GPU-based Algorithms for cutting deformable objects in implicit simulations

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1 Introduction

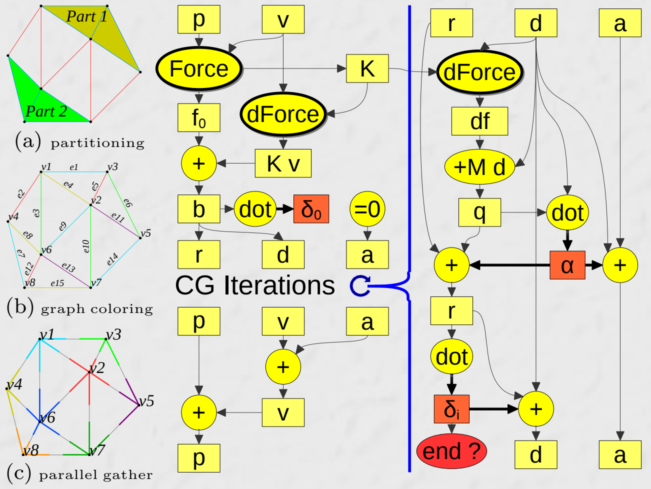
We present a series of contributions in the field of real-time simulation of soft tissue biomechanics. These contributions are based on the FEM Corotational model, and we rely on GPU implementations to enable real time computations.

2 GPU-Based Conjugate gradient

Implicit time integration allows large time steps for arbitrarily stiff objects, but it requires to solve a linear system :

$$\mathbf{A}x = b \tag{1}$$

Where $\mathbf{A}$ changes at each time step. We solve this system with a Conjugate Gradient algorithm (CG) that has been implemented on GPU. We parallelized the matrix operations directly on the original object mesh [1], which avoids the assembly of the matrix, and reduces consumed bandwidth.



3 Preconditioning techniques

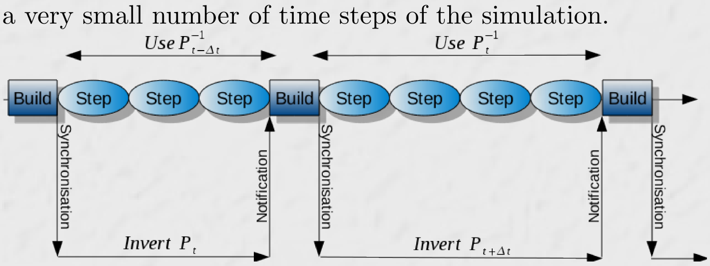
One critical issue when using iterative solvers is the choice of an efficient preconditioner:

$$\mathbf{P}\mathbf{A}x = \mathbf{P}b, \quad \text{such as} \quad \mathbf{P}\mathbf{A} \approx \mathbf{I} \tag{2}$$

We proposed to desynchronise the computation of the preconditioner from the simulation loop [2], using a separate CPU thread. This approach allows to use in a real-time context, prohibitively expensive preconditioners such as an exact factorization of the system:

$$\mathbf{A} = \mathbf{LDL}^T \tag{3}$$

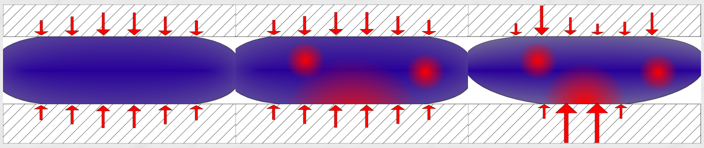
The resulting lower-triangular matrix $\mathbf{L}$ remains sparse, which enables the method to be used with fine meshes. With an optimized library, the factorization can be updated after a very small number of time steps of the simulation.



4 Preconditioner for contacts

To simulate contacts we rely on Signorini's law to build a Linear Complementary Problem (LCP):

$$\vec{\delta} = \mathbf{J}\mathbf{A}^{-1}\mathbf{J}^T\vec{\lambda} + \vec{\delta}_0 \quad \text{with} \quad 0 \leq \vec{\delta} \perp \vec{\lambda} \geq 0 \tag{4}$$



The computation of $\mathbf{A}^{-1}$ is very time consuming. Our proposition is to use the preconditioner as an approximation of this matrix [3]:

$$\mathbf{J}\mathbf{A}^{-1}\mathbf{J}^T \approx \mathbf{J}\left(\mathbf{LDL}^T\right)^{-1}\mathbf{J}^T \tag{5}$$

We compute this matrix in two steps:

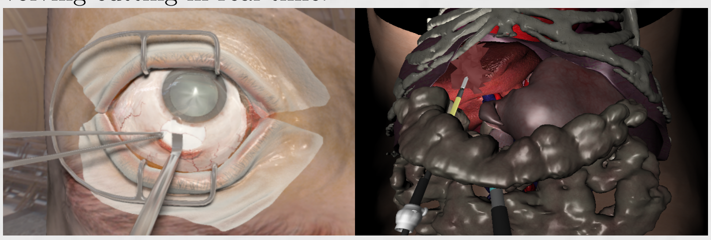
$$\mathbf{S} = \left(\mathbf{LDL}^T\right)^{-1}\mathbf{J}^T \tag{6}$$

$$\mathbf{W} = \mathbf{J}\ \mathbf{S} \tag{7}$$

The computation of each column of $\mathbf{S}$ is obtained by soling two sparse triangular systems, which is equivalent to an application of the preconditioner. However, each column can be processed in parallel, and we proposed a parallel approach to process multiple sparse triangular solve on GPU.

5 Simulation of cutting

Cutting involves topological modifications, which dramatically changes the spectrum of $\mathbf{A}^{-1}$ and affects the convergence rate of the Krylov algorthims. We propose a method to keep track of the evolving cut, and apply appropriate low-rank modifications of the preconditioner at each time step. Finally, we managed to simulate several medical surgery involving cutting in real time.



References

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[2] H. Courtecuisse, J. Allard, C. Duriez, and S. Cotin. Asynchronous preconditioners for efficient solving of non-linear deformations. In *Virtual Reality Interactions and Physical Simulations (VRIPhys)*, 2010.

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Acknowledgement

The European Research Council Starting Independent Research Grant (ERC Stg grant agreement No. 279578) entitled "Towards real time multiscale simulation of cutting in non-linear materials with applications to surgical simulation and computer guided surgery" is gratefully acknowledged.
We also thank the NSF for the fellowship given by Sukumar.

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