String Matching Algorithms

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Outline

- Problem definition
- Naïve algorithm
- Knuth-Morris-Pratt algorithm
- Boyer-Moore algorithm

Problem

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  che la dritta via era smarrita…

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  0011110101011010011000110101111011010111
  0110111001001010101011111011110110000101
  1011000010111111011110011 000011111000100
  1001010010111011101011011110101001100101
  0010111001000011111110010011011101011010
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- Both T and P can be modeled as arrays
  - $T[1 \ldots n]$ and $P[1 \ldots m]$
- Pattern P occurs with ***shift*** s in T iff
  - $0 \leq s \leq n - m$
  - $T[s + i] = P[i]$ for all positions $1 \leq i \leq m$

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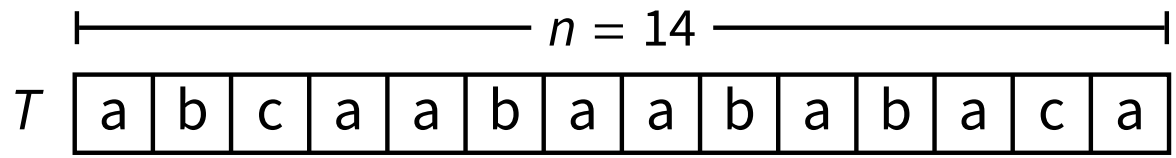

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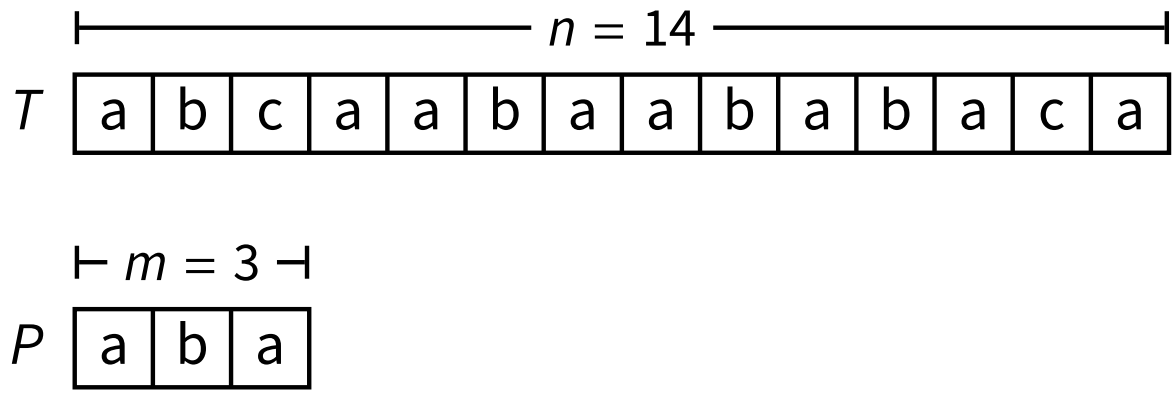

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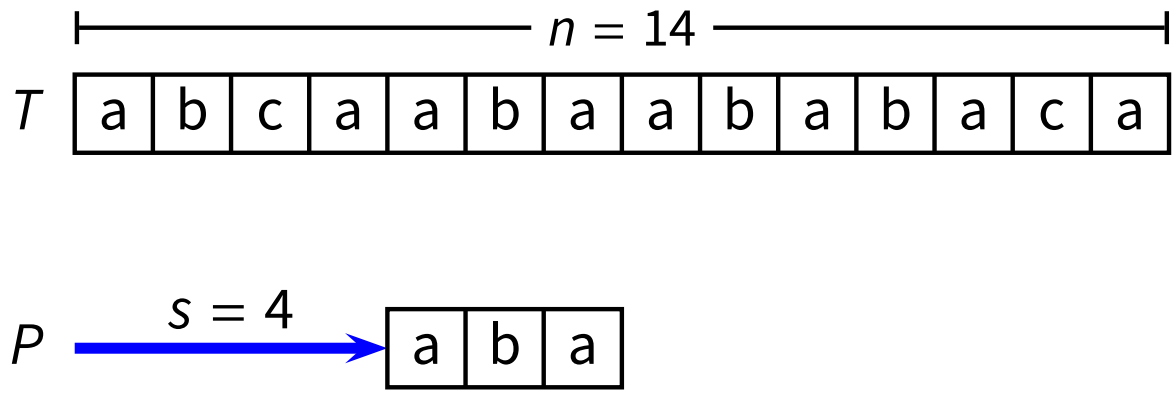

