Polygonal Finite Elements: Cubature and application to Reissner – Mindlin Plates

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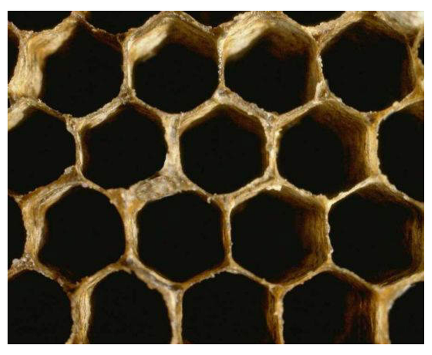
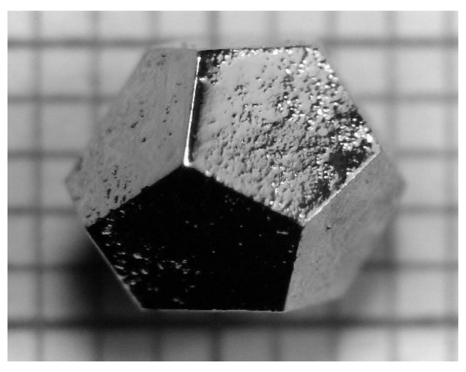
ABSTRACT

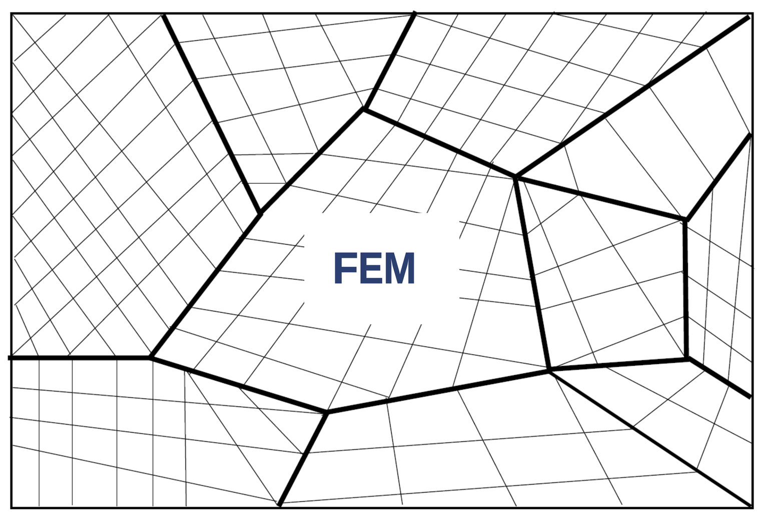
The use of polygonal elements with more than four sides can provide flexibility and better accuracy[1]. A brief overview of different cubature rules over arbitrary polygons is given. Polygonal finite elements with Wachspress interpolants are employed to study the response of plates based on first order shear deformation theory. A technique is outline to suppress shear locking.

