Representing and Searching Sets of Strings

Antonio Carzaniga

Faculty of Informatics
Università della Svizzera italiana

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Outline

- Radix search
- Ternary search tries

Sets of Strings

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- Several very important applications

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  E.g.,

  - dictionary (of words)
  - symbol table in a compiler
  - all kinds of key-based index
  - …

Symbol Table

- Operations

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  - insert(Key)
  - delete(Key)
  - search(Key)
  - min()
  - max()

Dictionary

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Dictionary

- Operations
  - insert(Key)
  - search(Key)
- No *delete* operation
- Built once and searched many times

Binary Search

```
BINARYSEARCH(A, K)
 1  first = 1
 2  last = length(A)
 3  while first ≤ last
 4        x = ⌈(first + last)/2⌉
 5        if A[x] == K
 6              return TRUE
 7        elseif first == last
 8              return FALSE
 9        elseif A[x] > K
10              last = x − 1
11        else fisrt = x + 1
12  return FALSE
```

```
TREE-SEARCH(T, K)
1  x = T.root
2  while x ≠ NIL and K ≠ x.key
3        if K < x.key
4              x = x.left
5        else x = x.right
6  if x ≠ NIL
7        return TRUE
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- So, the complexity of **BinarySearch**(A, K) is $O(m \log n)$

What About a Hash Table

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1  L = T[h(K)]
2  return LIST-SEARCH(L, K)
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HASH-SEARCH(T, K)
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  - and for the hash functions

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  - so, **StringEquals**$(A[x], K)$ is likely to iterate for nearly m steps
  - problem is, **StringEquals**$(A[x], K)$ is likely to go through the same prefix of K many times
- So, since $m = \Theta(\log N)$, and **BinarySearch**(A, K) uses $\Theta(\log N)$ comparisons each one running in $O(m)$:

$$T(N, m) = O(\log^2 N)$$

A New Data Structure

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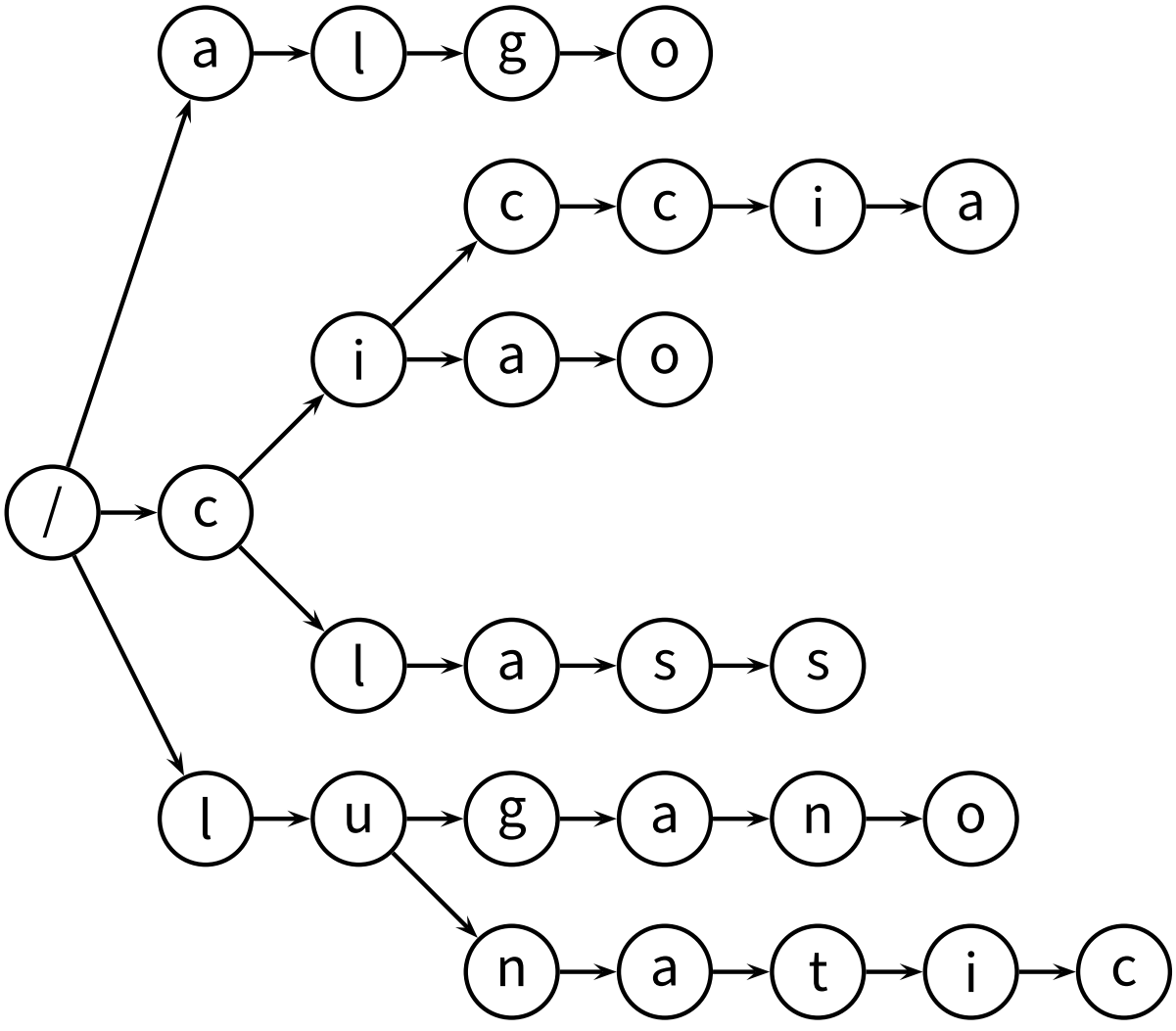

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- *Question:* how do we represent nodes and links?
  - one way would be to hold $|\Sigma|$ links
  - one for each character of the given alphabet Σ

Radix Trie

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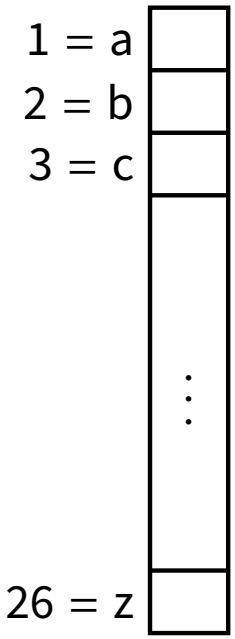

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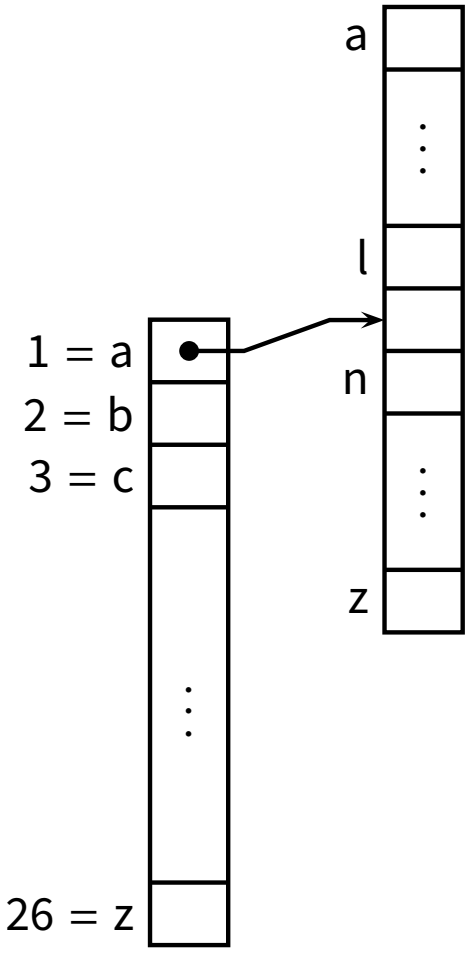

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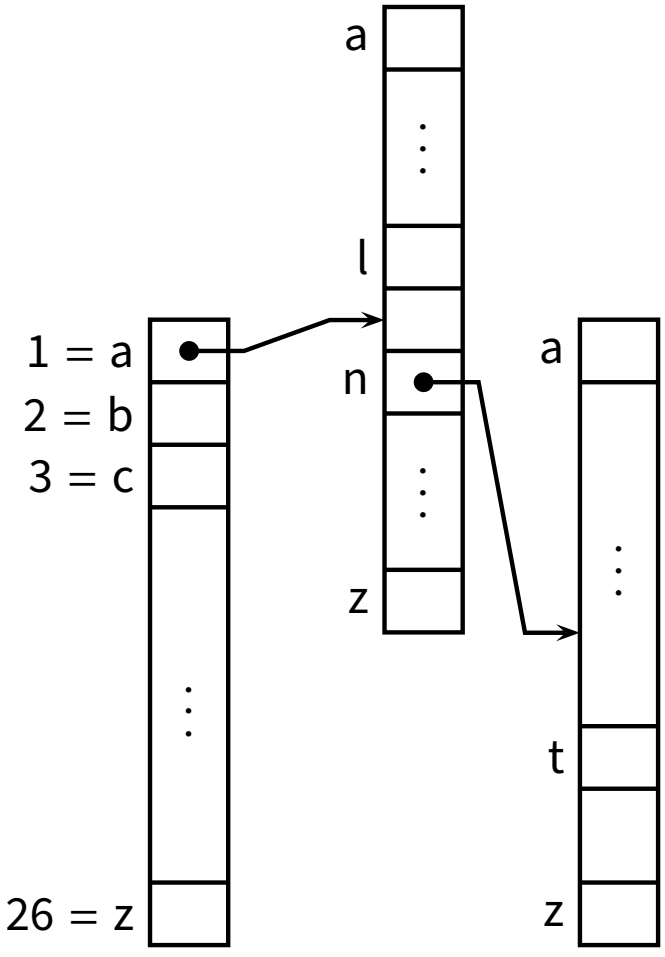

