Università della Svizzera italiana

Faculty of Informatics

Blended barycentric coordinates

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Introduction

Given an arbitrary polygon P with vertices $v_1, \dots, v_n$, we can write any point $v \in \mathbb{R}^2$ as an affine combination of these vertices

$$v = \sum_{i=1}^{n} b_i(v) v_i \qquad \text{with} \qquad \sum_{i=1}^{n} b_i(v) = 1$$

The weights $b_i(v)$ are the *barycentric coordinates* of v.

Types of coordinates

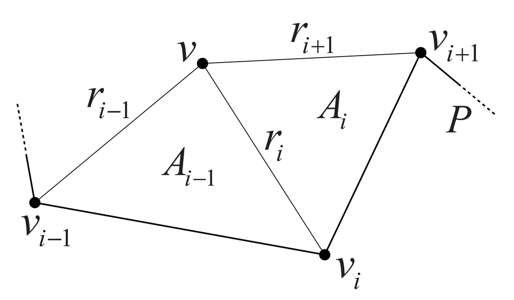
We can define three different types of coordinates by specifying the weight functions

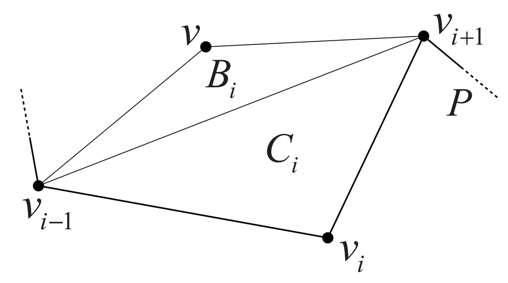
$$w_i = d_{i-1}A_{i-2} - d_i B_i + d_{i+1}A_{i+1}$$

and choosing functions d_i to be

$$d_i^{WP} = \frac{1}{A_{i-1}A_i} \qquad d_i^{MV} = \frac{r_i}{A_{i-1}A_i} \qquad d_i^{Met} = \frac{1}{C_i q_{i-1} q_i}$$

where $q_i = r_i + r_{i+1} - \|v_{i+1} - v_i\|$ and





Finally by normalizing weight functions w_i as follows

$$b_i = \frac{w_i}{W}, \qquad W = \sum_{i=1}^{n} w_i$$

we get coordinates b_i.

Shared properties

1. Affine precision
2. Partition of unity
3. Lagrange property
4. Linearity along edges
5. Simple closed form

Task

To obtain barycentric coordinates that are well-defined everywhere, have at least C^1- continuity at the vertices and have a simple closed form.

Blended WP and MV coordinates

To reach the goal we can blend Wachspress and Mean Value coordinates. They have all necessary properties except C^1- continuity if three or more vertices are collinear.

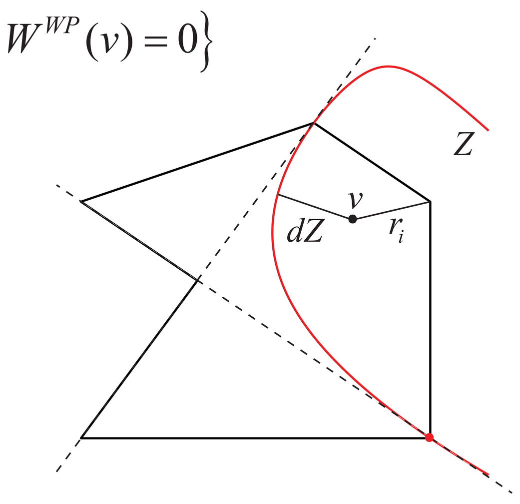
$$b_i^{BL} = \mu b_i^{WP} + (1-\mu) b_i^{MV}$$