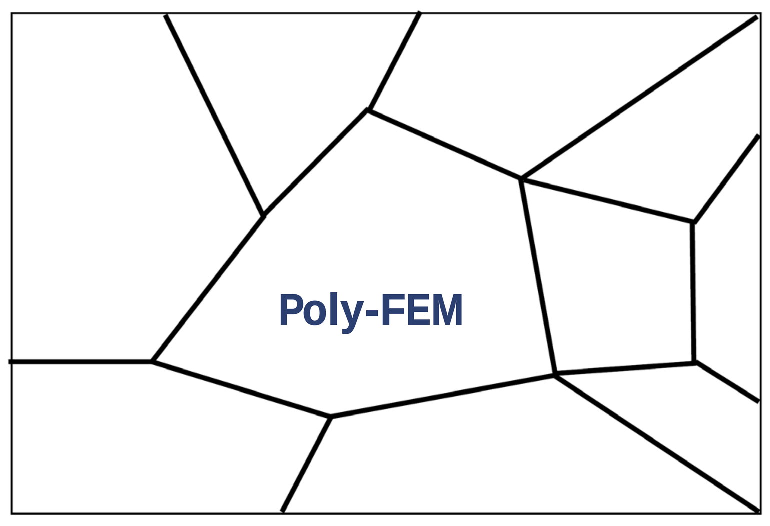
POLYGONS IN NATURE


Honey comb


Animal skin covered with polygons

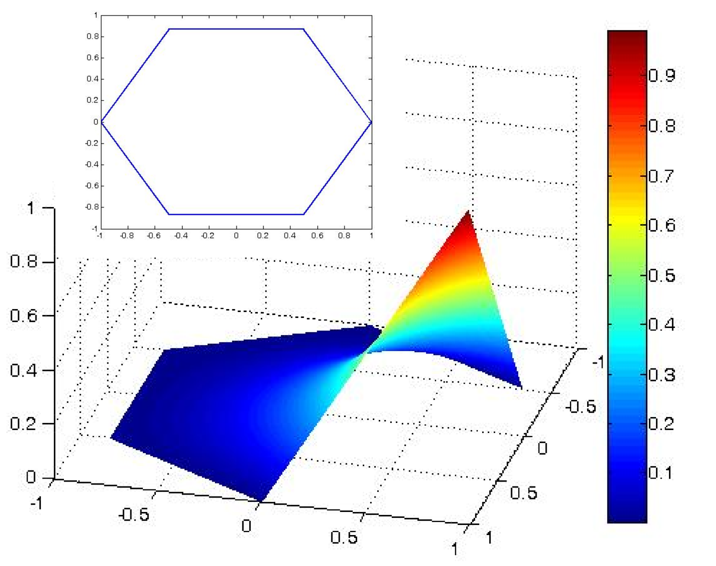

Ho-Mg-Zn quasi crystal

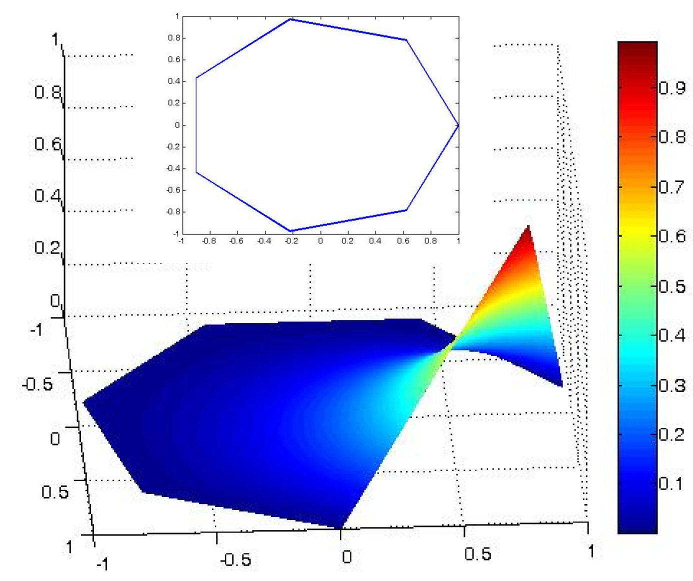




APPROXIMATION OVER POLYGONS[2]

- o Using length and area measures – Wachspress interpolants
- o Natural neighbour interpolants
- o Maximum entropy approximant
- o Barycentric coordinates