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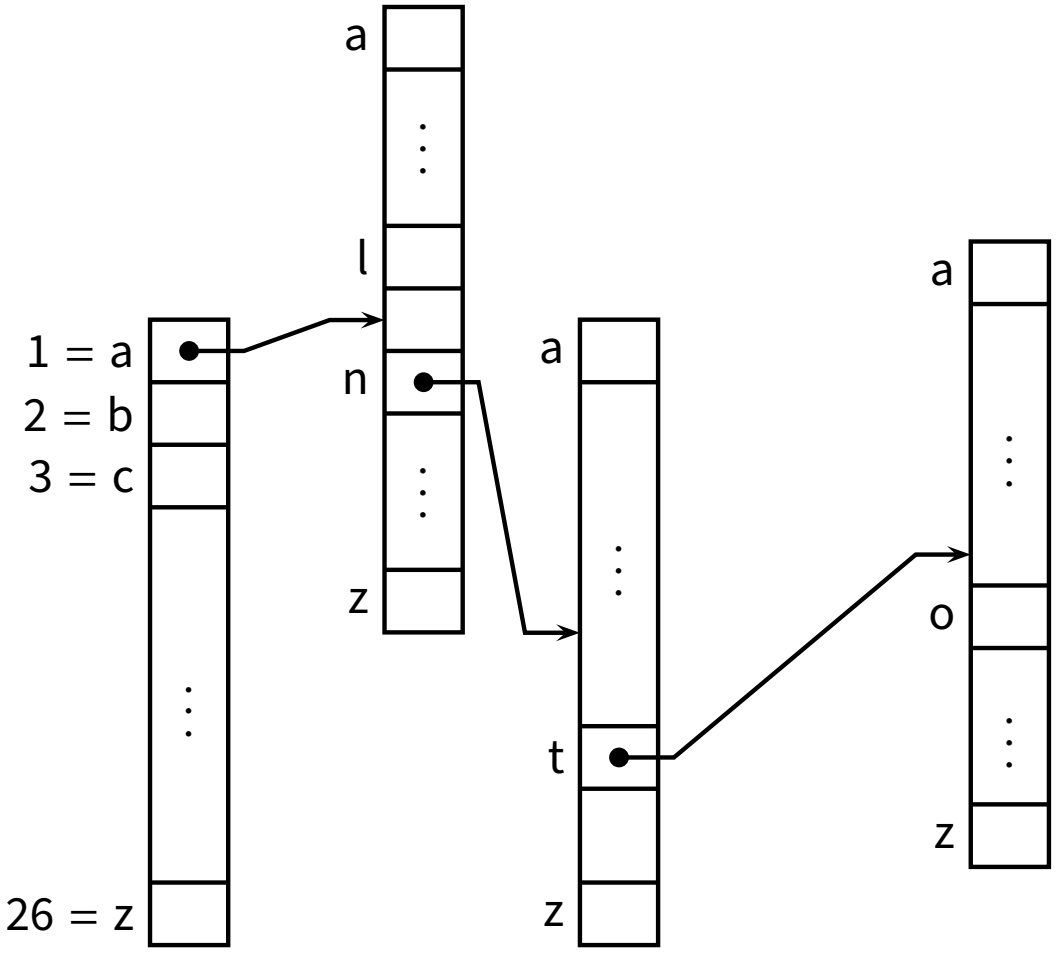

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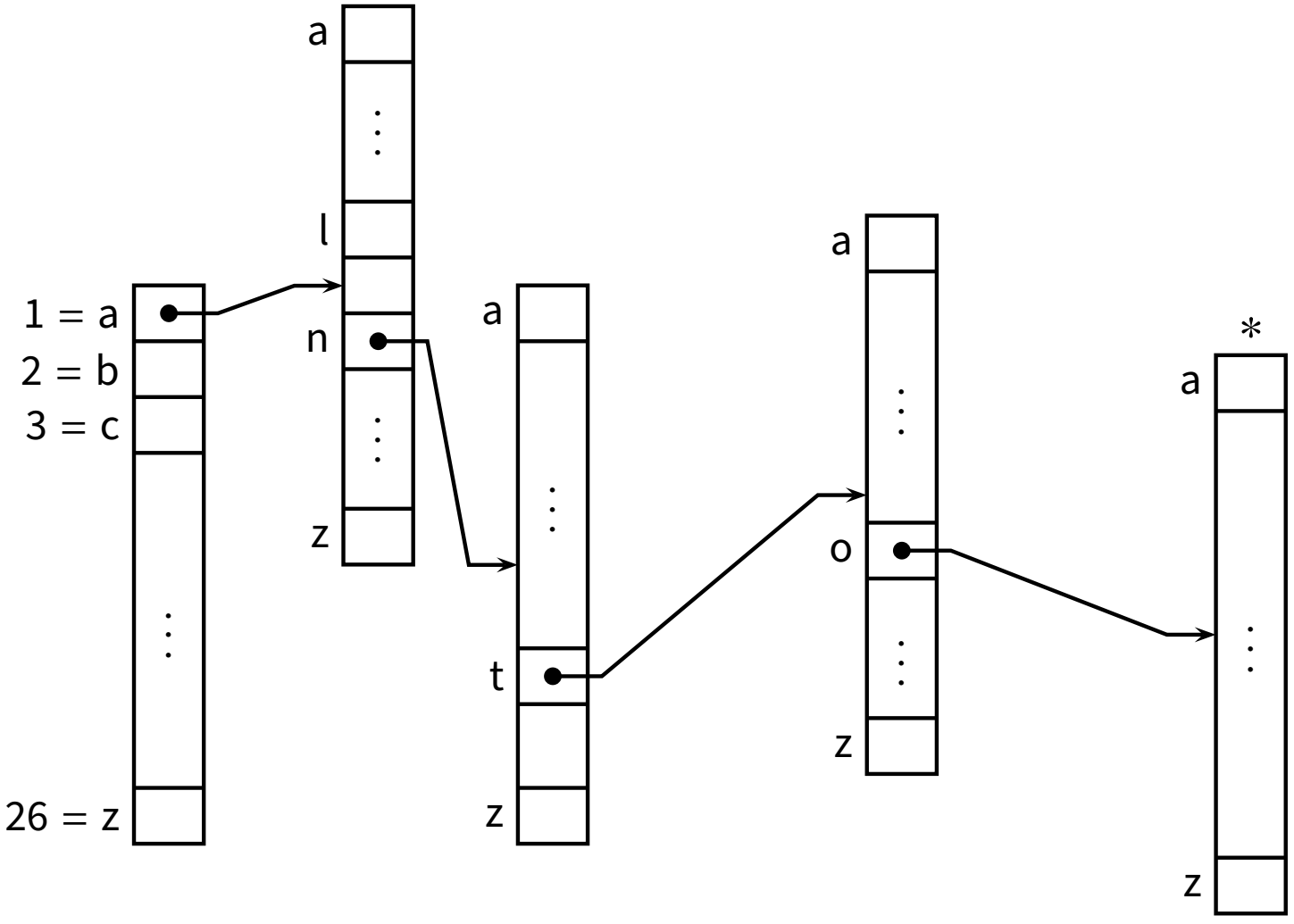

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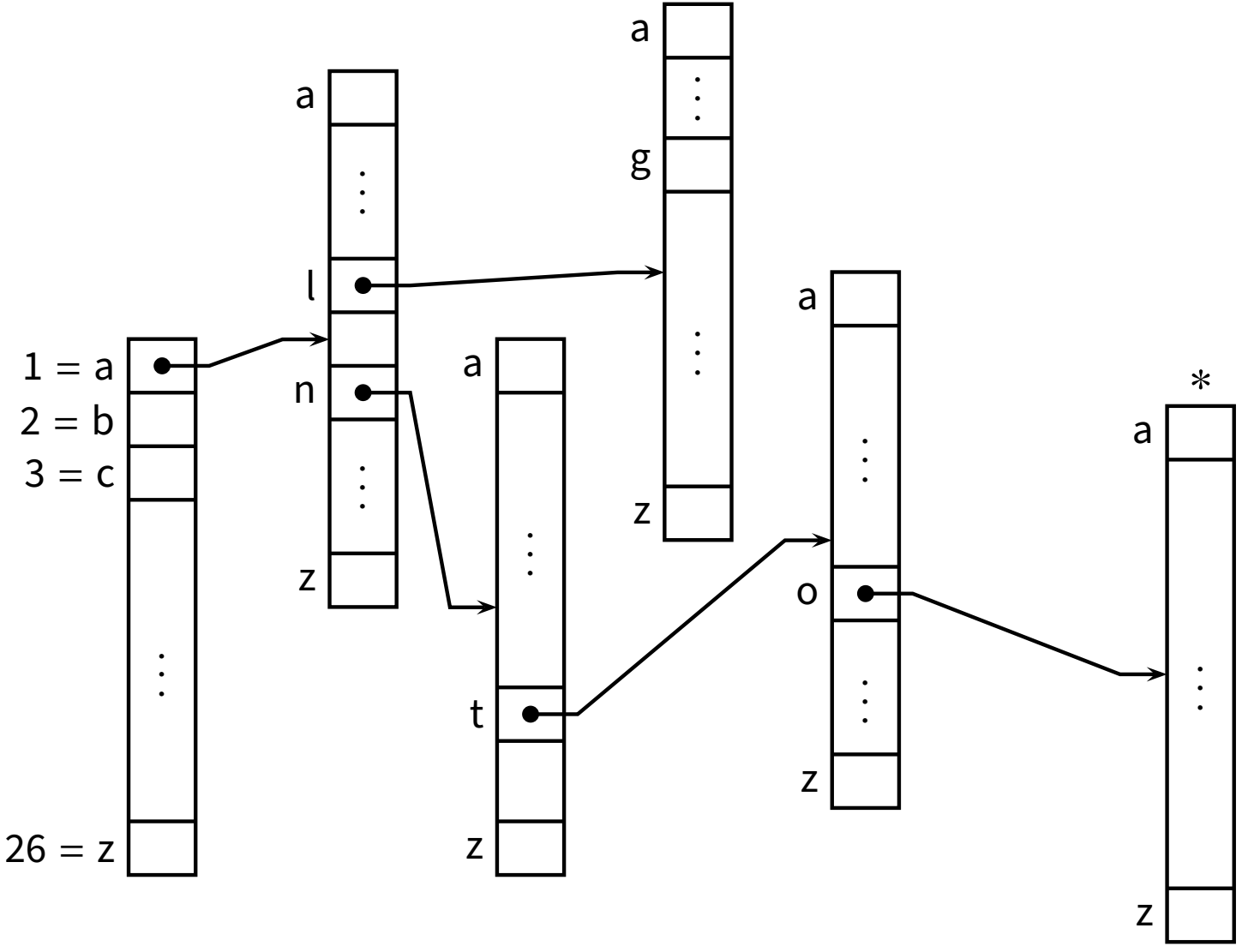

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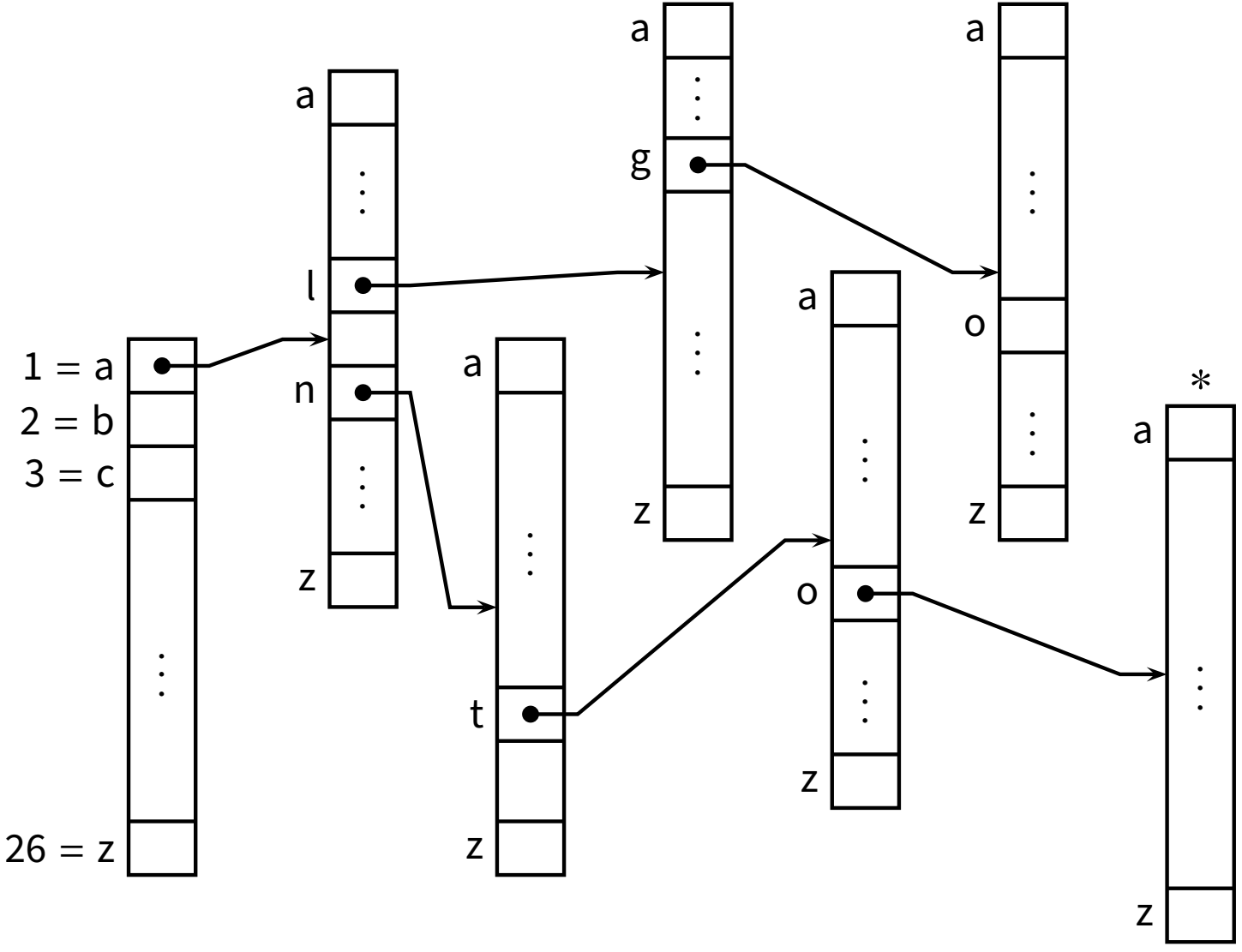

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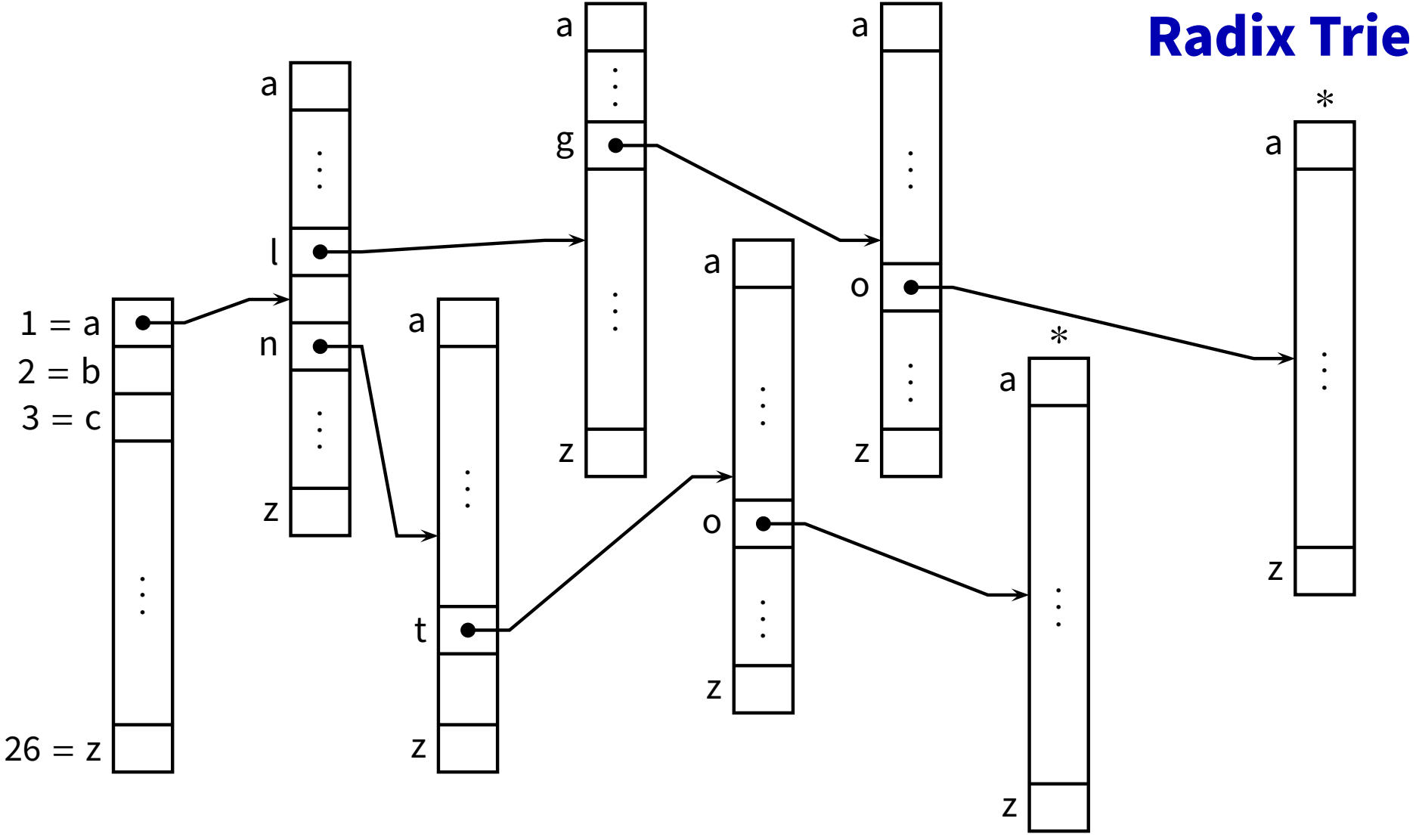

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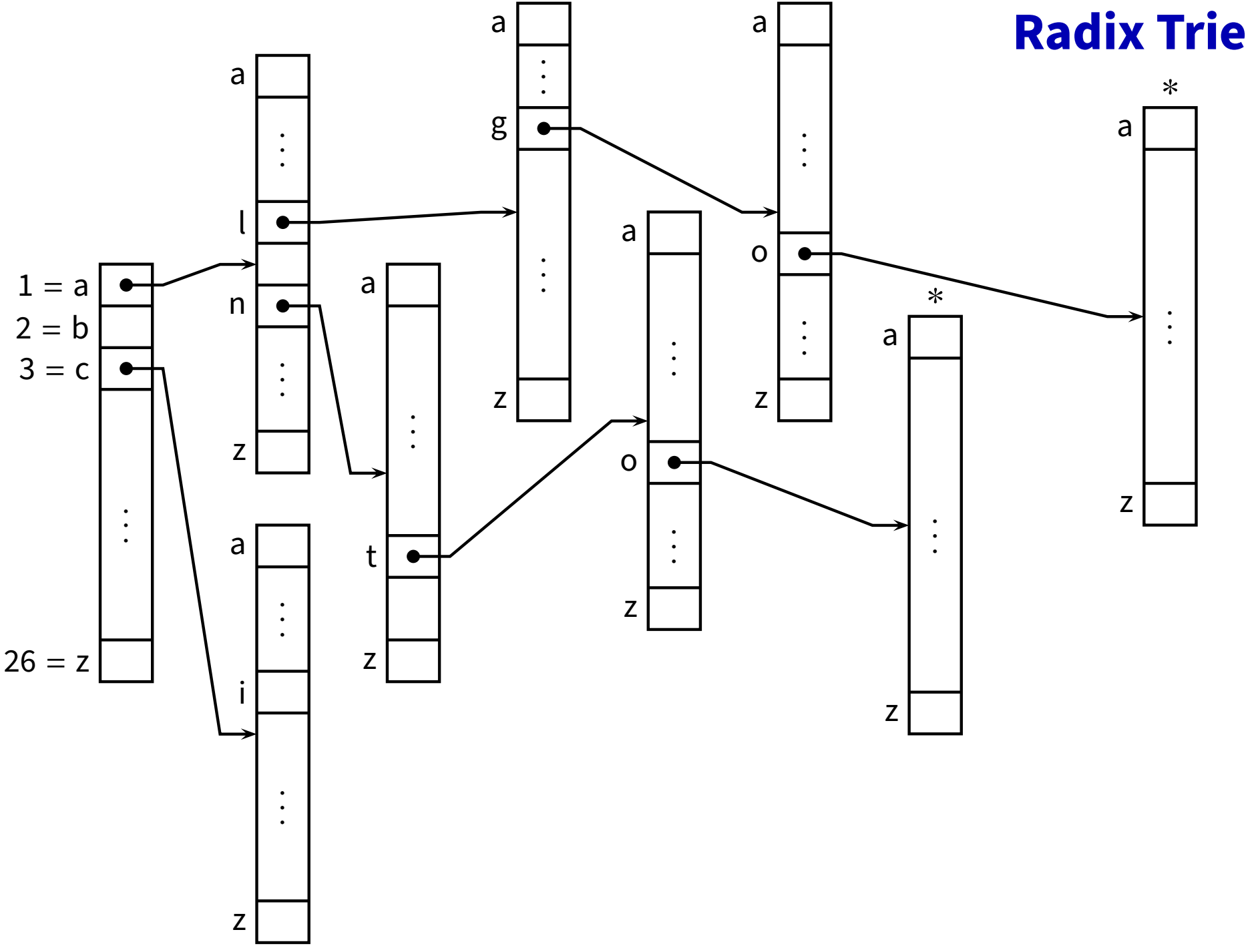

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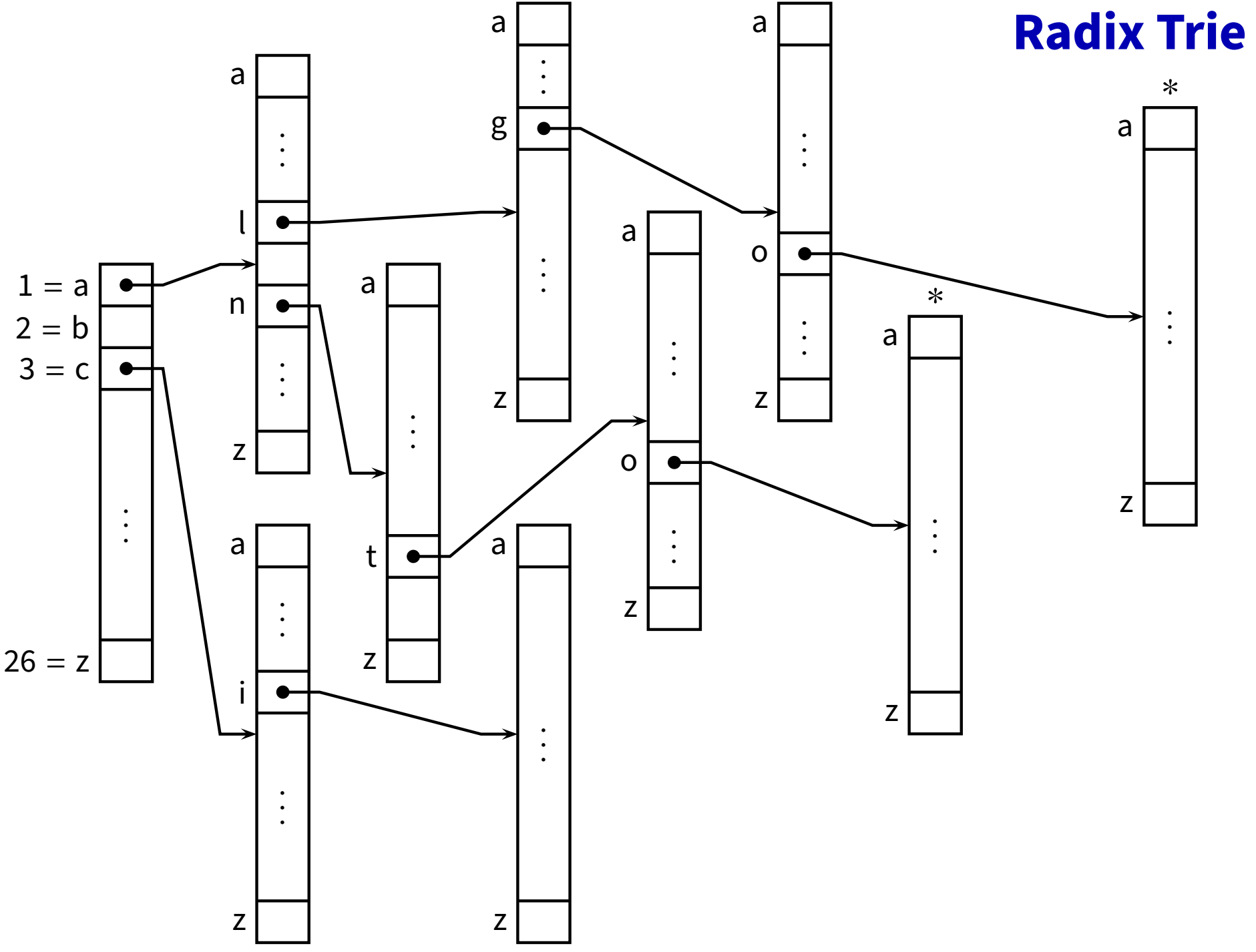

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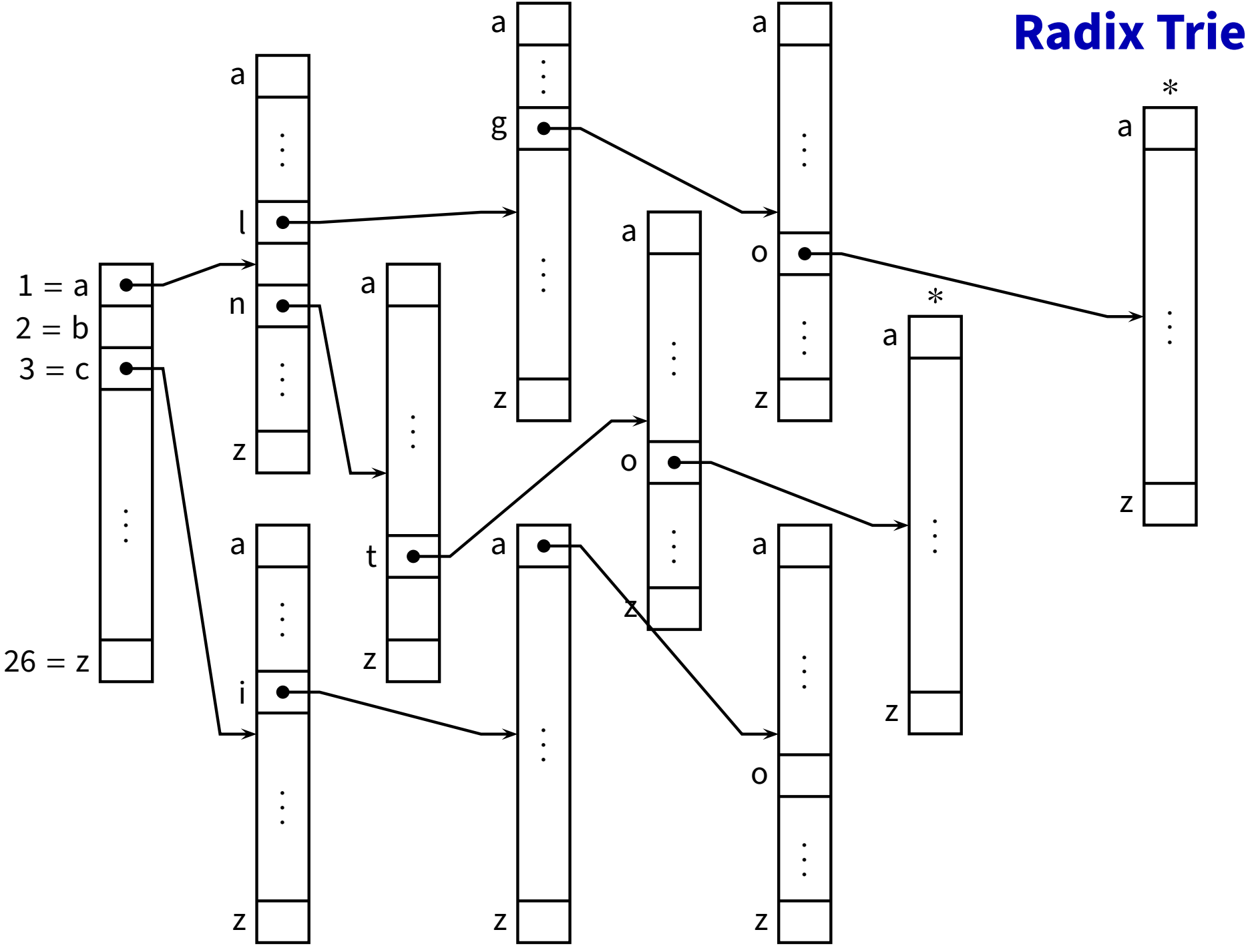

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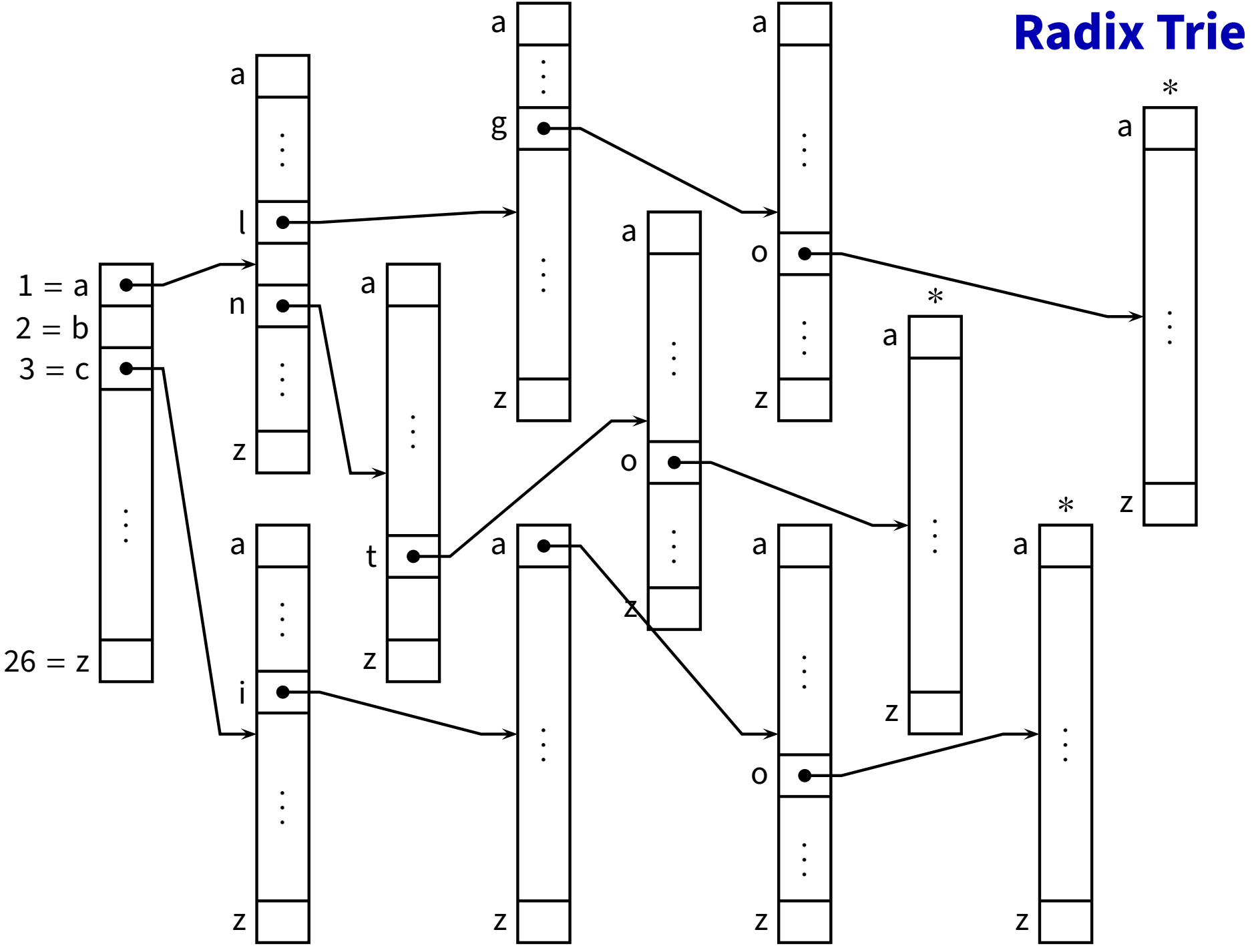

Radix Search

- Every element x has an array of links $x.links$
  - e.g., in "radix-256," an element represents a *byte* in a string (of bytes)
- Every element x has a $x.value$ that is TRUE if that prefix corresponds to a string in the dictionary
  - this is to distinguish an entire word from a prefix

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```
RADIXSEARCH(Root, K)
1  n = Root
2  for i = 1 to length(K)
3        if n.links[K[i]] == NIL
4              return FALSE
5        else n = n.links[K[i]]
6  return n.value
```

