A NONLOCAL DAMAGE MECHANICS FORMULATION FOR SIMULATING CREEP FRACTURE IN ICE SHEETS AND GLACIERS

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1. Introduction

- Creep fracture plays an important role in iceberg calving from glaciers and in the catastrophic collapse of ice shelves.
- A better understanding of creep fracture is required to predict the sea level change and its impact on society.
- Current theoretical models based on linear elastic fracture mechanics are too simplistic.
- Rigorous computational models can help us gain new insights into the relevant mechanisms behind ice sheet break up.

2. Objectives

1. Develop a three-dimensional thermo-viscoelastic constitutive model for polycrystalline ice
2. Devise a strain space nonlocal formulation to simulate structural failure due to creep fracture
3. Calibrate and validate the model using experimental data.
4. Test the accuracy and consistency of numerical results using benchmark examples and then make predictions.

3. Viscoelastic Constitutive Model

Additive decomposition: Assuming small strains [1],

$$\epsilon_{kl} = \epsilon^{\mathrm{e}}_{kl} + \epsilon^{\mathrm{d}}_{kl} + \epsilon^{\mathrm{v}}_{kl}. \qquad (1)$$

Stress-strain relations: The elastic, delayed elastic and viscous strain components are given by,

$$\tilde{\sigma}_{kl} = \frac{E}{3(1-2\nu)}\,\epsilon^{\mathrm{e}}_{ii}\delta_{kl} + \frac{E}{(1+\nu)}\left(\epsilon^{\mathrm{e}}_{kl} - \frac{1}{3}\epsilon^{\mathrm{e}}_{ii}\delta_{kl}\right), \qquad (2)$$

$$\dot{\epsilon}^{\mathrm{d}}_{kl} = A\left(\frac{3}{2}K\tilde{\sigma}^{\mathrm{dev}}_{kl} - \epsilon^{\mathrm{d}}_{kl}\right), \qquad (3)$$

$$\dot{\epsilon}^{\mathrm{v}}_{kl} = \frac{3}{2}\,K_N\left(\frac{3}{2}\tilde{\sigma}^{\mathrm{dev}}_{mn}\tilde{\sigma}^{\mathrm{dev}}_{mn}\right)^{(N-1)/2}\tilde{\sigma}^{\mathrm{dev}}_{kl}. \qquad (4)$$

Temperature dependence: The relation for K_N is,

$$K_N(T) = K_N(T_{\mathrm{m}})\exp\left(\frac{-Q}{R}\left(\frac{1}{T} - \frac{1}{T_{\mathrm{m}}}\right)\right). \qquad (5)$$

4. Nonlocal Continuum Damage Formulation

Effective stress concept: We define a transformation,

$$\tilde{\sigma}_{ij} = M_{ijkl}\,\sigma_{kl}, \qquad (6)$$

$$M_{ijkl} = \tfrac{1}{2}(\omega_{ik}\delta_{jl} + \omega_{jk}\delta_{il}),\;\; \omega_{ik}(\delta_{kj} - D_{kj}) = \delta_{ij}. \qquad (7)$$

Damage rate: In a small strain Lagrangian framework,

$$\dot{D}_{ij} = \begin{cases} f_{ij}, & \text{if} \quad \max\{\epsilon_{ij}\} \geq \epsilon_{th}, \\ 0, & \text{if} \quad \max\{\epsilon_{ij}\} < \epsilon_{th}. \end{cases} \qquad (8)$$

Damage evolution function: The generalized form is,

$$f_{ij} = B\langle\chi\rangle^r\left(\omega_{mn}\xi^{(1)}_m\xi^{(1)}_n\right)^{k_\sigma}\left[(1-\gamma)\,\delta_{ij} + \gamma\xi^{(1)}_i\xi^{(1)}_j\right], \qquad (9)$$

$$\chi = \alpha\tilde{\sigma}^{(1)} + \beta\sqrt{\frac{3}{2}\tilde{\sigma}^{\mathrm{dev}}_{mn}\tilde{\sigma}^{\mathrm{dev}}_{mn}} + (1-\alpha-\beta)\tilde{\sigma}_{kk}. \qquad (10)$$