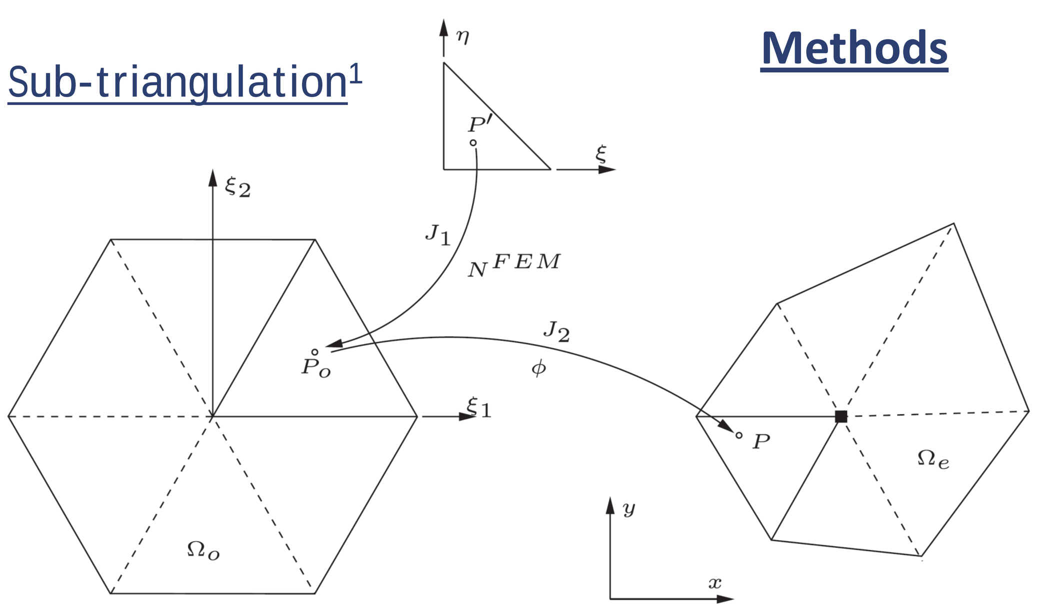


CUBATURE OVER POLYGONS

$$\sum_{(\xi,\eta)\in\Xi_{2n-1}} w_{\xi,\eta}\, f(\xi,\eta) \approx \iint_\Omega f(x,y)\,dxdy\ ,\quad \Omega\subset\mathbb{R}^2 \ \text{polygon}$$

Sub-triangulation[1]

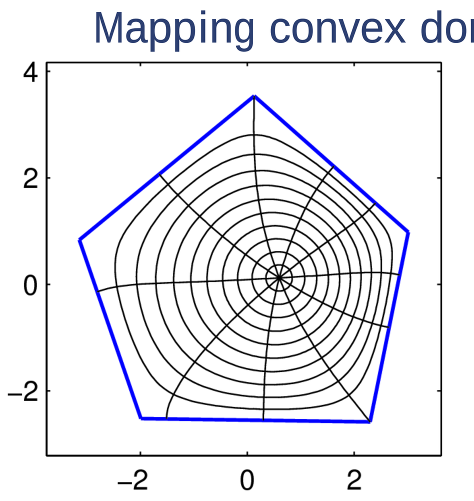
Methods

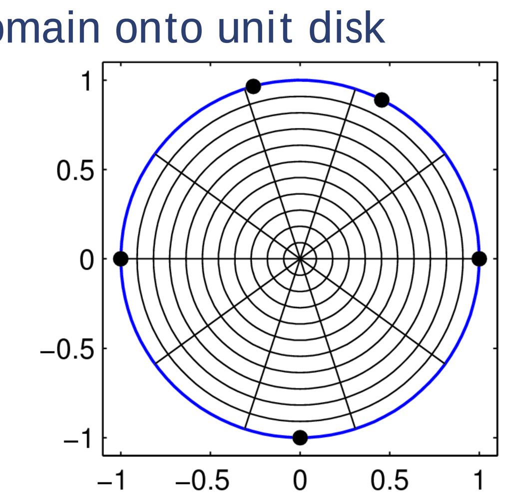


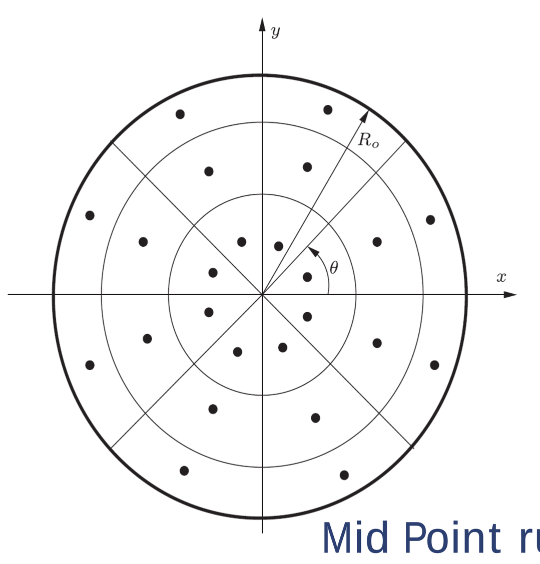
Cubature over triangles
- o Symmetry quadrature
- o Generalized quadrature
- o Dunavant rule
- o Fekete points

Schwarz Christoffel Mapping[3] $f(z) = A\int^z \prod_{j=1}^{n} (z'-z_j)^{-\beta_j}\, dz' + B$