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  - a better characterization (*Exercise:* figure this out!):

$$S(N, m) = \Theta\left(|\Sigma| \left[\frac{N-1}{|\Sigma|-1} + N\left(m - \frac{\log N}{\log |\Sigma|}\right)\right]\right)$$

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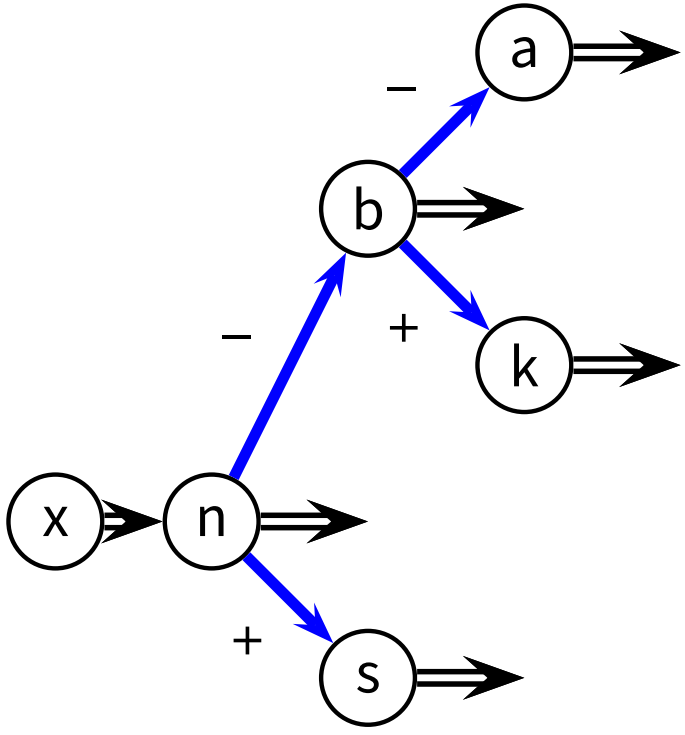

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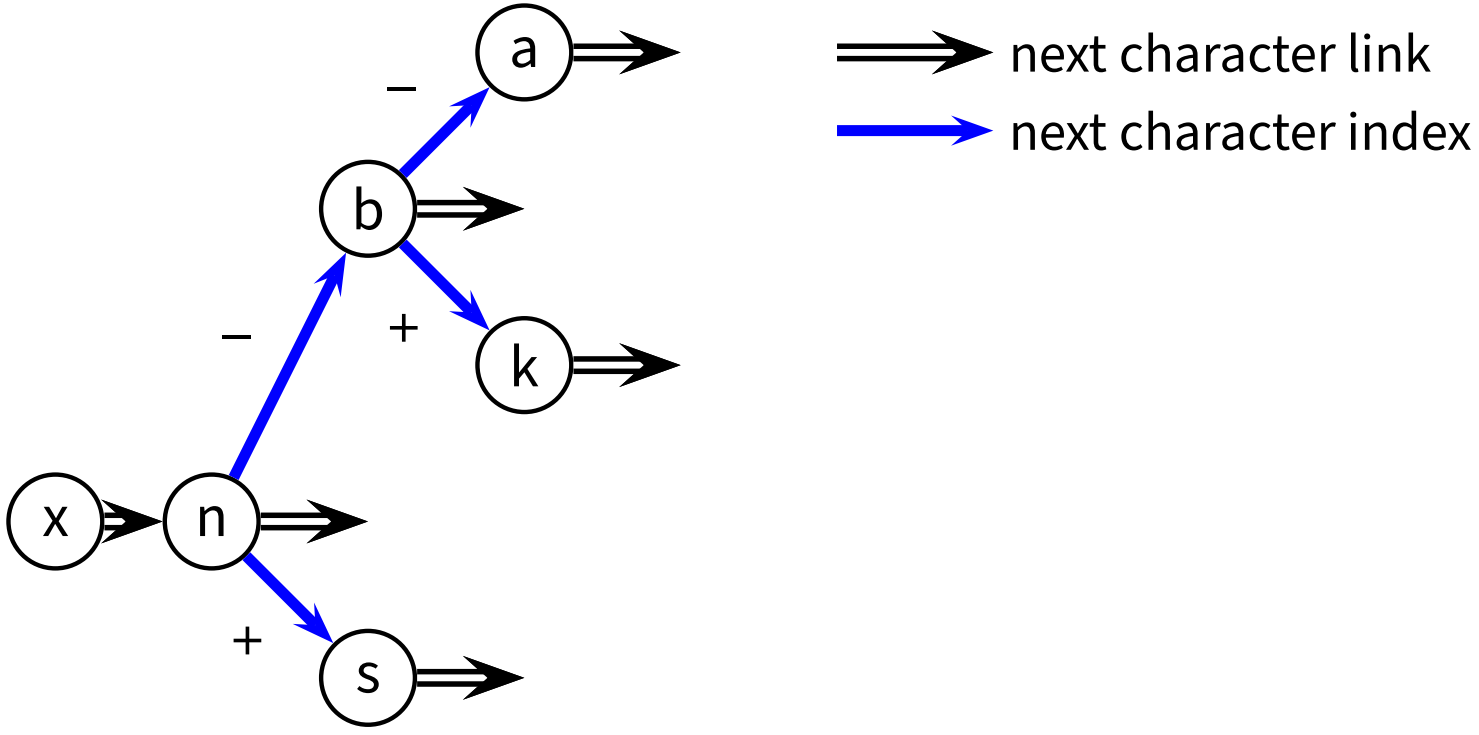

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- *n.value* is the *value* to which *n* maps to; if the TST is a dictionary, then *n.value* is *true* iff the prefix represented by *n* is a key in the dictionary

- A node *n* has three links
  - *n.lower* links to a node representing a “lower” character at the same position
  - *n.higher* links to a node representing a “higher” character at the same position
  - *n.equal* links to a node representing a character in the next position

Example

“culture”

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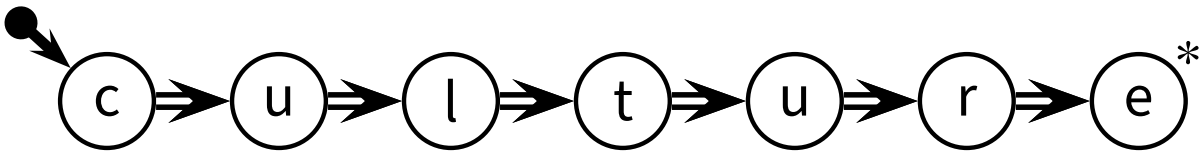
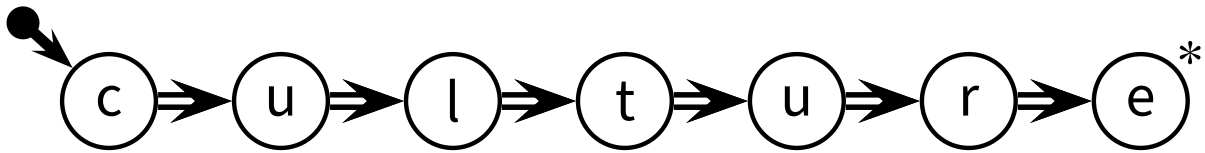