Tension-compression asymmetry: The different behavior of ice under compression and tension is captured by $k_{\boldsymbol{\sigma}}$ as,

$$k_{\boldsymbol{\sigma}} = \begin{cases} [k_1 + k_2|\sigma_{ii}|], & \text{for} \quad 0 \leq \sigma_{ii} \leq 1\,\mathrm{MPa}, \\ -[k_3 + k_4|\sigma_{ii}|], & \text{for} \quad -3\,\mathrm{MPa} \leq \sigma_{ii} < 0. \end{cases} \qquad (11)$$

Temperature dependence: The relation for B is,

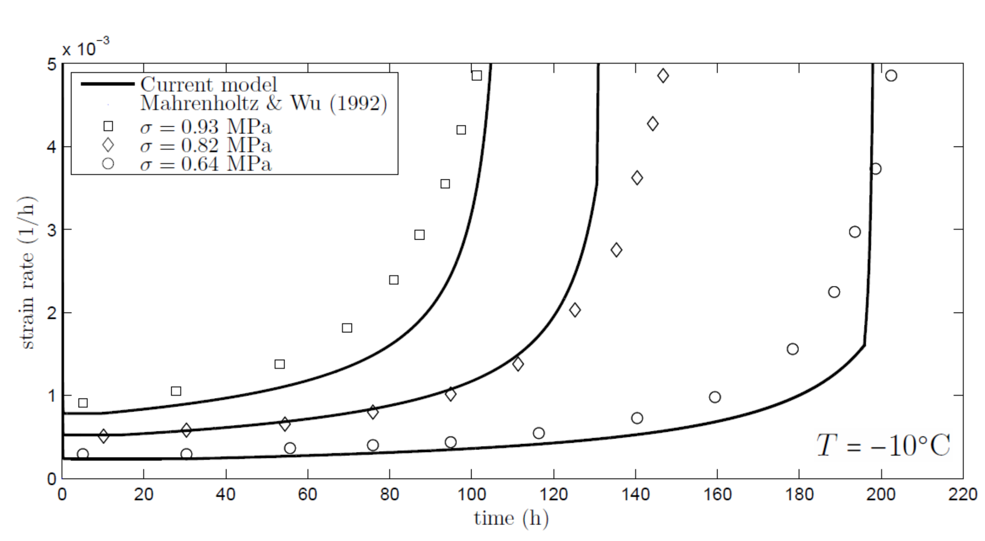
$$B(T) = B(T_{\mathrm{m}})\exp\left(\frac{-U}{R}\left(\frac{1}{T} - \frac{1}{T_{\mathrm{m}}}\right)\right). \qquad (12)$$

Nonlocal damage rule: The nonlocal damage rate at $\boldsymbol{X}^k$ is [2],

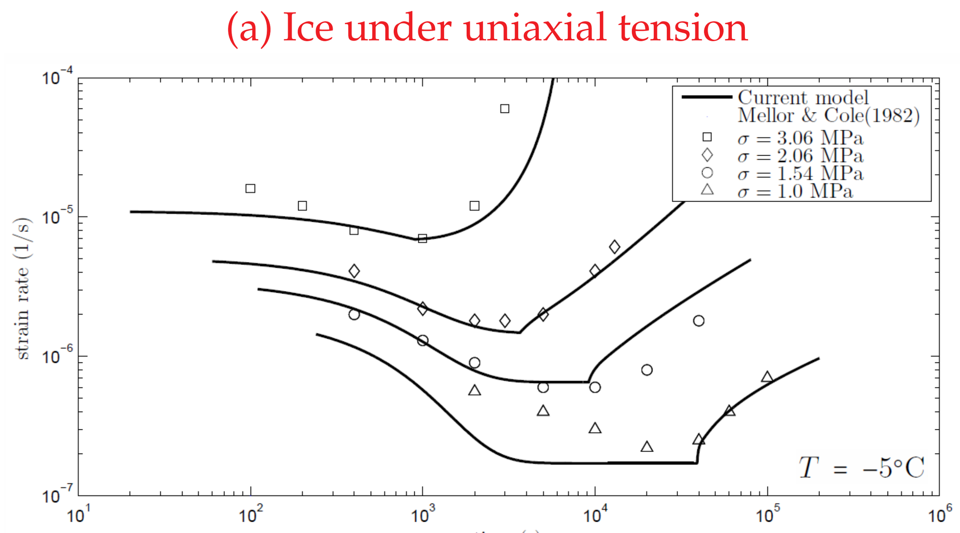
$$\dot{D}^{\mathrm{nl}}_{ij}(\boldsymbol{X}^k) = \frac{\sum_{l=1}^{N_{\mathrm{GP}}}\Phi(\boldsymbol{X}^k,\boldsymbol{X}^l)\dot{D}_{ij}(\boldsymbol{X}^l)}{\sum_{l=1}^{N_{\mathrm{GP}}}\Phi(\boldsymbol{X}^k,\boldsymbol{X}^l)}, \qquad (13)$$

$$\Phi(\boldsymbol{X}^k,\boldsymbol{X}^l) = \exp\left(\frac{2\|\boldsymbol{X}^k - \boldsymbol{X}^l\|^2}{l_c^2}\right). \qquad (14)$$