$$\mu = \frac{dZ^2}{dZ^2 + dV^2}$$

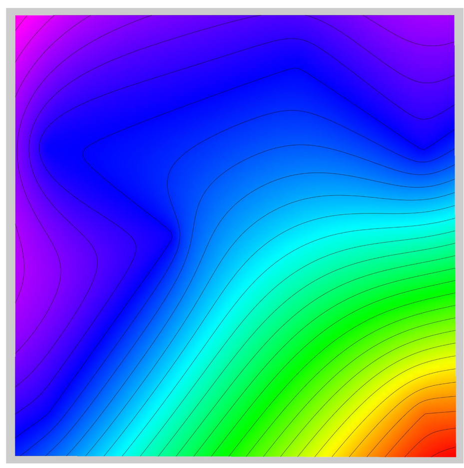
where

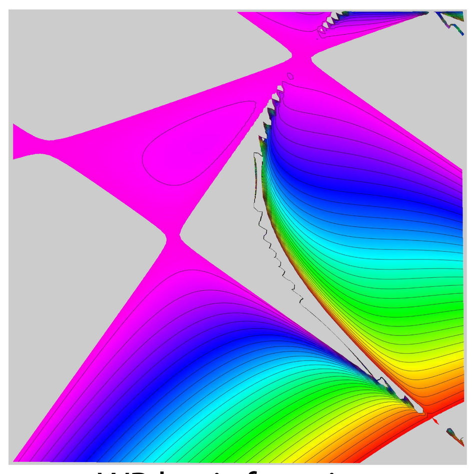
$$dZ = dist(v, Z) = \frac{W^{WP}}{\nabla W^{WP}}, \quad Z = \{v : W^{WP}(v) = 0\}$$

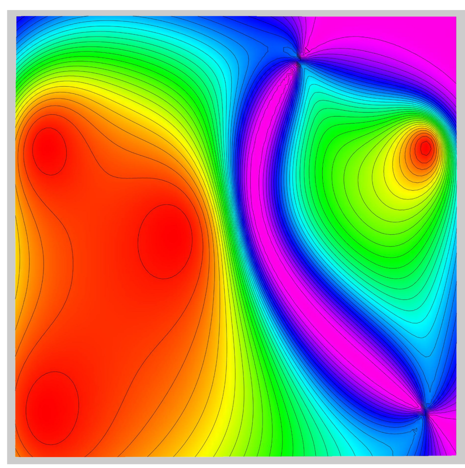
$$dV = \frac{R}{\nabla W^{WP}}, \quad R = \prod_{i=1}^{n} r_i$$

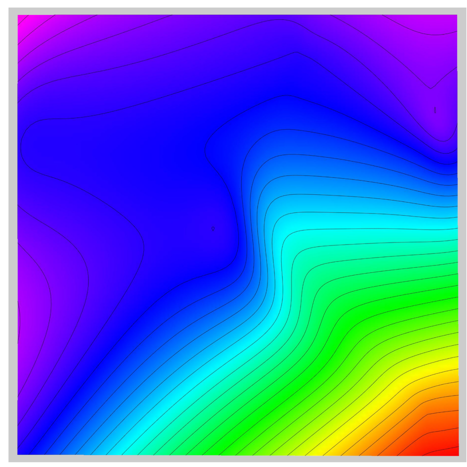


Then


MV basis function


WP basis function


Blend function μ


Blended basis function

Non shared properties

Type of coordinates	**Poles**	**Positivity inside convex polygons**	**Continuity at the vertices**	**Invariance**
$b_i(v)$	$W(v) = 0$	$b_i(v) \geq 0$	C^m	If $P' = \varphi(P)$ then $b_i(v) = b_i'(\varphi(v))$
Wachspress	Yes	Yes	C^{1*}	φ - affine
Mean Value	No	Yes	C^0	φ - similarity
Blended WP and MV	No	Yes	C^{1**}	φ - similarity
Metric	No	No	C^1	φ - similarity
Blended MV and Metric***	No	Yes	C^1	φ - similarity

* Only for convex polygons. For concave polygons they are discontinuous at the zero set of W.
** These coordinates are C^1 continuous only if P does not contain 3 or more collinear vertices.
*** Future work.

Future work

By blending Mean Value and Metric coordinates we can avoid the problem with collinear vertices.

References

[1] K. Hormann, M. S. Floater: Mean value coordinates for arbitrary planar polygons. ACM Transactions on Graphics, 25(4):1424-1441, 2006.

[2] N. Sukumar, E. A. Malsch: Recent advances in the construction of polygonal finite element interpolants. Archives of Computational Methods in Engineering, 13(1):129-163, 2006.

[3] G. Taubin: Distance approximations for rasterizing implicit curves. ACM Transactions on Graphics, 13(1):3-42, 1994.