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On the shape-preserving property of barycentric coordinates

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Summary

Barycentric coordinates are often used in cage-based space deformation. **Shape-preserving** is an important property of space deformation. We review some barycentric coordinates with emphasis on their shape-preserving property and we try to improve them.

Review

Given a cage P with vertices $V = \{\mathbf{v}_i\}_{i \in I_V}$ and faces $T = \{t_j\}_{j \in I_T}$.
Let P' be the deformed cage with deformed vertices $V' = \{\mathbf{v}'_i\}_{i \in I_V}$ and faces $T' = \{t'_j\}_{j \in I_T}$,

Traditional barycentric coordinates

E. g.: Mean Value Coordinates (MVC) and Harmonic Coordinates.
Express a point $\mathbf{x}$ inside P as

$$\mathbf{x} = F(\mathbf{x}; P) = \sum_{i \in I_V} \varphi_i(\mathbf{x}) \mathbf{v}_i$$

with suitable coordinate functions $\varphi_i(\cdot)$.
The deformation is defined by

$$\mathbf{x} \mapsto \mathbf{x}' = F(\mathbf{x}; P') = \sum_{i \in I_V} \varphi_i(\mathbf{x}) \mathbf{v}'_i \,.$$

Properties: interpolatory, not shape-preserving

Green Coordinates (GC)

Express a point $\mathbf{x}$ inside P as

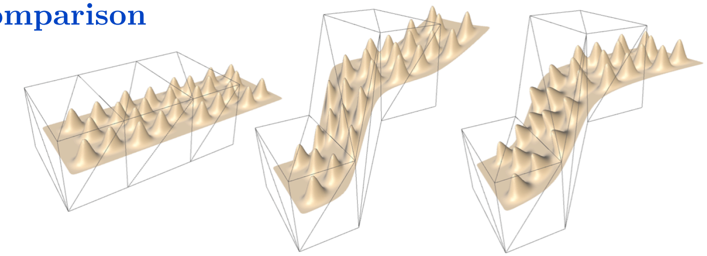
$$\mathbf{x} = F(\mathbf{x}; P) = \sum_{i \in I_V} \phi_i(\mathbf{x}) \mathbf{v}_i + \sum_{j \in I_T} \psi_j(\mathbf{x})\, \mathbf{n}(t_j)$$

with suitable functions $\phi_i(\cdot)$ and $\psi_j(\cdot)$ and outward normal function $\mathbf{n}(\cdot)$.
The deformation is defined by

$$\mathbf{x} \mapsto \mathbf{x}' = F(\mathbf{x}; P') = \sum_{i \in I_V} \phi_i(\mathbf{x}) \mathbf{v}'_i + \sum_{j \in I_T} \psi_j(\mathbf{x})\, s_j\, \mathbf{n}(t'_j) \,.$$

Properties: shape-preserving, not interpolatory

A comparison



Original model — Deformed model using MVC — Deformed model using GC

From [Lipman et al., SIGGRAPH 2008, Green Coordinates, Fig. 3].

Improved barycentric coordinates

1. Refine the cage P and get $\bar{P}$ with vertices $\bar{V} = \{\bar{\mathbf{v}}_i\}_{i \in I_{\bar{V}}} \supseteq V$ and faces $\bar{T} = \{\bar{t}_j\}_{j \in I_{\bar{T}}}$.
2. Express a point $\mathbf{x}$ inside P as

   $$\mathbf{x} = F(\mathbf{x}; \bar{P}) = (1 - \alpha) \sum_{i \in I_{\bar{V}}} \phi_i(\mathbf{x}) \bar{\mathbf{v}}_i + \alpha \sum_{j \in I_{\bar{T}}} \psi_j(\mathbf{x})\, \mathbf{n}(\bar{t}_j)$$

   with suitable functions $\phi_i(\cdot)$, $\psi_j(\cdot)$, outward normal function $\mathbf{n}(\cdot)$ and parameter $\alpha \in [0,\ 1]$.
3. Compute the deformed cage $\bar{P}'$ with vertices $\bar{V}' = \{\bar{\mathbf{v}}'_i\}_{i \in I_{\bar{V}}}$ and faces $\bar{T}' = \{\bar{t}'_j\}_{j \in I_{\bar{T}}}$:
   - for *constrained vertices* $\bar{\mathbf{v}}_i$, i. e., $\bar{\mathbf{v}}_i \in V$ and $\bar{\mathbf{v}}'_i$ is to be interpolated: set

     $$\bar{\mathbf{v}}'_i = \mathbf{v}'_i \,.$$

   - for all other *unconstrained vertices* $\bar{\mathbf{v}}_i$: compute $\bar{\mathbf{v}}'_i$ by minimizing some energy, e. g., stretching energy + bending energy.
4. The deformation is defined by

   $$\boxed{\mathbf{x} \mapsto \mathbf{x}' = F(\mathbf{x}; \bar{P}') = (1 - \alpha) \sum_{i \in I_{\bar{V}}} \phi_i(\mathbf{x}) \bar{\mathbf{v}}'_i + \alpha \sum_{j \in I_{\bar{T}}} \psi_j(\mathbf{x})\, s_j\, \mathbf{n}(\bar{t}'_j) \,.}$$

Advantage: shape-preserving, interpolatory (if $\alpha = 0$)
Disadvantage: In step 3 a nonlinear optimization may occur, which may not converge and may increase computing time.