5. Model Calibration

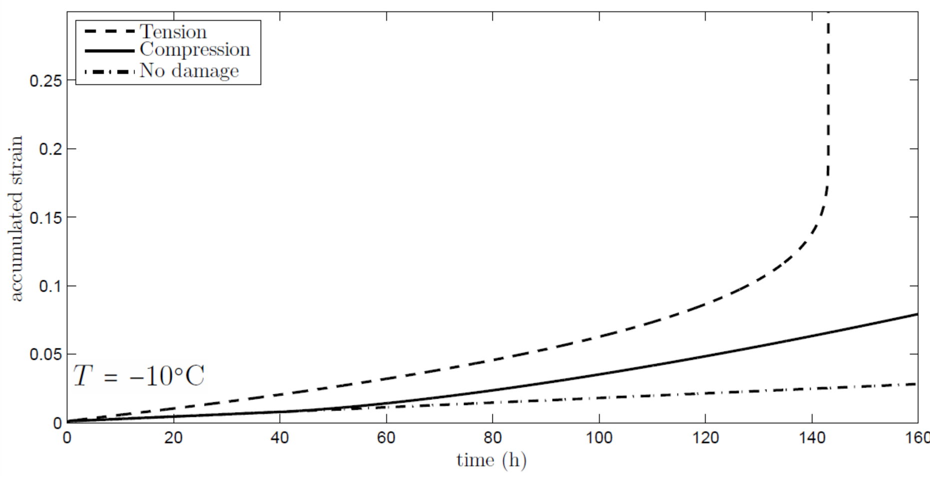


(a) Ice under uniaxial tension

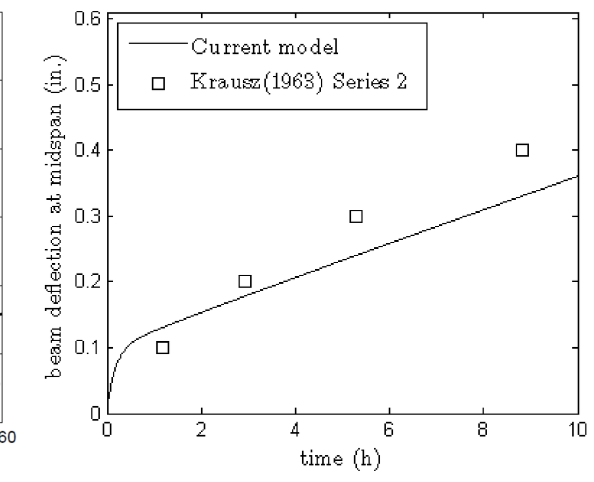


(b) Ice under uniaxial compression

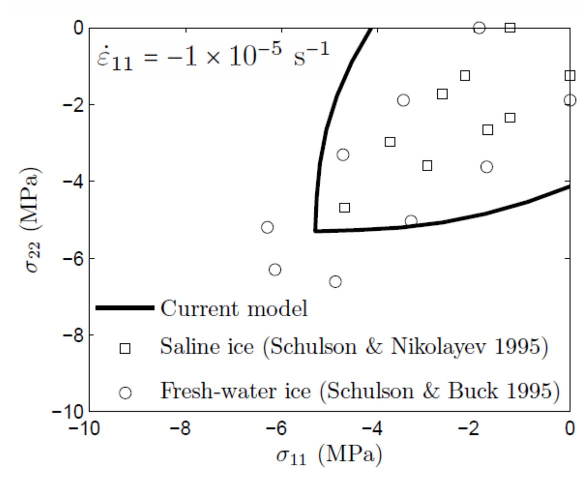
6. Model Validation



(d) Uniaxial loading $|\sigma| = 0.8$ MPa



(e) Four point bending

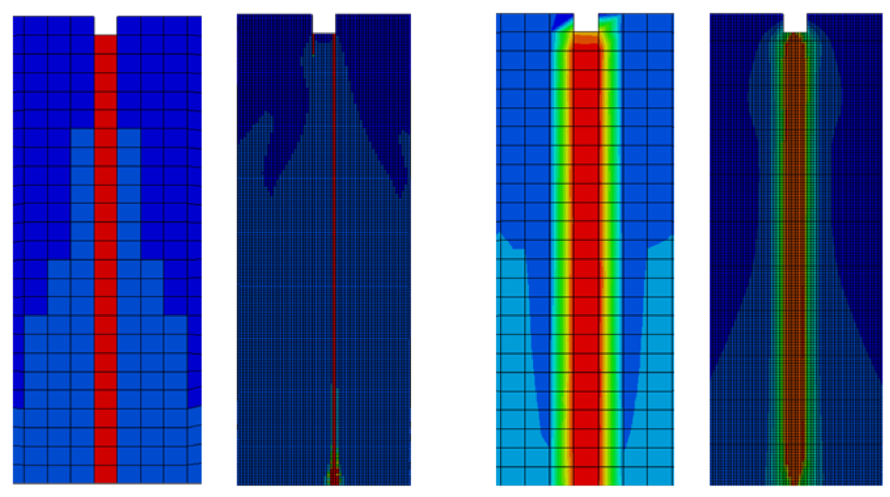


(f) Biaxial compression

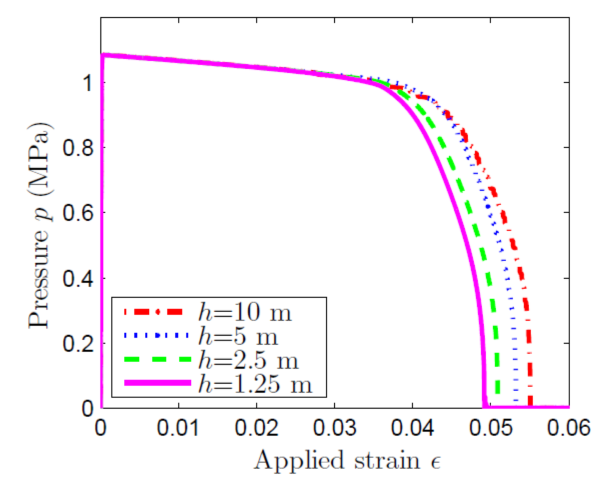
The above figures show the numerical results obtained for an ice slab subjected to various loading conditions. From figure (d), it is evident that after 140 hours ice under uniaxial tension exhibits sudden rupture whereas no such failure is observed under uniaxial compression, even at a later time. Figure (e) shows the good agreement between model results and experimental