Mapping convex domain onto unit disk





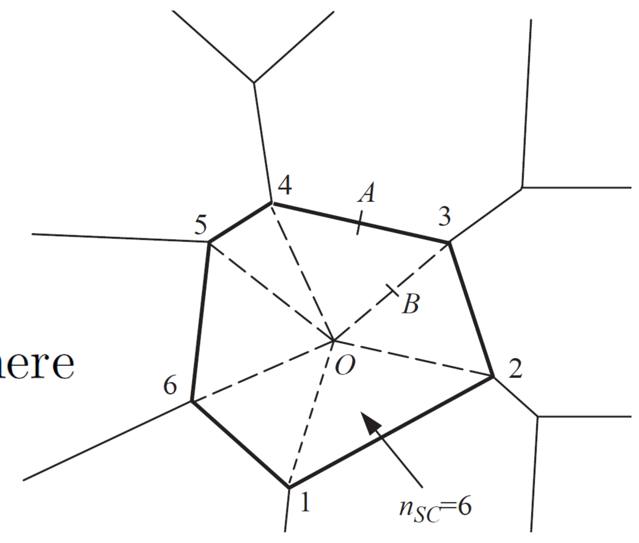


Mid Point rule

Smoothed Finite Element Method[4] $\tilde{\varepsilon}^h_{ij}(\mathbf{x}_C) = \int_{\Omega^h} \varepsilon^h_{ij}(\mathbf{x})\Phi(\mathbf{x}-\mathbf{x}_C)d\mathbf{x}$

$$\Phi \geq 0 \quad \text{and} \quad \int_{\Omega^h} \Phi(\mathbf{x})d\mathbf{x} = 1$$

$$\Phi = \frac{1}{A_C} \quad \text{in} \quad \Omega_C \quad \text{and} \quad \Phi = 0 \quad \text{elsewhere}$$



Green – Gauss Quadrature[5]

$$\iint_\Omega f(x,y)\,dxdy = \oint_{\partial\Omega} \mathcal{F}(x,y)\,dy\ ,\quad \mathcal{F}(x,y) = \int f(x,y)\,dx$$

APPLICATION TO REISSNER-MINDLIN PLATES

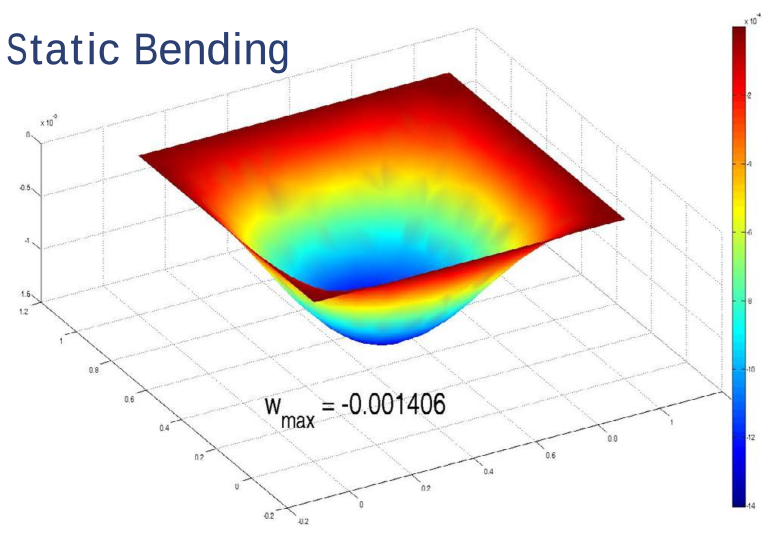
Displacement field

$$\begin{aligned} u(x,y,z,t) &= u_o(x,y,t) + z\theta_x(x,y,t) \\ v(x,y,z,t) &= v_o(x,y,t) + z\theta_y(x,y,t) \\ w(x,y,z,t) &= w_o(x,y,t) \end{aligned}$$

Modified shear correction

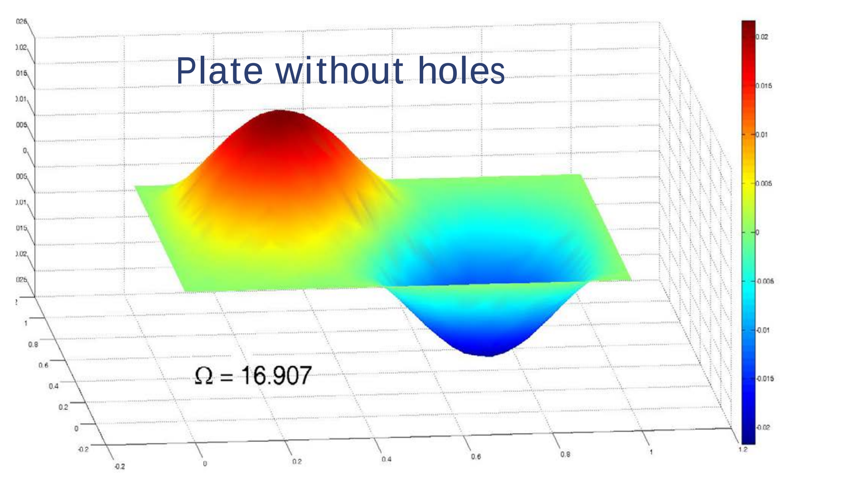
$$\kappa_e = \kappa \frac{(t/\beta l_e)^2}{\left\{1 + (t/\beta l_e)^{2n}\right\}^{1/n}}$$