Example

“lugano”



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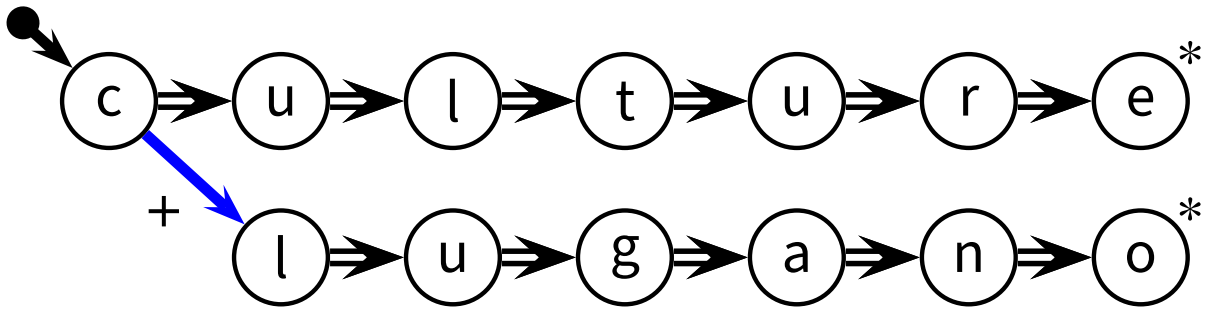

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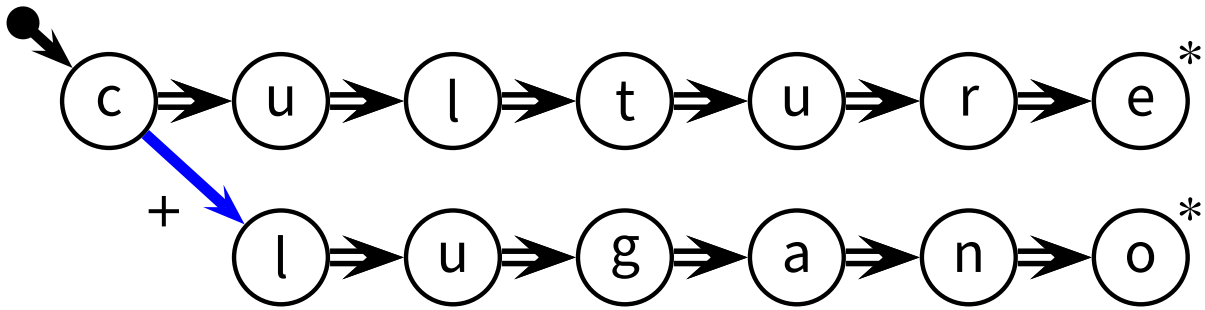

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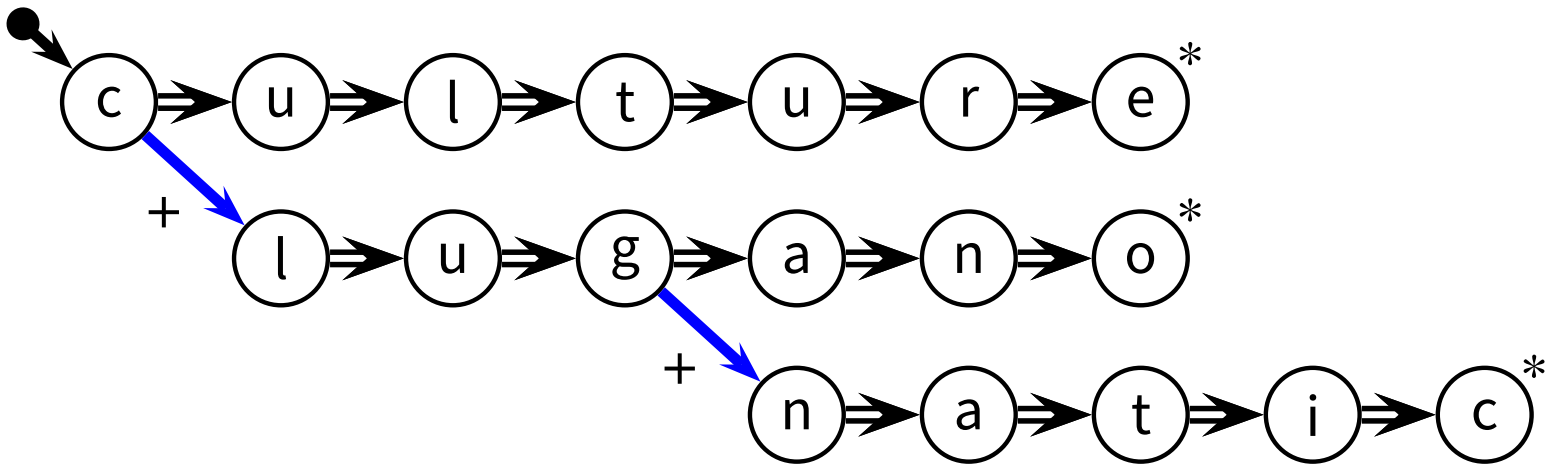

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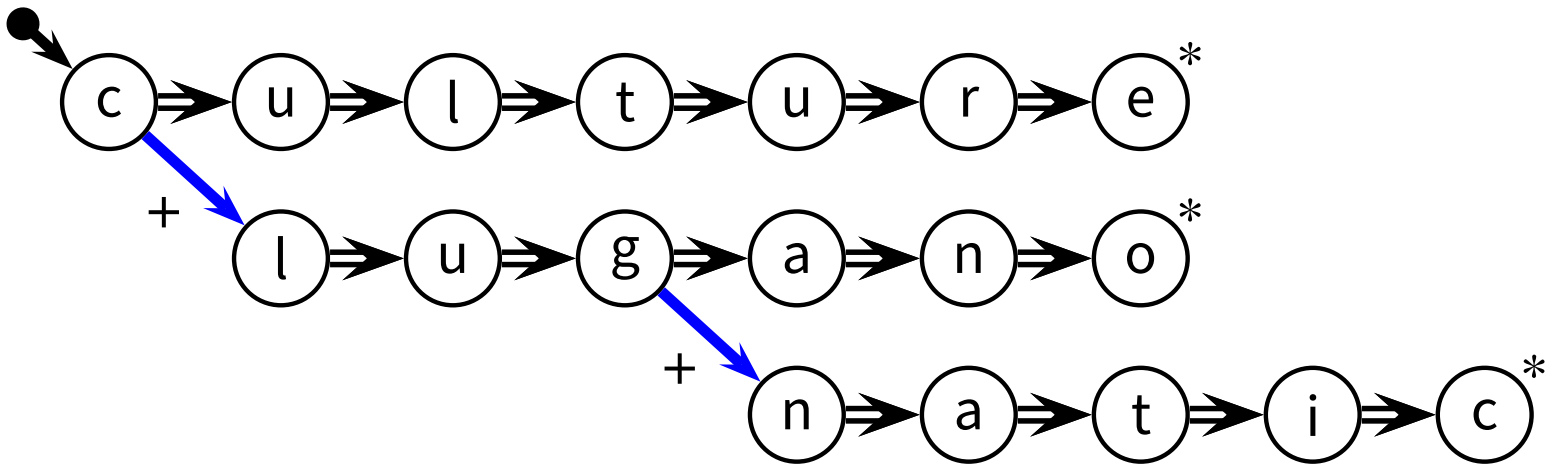
“ciao”



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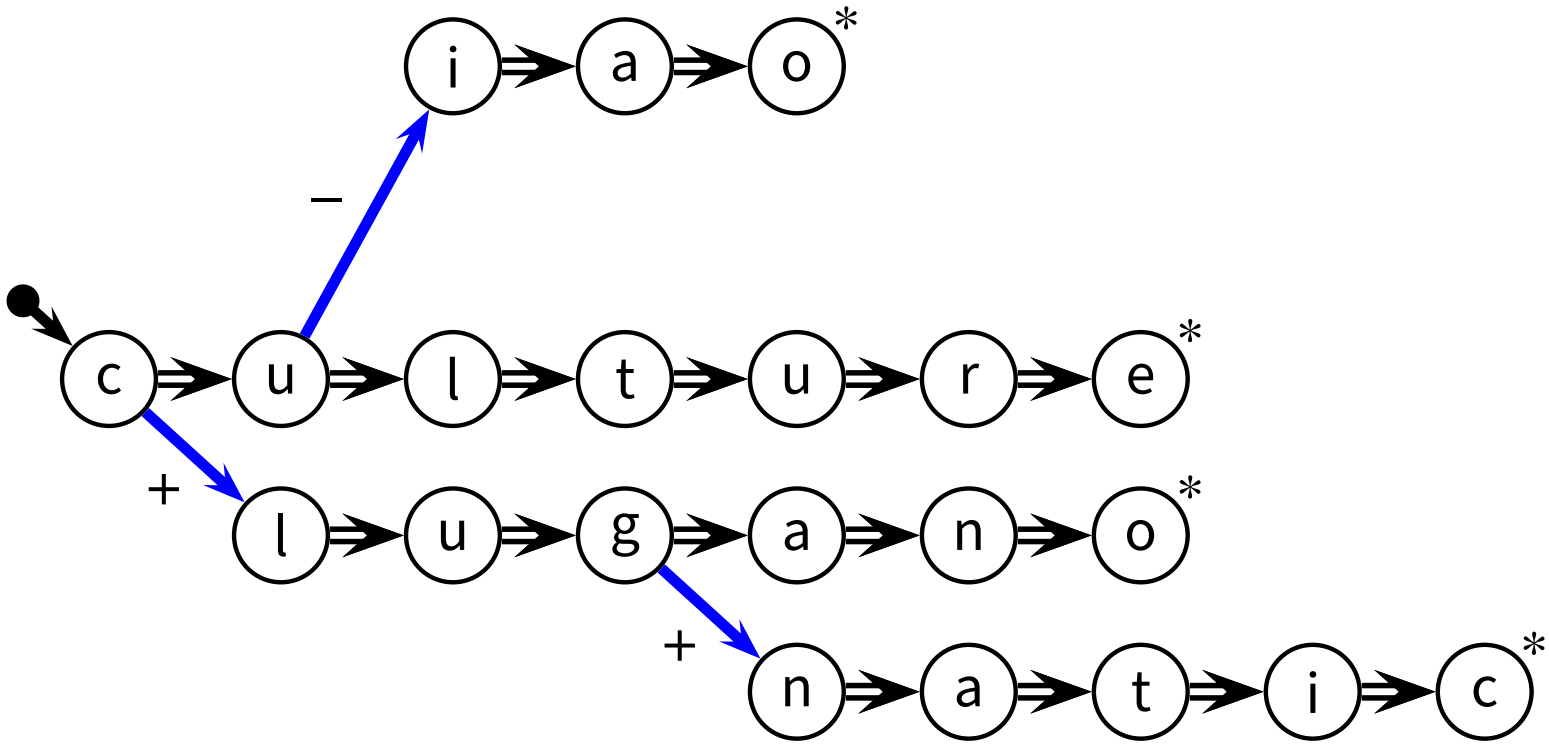