data for the four point bending test in the initial stages of creep. From figure (f), it is clear that the model is able to capture the increase in ice strength under biaxial compression that is observed experimentally, quite well.

7. Numerical Results and Discussion

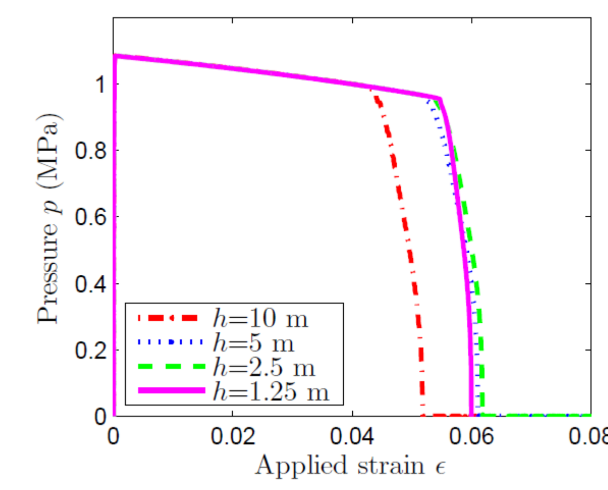
Figures (g)–(i) show the simulated crack paths for **pure mode I creep fracture** of an ice slab subjected to a uniform applied tensile strain. The local damage model constricts the damage zone to be one element wide. As we decrease the mesh size h the damage zone width decreases and the collapse occurs early at lesser strain, which is not physical. The nonlocal integral damage model introduces a material length scale l_c, yielding thermodynamically consistent numerical results.



