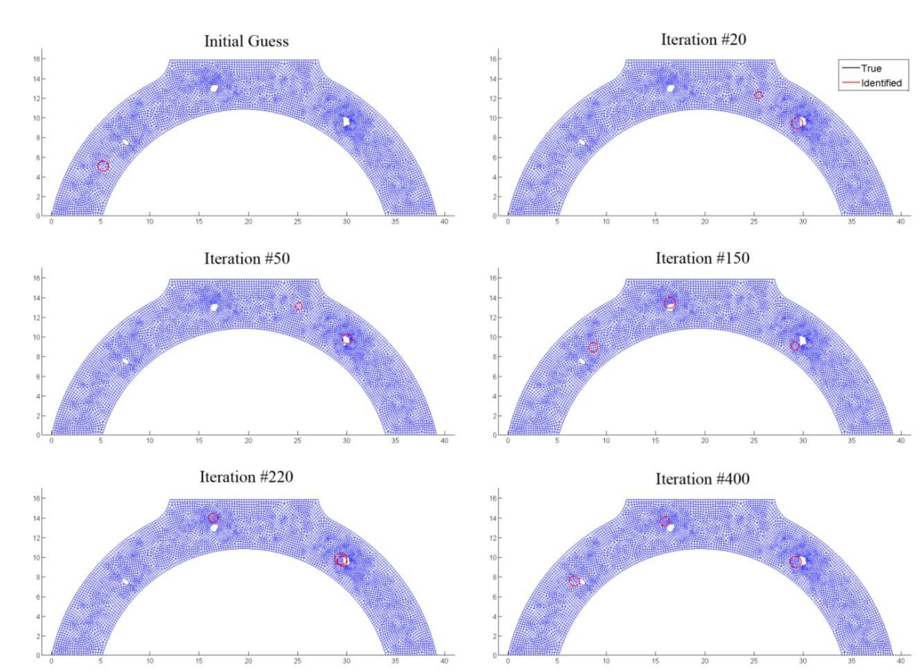


Figure 7. Mesh (3429 nodes) and boundary conditions for XFEM forward problem

Figure 8. Snapshots of the XFEM-EABC evolutionary process in the identification of three non-regular shaped voids (the number of flaws is unknown a priori)

$n_t = 4$

The identified number of flaws is 3.



RESULTS (Continued)

The performance of the XFEM-EABC algorithm is robust and efficient even with the artificial noise in strain measurements caused by irregular shaped voids and the numerical/modeling difference between the FEM and the XFEM with different meshes

Figure 9. Detection of three non-regular shaped voids under condition of number of flaws unknown: XFEM-EABC convergence of (a) objective function evaluation and (b) number of flaws

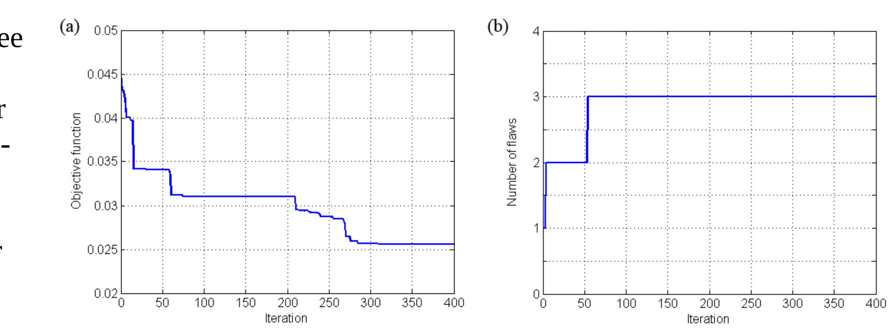
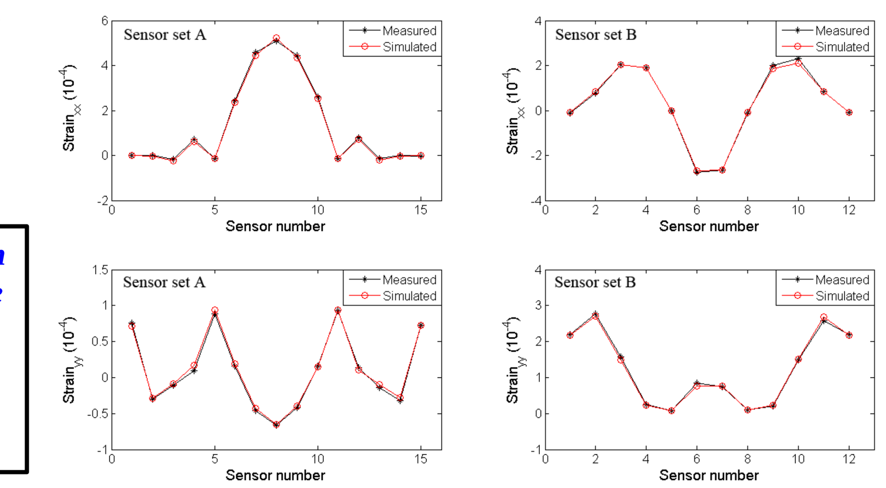


Figure 10. Comparison of measured strains and simulated (estimated or predicted) strains

The proposed algorithm is able to truly converge to the number of flaws and approximates the irregular shaped flaws quite well.



CONCLUSIONS & FUTURE WORK

This work presents a novel computational scheme, namely topologically adaptive XFEM-EABC algorithm, for solving inverse problems for identification of multiple flaws in structures through a limited number of strain measurements. Numerical results show the performance of the XFEM-EABC algorithm is robust even with artificial noise involved in measurements. Overall, the satisfactory results are encouraging for potential implementation of this algorithm in the field of multi-void flaws detection.

Nevertheless, experimental studies and real applications are required for further examination of the proposed strategy regarding different sensor availabilities and placements. Continuous work will also further explore the detection of multiple flaw geometries besides voids, such as cracks.

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