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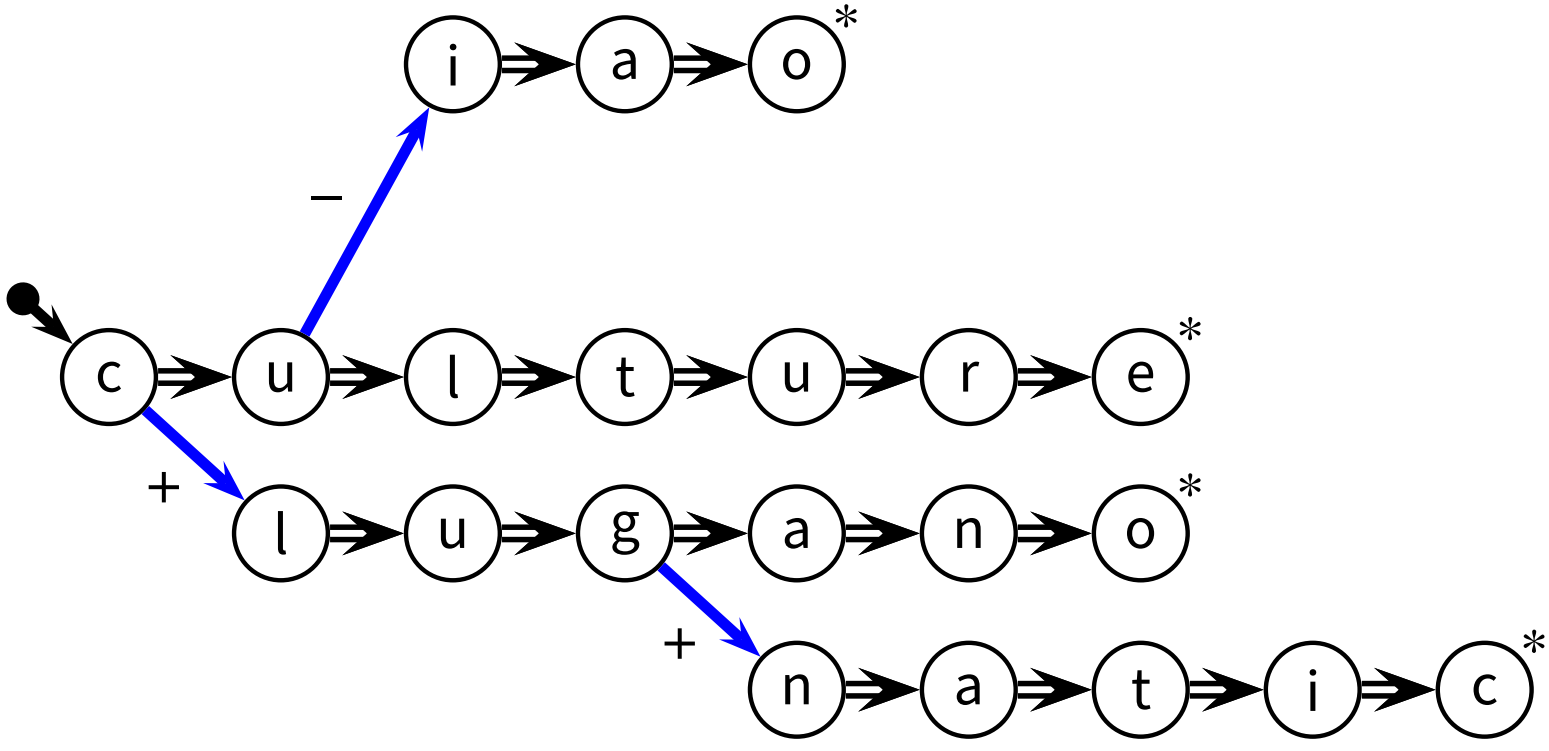

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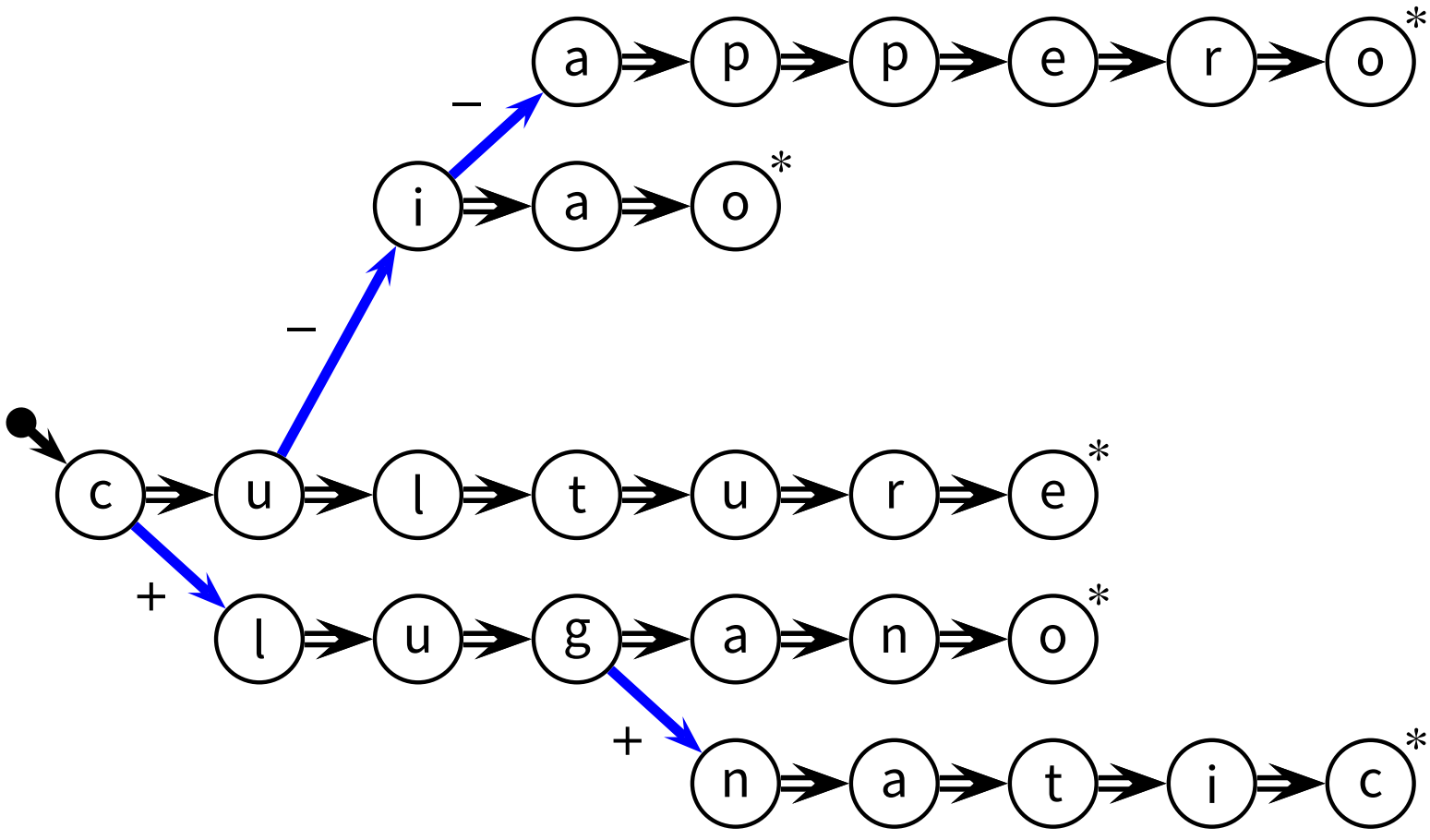
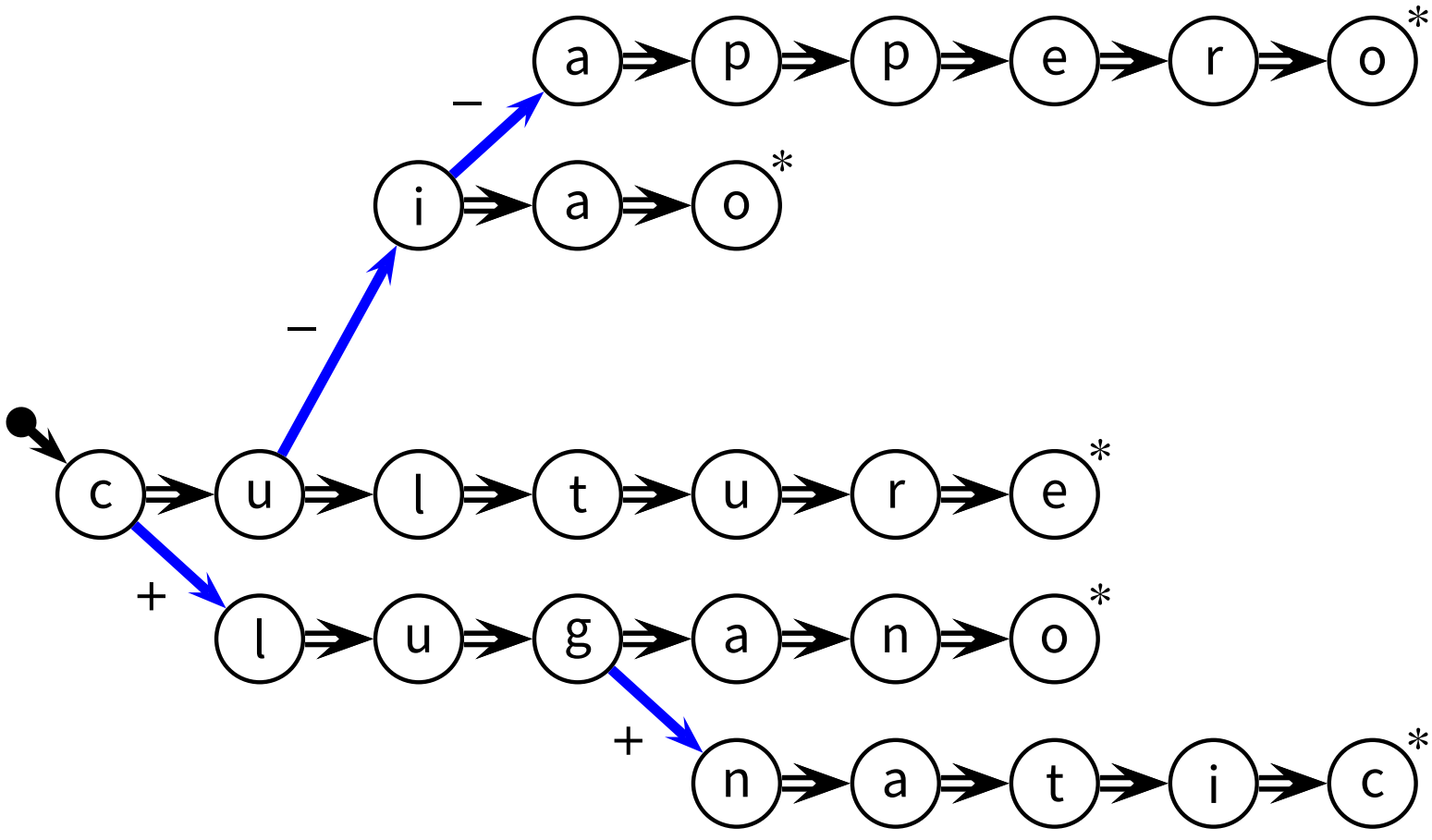


Example

“class”



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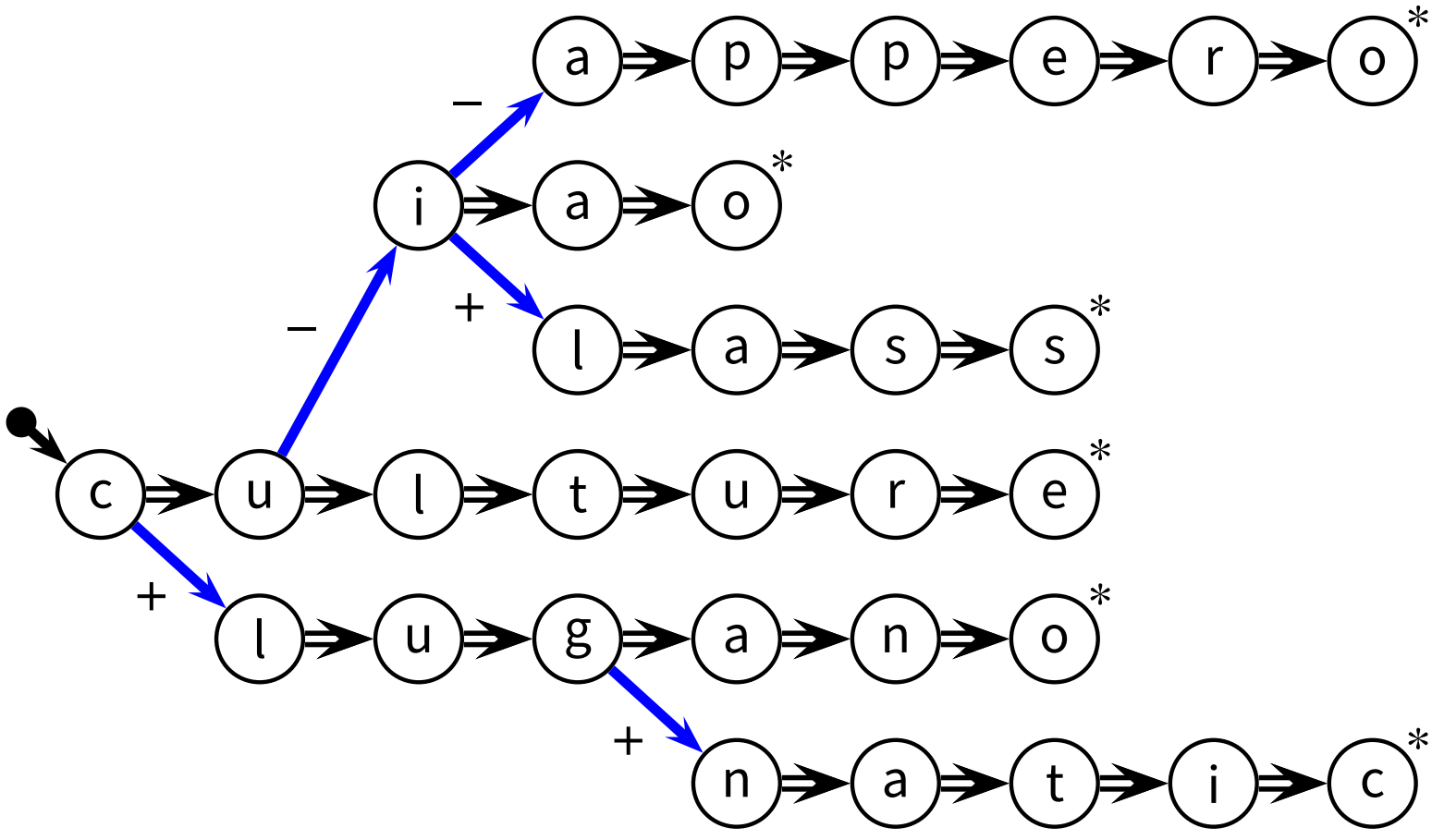