(g) Local vs. nonlocal damage

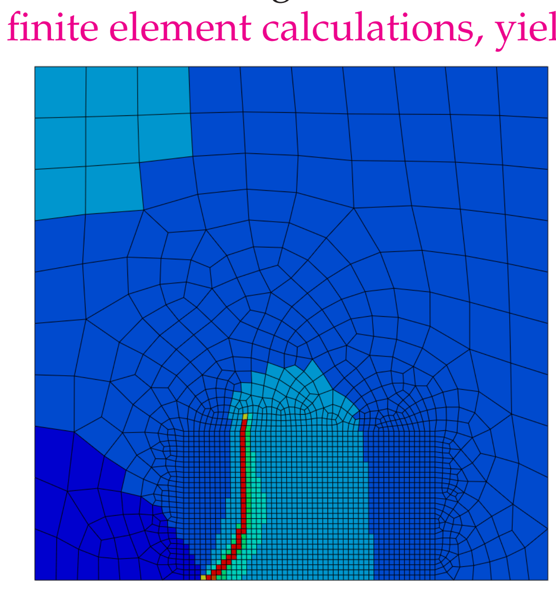


(h) Local damage

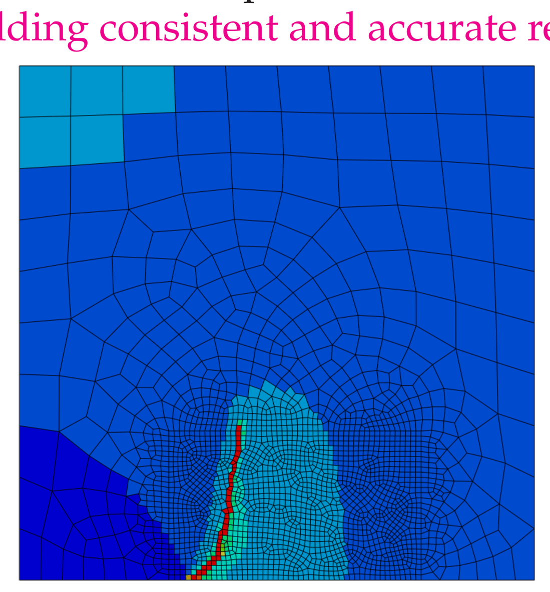


(i) Nonlocal damage, $l_c = 7$ m

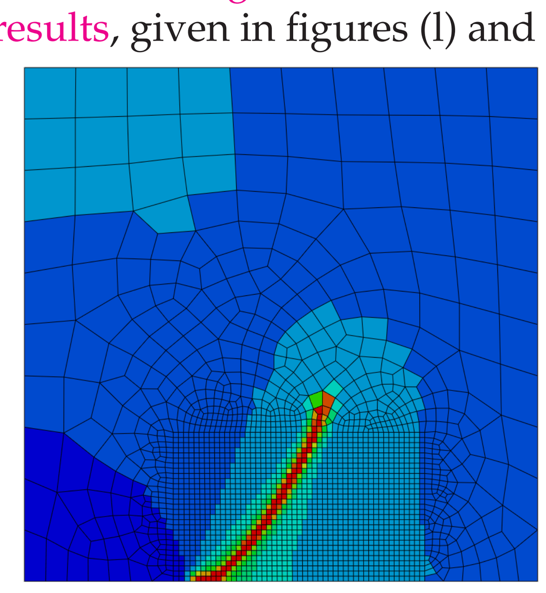
Figures (j)–(m) show the simulated crack paths for **mixed mode creep fracture** of a square ice plate with a center crack subjected to biaxial tension. Only 1/4th of the domain is modeled due to symmetry. It is clear from figures (j) and (k) that the local damage model results are mesh dependent. The nonlocal damage model alleviates the directional mesh bias of finite element calculations, yielding consistent and accurate results, given in figures (l) and (m).