Static Bending



Free Vibration

Plate without holes



Panel Flutter

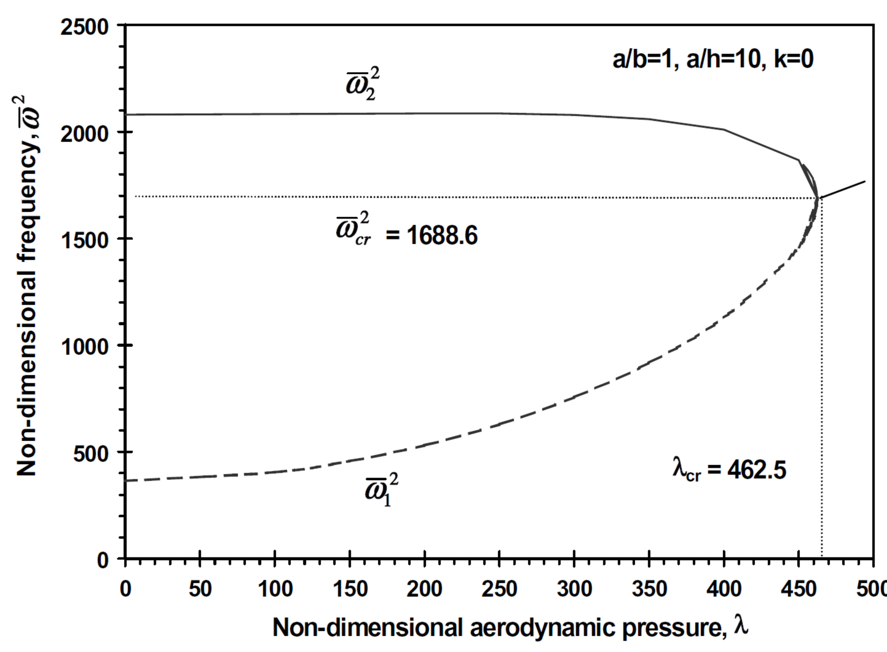
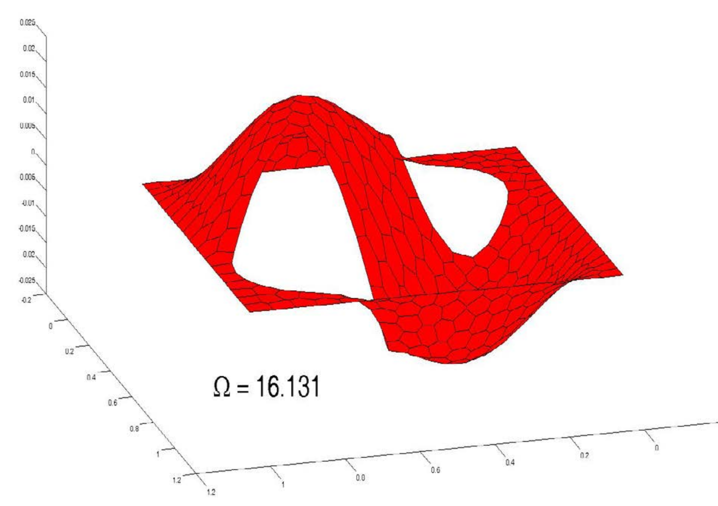


Plate with holes



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[3]S Natarajan, *et al., Numerical integration over arbitrary polygons, IJNME, 80 (2009), 103-134.*
[4]KY Dai, *et al., An n-sided polygonal smoothed finite element method, FEAD, 43 (2007), 847-860.*
[5]A Smmoriva and M Vianello, *Product Gauss cubature over polygons based on Green's integration formula, BIT Numerical methods, 47 (2007), 441-453.*
[6]Picture source: http://en.wikipedia.org/wiki/Polygon

Of Interest

D Kumar Patel, D Roy Mahapatra, *Polygonal finite element based scheme to model honeycomb sandwich panels.* NSF Workshop poster presentation, Columbia University, July 25-27, 2012.

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