Example

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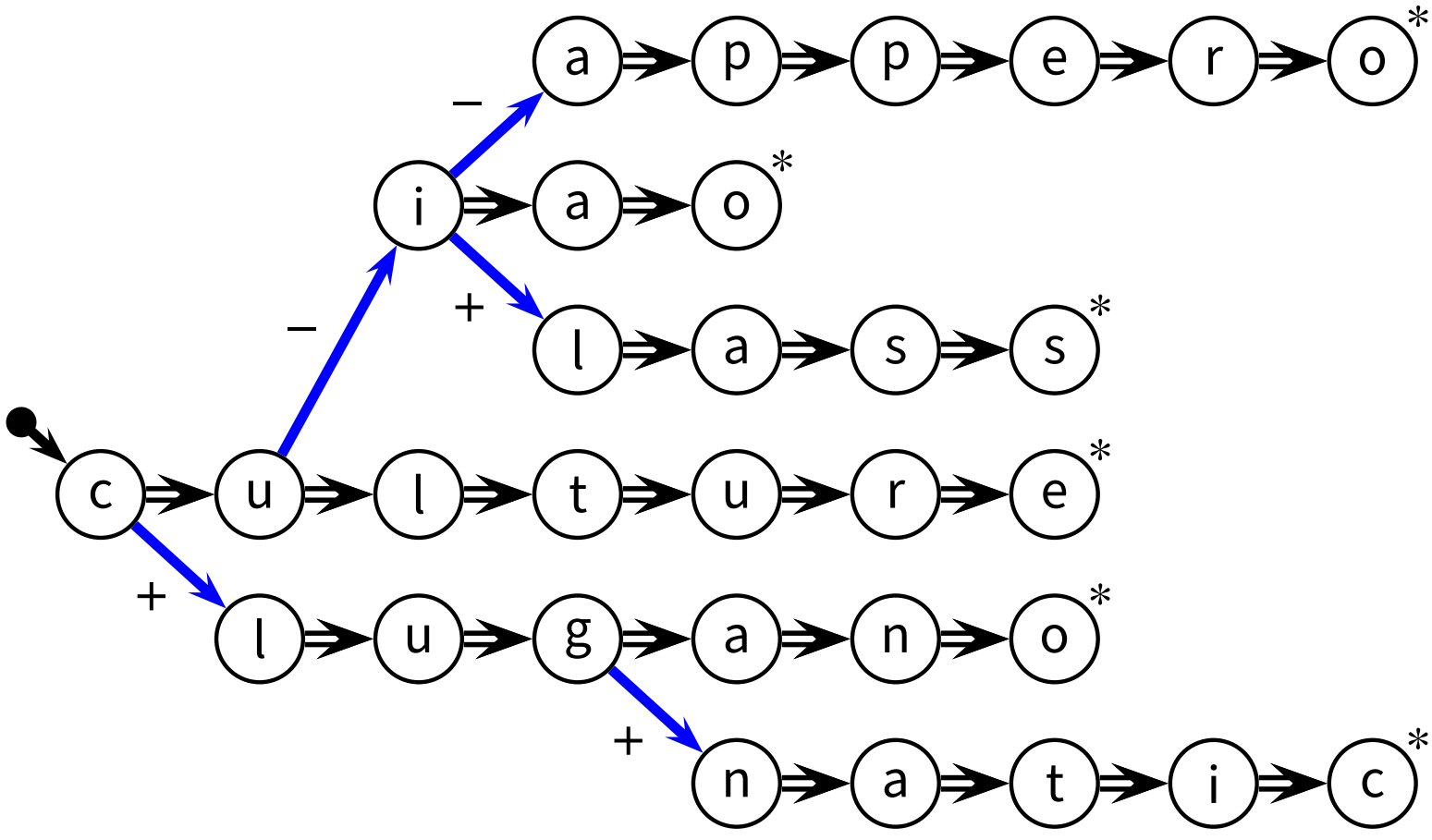

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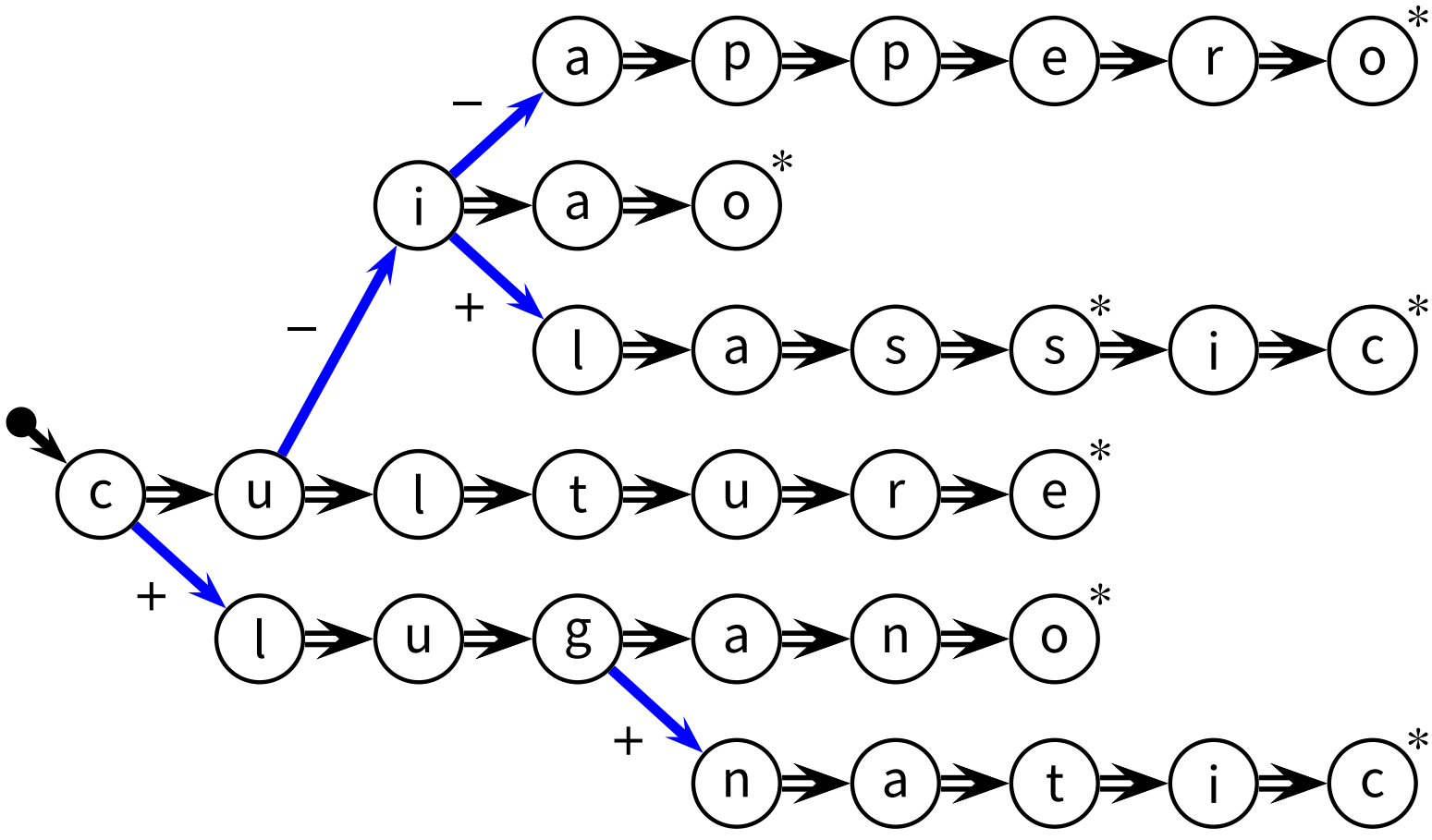
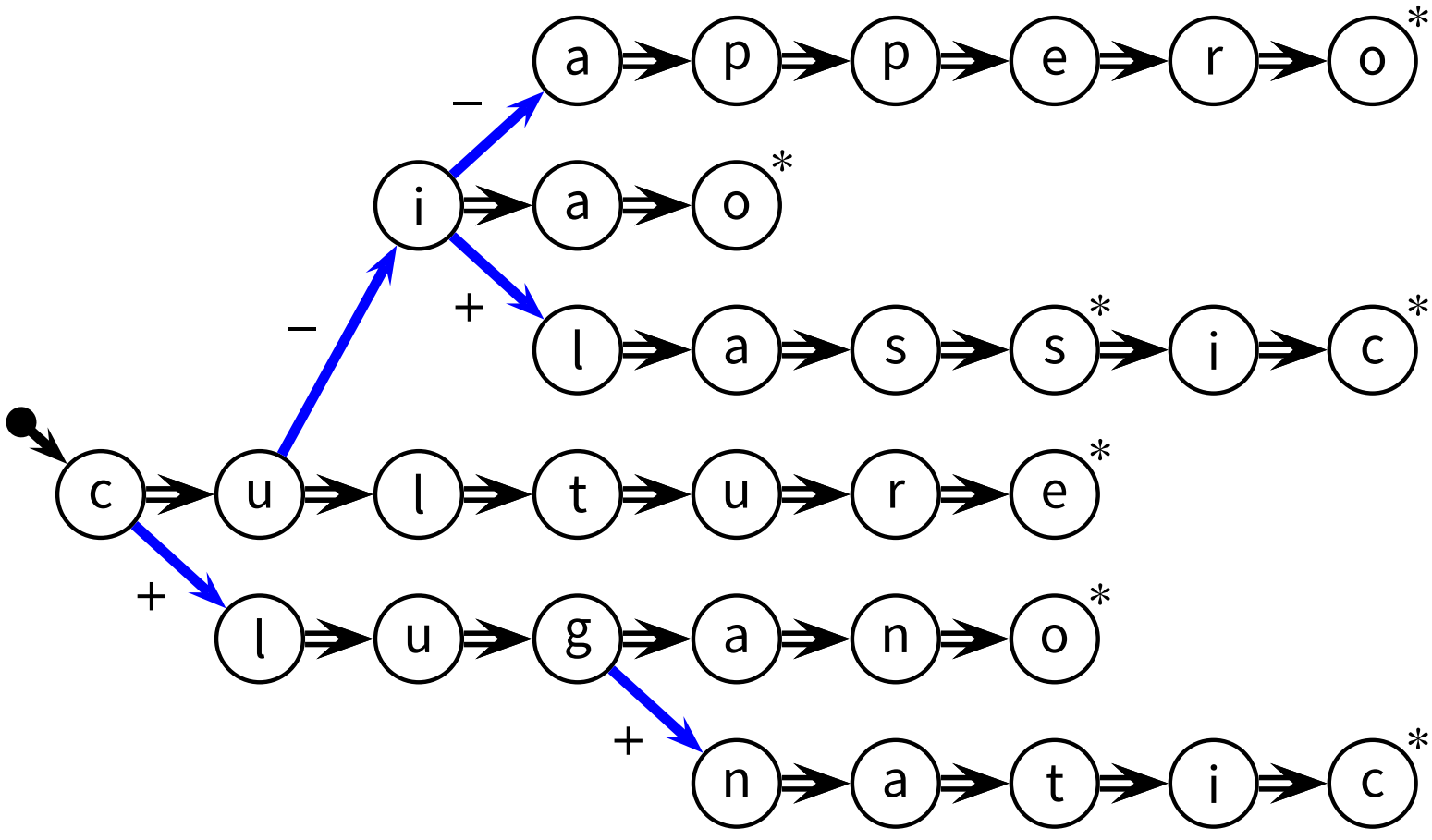