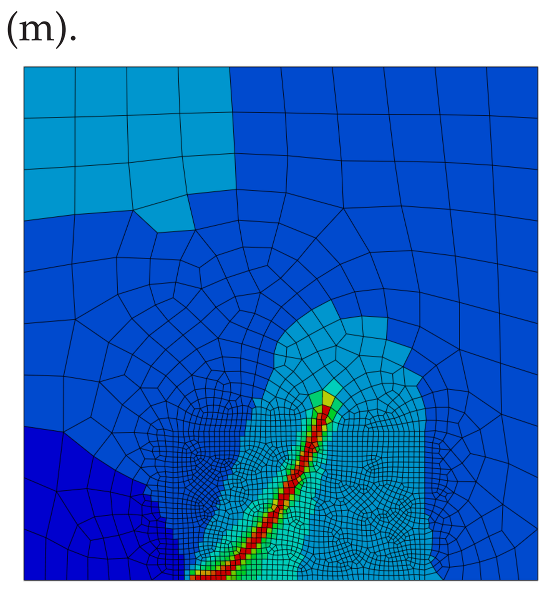
(j) Local, structured



(k) Local, unstructured

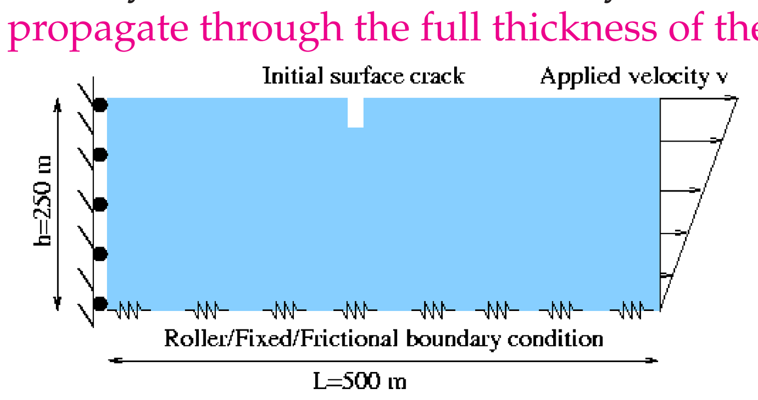


(l) Nonlocal, structured

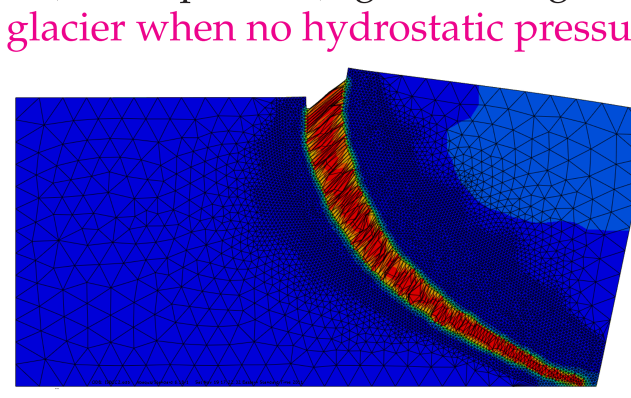


(m) Nonlocal, unstructured

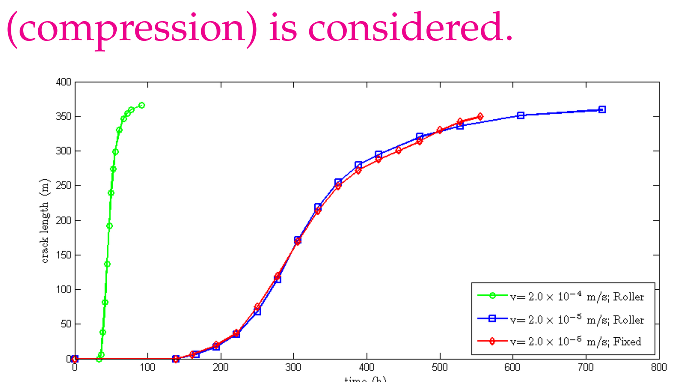
Next, we use the nonlocal damage model to simulate **crevasse formation in idealized glaciers** (ice slabs) considering depth varying ice flow velocity profiles. The set up of the simulation is given in Figure (n) and the predicted fracture pathway for v$= 2.0 \times 10^{-4}$ m/s is shown in Figure (o). Next, we estimate the crack propagation rates by varying the flow velocity and the bottom boundary condition (free slip/fixed), given in Figure (p). The results indicate that fractures can propagate through the full thickness of the glacier when no hydrostatic pressure (compression) is considered.



