METAL FATIGUE IN DENTAL IMPLANTS:
A MARKOFF CHAIN- STOCHASTIC FINITE ELEMENT FORMULATION

María Prados-Privado

ABSTRACT

Random distribution of loads and constitutive properties are incorporated here in the Coffin-Manson damage model. Based on the Markoff chain, Bogdanoff-Kozin probabilistic models are used.

The probabilistic transition matrix (PTM), which is obtained from *Stochastic Finite Elements* associated with the expansion of random fields, leads to the cumulative damage density function that are used in fatigue life predictions. An example on dental implants is demonstrated here.

INTRODUCTION

Fatigue of metal components is currently recognized as one of the main causes of failure of structural elements. Three different stages can be considered during the fatigue process: cracks nucleation; cracks propagation and final failures of components.

Models developed to study the crack nucleation process are mainly based on the local strain approach [1]. Uncertainties on material properties, dimensions of the structural element and load history have a decisive influence on the fatigue process and therefore on the life of the structural component. All of this suggests to include the probabilistic character of the different variables from the very beginning. In this work one focuses only on the crack nucleation stage.

Stochastic Finite Element Method (SFEM) [2] has become an excellent tool to estimate the influence of the stochastic properties of loads, material properties and geometry on responses. In the present work, the crack nucleation stage in fatigue is considered as a cumulative damage problem, discrete in time and space, using the probabilistic models based on the theory of Markoff chains that was developed in the Bogdanoff-Kozin (B-K) models [3]. In this work, the construction of these models is established from the results of a series of SFEM analyses. Hence, as opposed to conventional formulations, it was not necessary to minimize any functional.

The proposed procedure consists of the construction of a cumulative damage B-K model from the SFEM results computed for every random variable e.g. : the material parameters, the applied loads. However, other random variables can also be included in the proposed scheme rather easily.

CONSTRUCTION OF THE CUMULATIVE DAMAGE B-K MODEL

The main goal of this section is to show how to construct a model, which takes into account under a simple mathematical structure, when the main sources of uncertainty appear in physical cumulative damage problems.

Main assumptions:

A B-K model [3] is defined by a series of basic initial assumptions. Some other assumptions may be added in order to complement or define a more specific type of B-K models. Among them, the most important are:

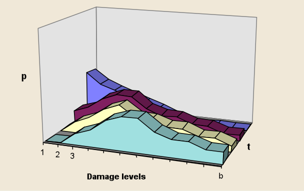
1.There exist repetitive "damage cycles" (DC) of constant severity.

2.The damage levels are discrete 1,2,..,j,..,b. The last level b is considered as a "failure state", that is, the life of the component is considered to be expired when level b is reached.

3.The cumulative damage in a DC depends only on that specific DC and the damage level at the beginning of that DC.

4.The damage level in one DC increases from that DC i to the next DC i+1 or remains in the initial DC i, being impossible to skip one or more damage states.

$$\mathbf{P}=\begin{bmatrix} p_1 & q_1 & 0 & 0 & \dots & 0 & 0 \\ 0 & p_2 & q_2 & 0 & \dots & 0 & 0 \\ 0 & 0 & p_3 & q_3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & p_{b-1} & q_{b-1} \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \end{bmatrix} \qquad \mathbf{p}_t=\mathbf{p}_0\mathbf{P}^t \qquad t=0,1,2,\dots$$



STOCHASTIC EXPRESSION FOR THE DAMAGE MODEL FOR THE NUCLEATION STAGE

Deterministic expression for fatigue life (nucleation stage, Coffin and Basquin-Manson) is summarized:

$$\frac{\Delta\varepsilon_{ep}}{2}=\frac{\sigma_f'}{E}\left(2N_f\right)^b+\varepsilon_f'\left(2N_f\right)^c$$

$\frac{\Delta\varepsilon_{ep}}{2}$: Elastic-plastic strain amplitude
σ_f': Fatigue strength coefficient
b: Fatigue strength exponent
ε_f': Fatigue ductility coefficient
c: Fatigue ductility exponent
E: Young modulus
N_f: Fatigue life

The objective is to obtain the mean value and variance of fatigue life from SFEM, in order to build the B-K model. Now all variables are considered to be random.

SFEM: THE PERTURBATION APPROACH

The aim is to obtain the response random fields with the First order Taylor expansions:

$$\mathbf{u}=\mathbf{u}^{\mathbf{0}}+\sum_{i=1}^{N}\mathbf{u}_{\mathbf{i}}^{\mathbf{I}}(\alpha_i-\alpha_i^0)+\dots \qquad \varepsilon_{\mathbf{e}}=\varepsilon_{\mathbf{e}}^{\mathbf{0}}+\sum_{i=1}^{N}\varepsilon_{\mathbf{ei}}^{\mathbf{I}}(\alpha_i-\alpha_i^0)+\dots$$

It's neccessary to obtain the sensitivities. Starting from the equilibrium equation:

K u = f

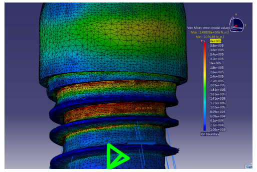
By taking the derivative of the last expression respect to random variables:

$$\frac{\partial\mathbf{K}}{\partial\alpha_i}\mathbf{u}+\mathbf{K}\frac{\partial\mathbf{u}}{\partial\alpha_i}=\frac{\partial\mathbf{f}}{\partial\alpha_i}$$

$\frac{\partial\mathbf{K}}{\partial\alpha_i}$ must be evaluated at the element level;
u obtained from previuos FE analisys (evaluated at mean values);
$\frac{\partial\mathbf{f}}{\partial\alpha_i}$ boundary conditions.

SOME RESULTS

Applied load has been taken from [4]. It includes the regular forces and the overloads from bruxism. The mean values of fatigue life is 2.47 x 10^{11} cycles (11.289,5 years), and with overloads the cycles decrease up to 1.2 x 10^8 (2.74 years).
The fatigue life (bruxism) is 34,5 million cycles (1.57 years) with a failure probability of 0.00713



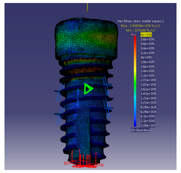
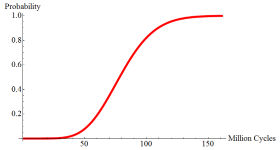


Table 1: Random Variables (INPUT)

Variable	Mean Value	Variance
Force (modulus)	423 N	169
σ_f	290 MPa	2.1025×10^{6}
ϵ_f	0.0789	1.5563×10^{-13}
b	-0.0377	4.0501×10^{-12}
c	-0.4025	5.0448×10^{14}

Table 2: Random Variables (OUTPUT)

Variable	Mean Value	Variance
Fatigue life (Cycles)	1.2044×10^{8}	5.044×10^{14}
b (PTM dim)	24	-
r	4.1887	-



CONCLUSIONS

-Within the context of fatigue lives for dental implants, this research demonstrates a drastic change that has severe implications for patients of bruxism and implant manufactures. Here, a finite element analysis in tandem with the Bogdanoff-Kozin probabilistic model (at the nucleation stage) has been carried out. :

- In general, the mean value driven design-analysis way overestimates (e.g. to the tune of thousand folds) fatigue lives in comparison with more realistic stochastic analysis.
- In particular, the fatigue life for dental implants, for persons who have the habit of bruxism, *drastically* decreases from 11,289.5 years to 2.74 years.

References

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CONTACT

María Prados-Privado	+(34) 91 488 9109
Rey Juan Carlos University	+(34) 600 437 964
Madrid (Spain)	mprados@unizar.es