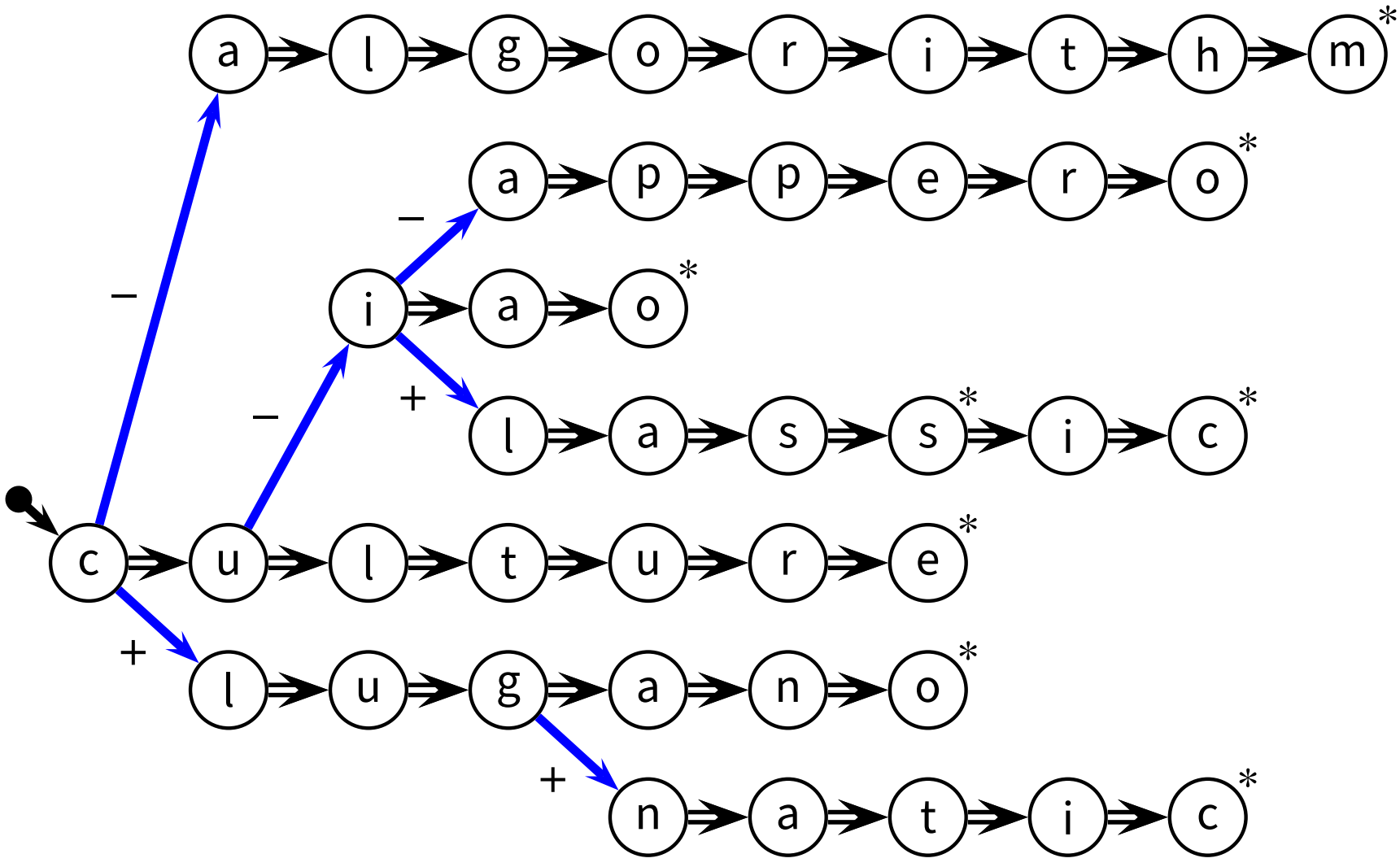
Example

“algorithm”



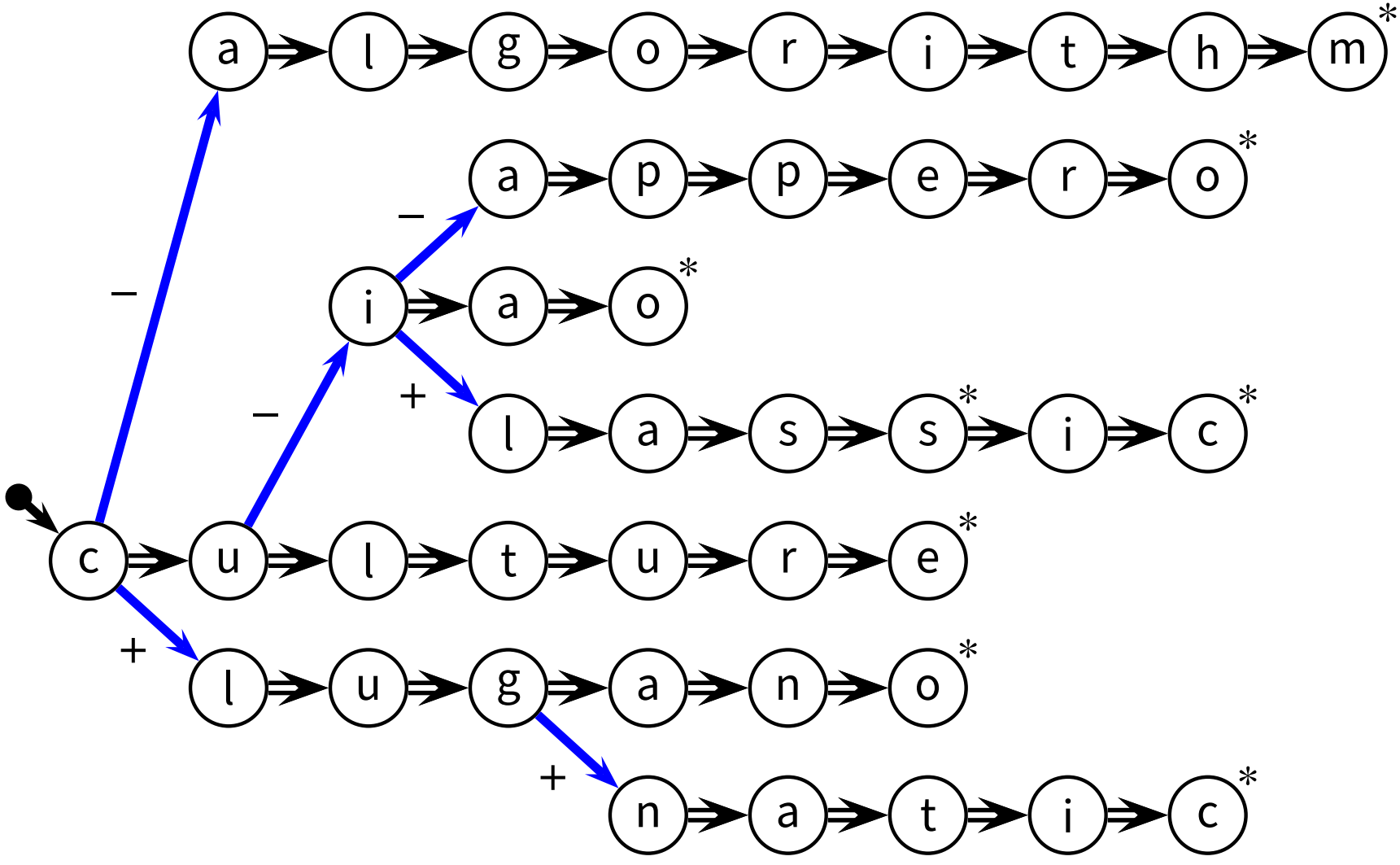
Example

“algorithm”



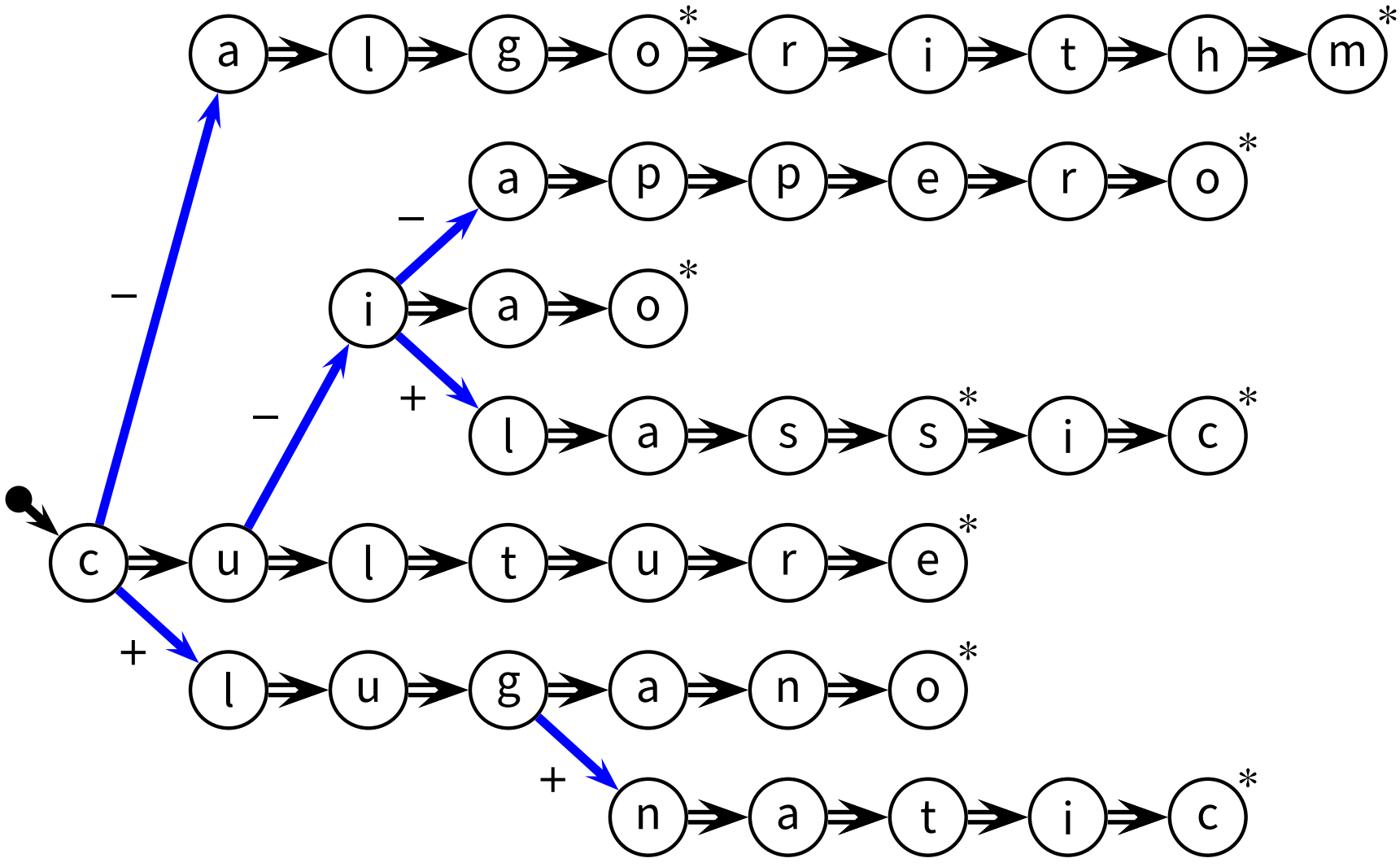
Example

“algo”



Example

“algo”



TST Search

TST Search

```
TSTSEARCH(T, K)
 1  for i = 1 to |K|
 2       if i > 1
 3            T = T.equal
 4       while T ≠ NIL and K[i] ≠ T.character
 5            if K[i] < T.character
 6                 T = T.lower
 7            else T = T.higher
 8       if T == NIL
 9            return FALSE
10  return n.value
```

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- Is it correct?

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- Is it correct? Not completely! (**Exercise:** fix it.)
- Complexity?

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```

- Is it correct? Not completely! (**Exercise:** fix it.)
- Complexity? Non-trivial...

TST Insertion

TST Insertion

- Recursion starts with $root = \textbf{TSTInsert}(root, K, 1)$

```
TSTInsert(T, K, i)
 1  if T == NIL
 2        T = NewNode(K[i])
 3  if K[i] < T.character
 4        T.lower = TSTInsert(T.lower, K, i)
 5  elseif K[i] > T.character
 6        T.higher = TSTInsert(T.higher, K, i)
 7  elseif K[i] == T.character
 8        if i < |K|
 9              T.equal = TSTInsert(T.equal, K, i + 1)
10        else T.value = TRUE
11  return T
```