(n) Schematic diagram

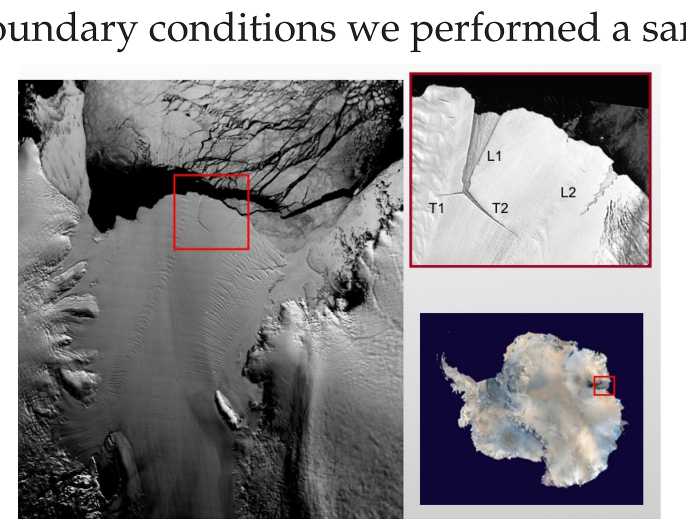


(o) v$= 2.0 \times 10^{-4}$ m/s

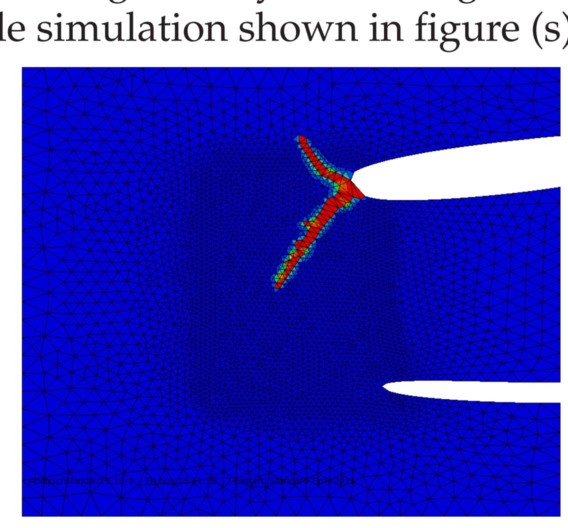


(p) Crack length vs time

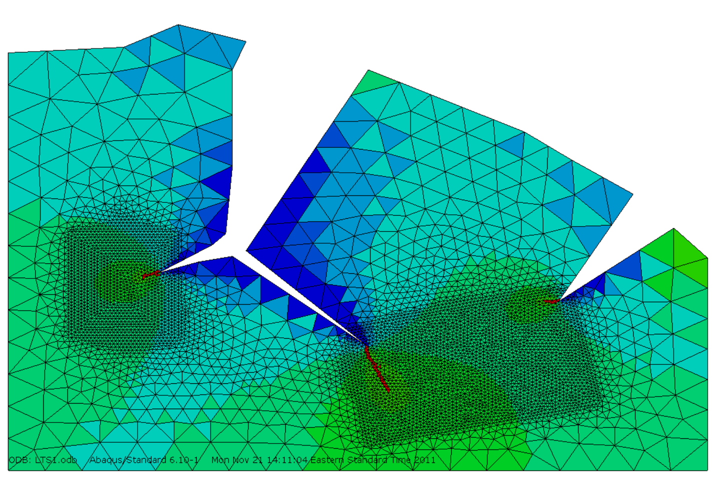
Figure (q) shows the satellite image of the **loose tooth system in the Amery ice shelf in Antarctica**. Figure (r) shows the numerical results obtained for the ice shelf approximated as a square plate with two existing cracks subjected to biaxial tension. Owing to the applied boundary conditions we can see the crack forking at the tip resembling the loose tooth system. Next, we meshed an extract of the actual geometry containing the loose tooth system and by assuming idealized boundary conditions we performed a sample simulation shown in figure (s).



(q) Loose tooth system, Amery ice shelf



(r) Idealized simulation



(s) Simulation using the actual geometry

8. Conclusions

- The constitutive model calibrated using uniaxial test data gives good results under multiaxial loading.
- The nonlocal damage formulation alleviates mesh dependence and yields thermodynamically consistent results.
- Numerical results demonstrate the viability of the proposed formulation for studying iceberg calving and rift propagation.
- In future, we shall model the influence of sea water level on crevasse formation so as to evaluate glacier stability.

9. Acknowledgements

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10. References

[1] R. Duddu and H. Waisman. *A temperature dependent creep damage model for polycrystalline ice.* Mech. Mat., 46 : 23-41, 2012.

[2] R. Duddu and H. Waisman. *A nonlocal continuum damage mechanics approach to simulation of creep fracture in ice sheets.* Comp. Mech., under review.