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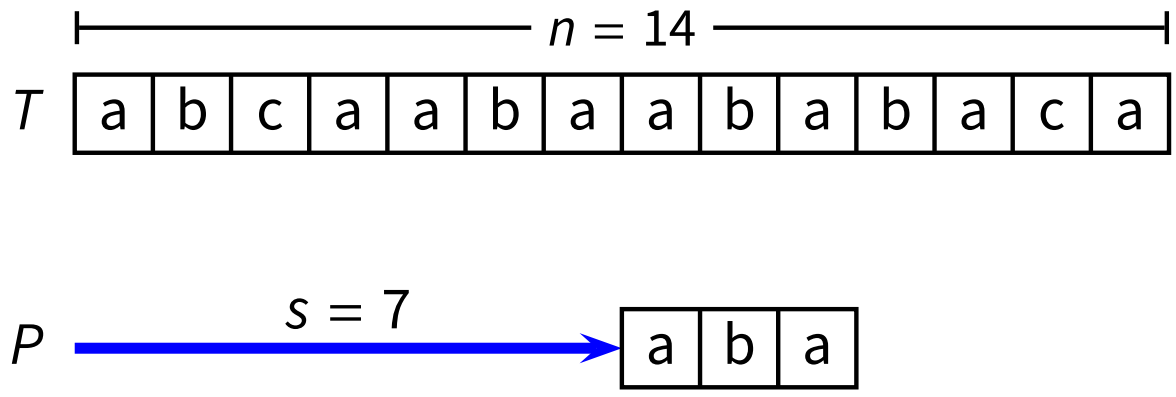

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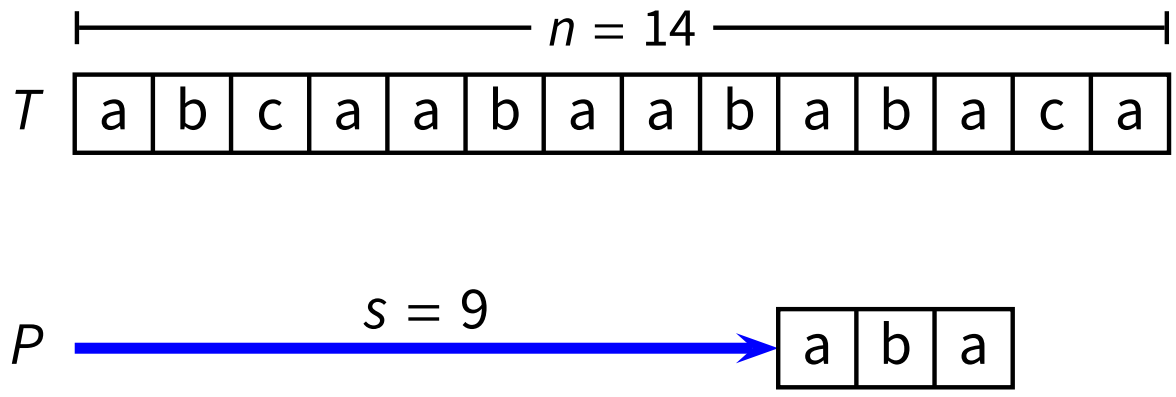

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3  for s = 0 to n − m
4      if SUBSTRING-AT(T, P, s)
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SUBSTRING-AT(T, P, s)
1  for i = 1 to length(P)
2      if T[s + i] ≠ P[i]
3          return FALSE
4  return TRUE
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So, $(n-m+1)m$ is a tight bound, so the (worst-case) complexity of **NAIVE-STRING-MATCH** is

$$\boxed{\Theta((n-m+1)m)}$$

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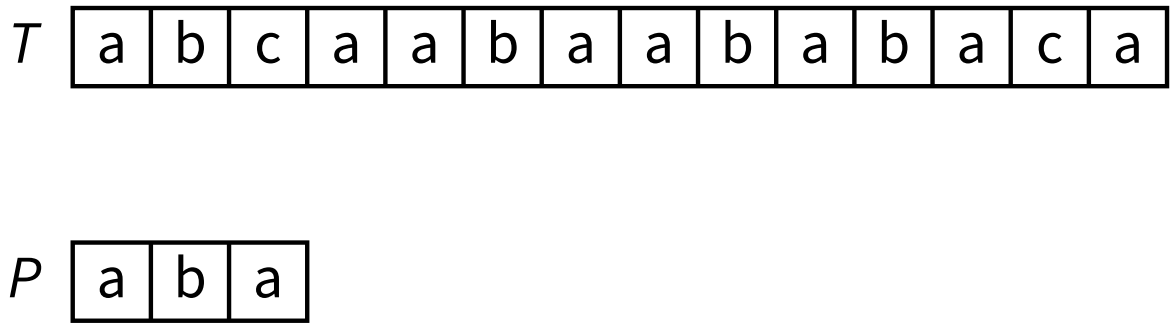

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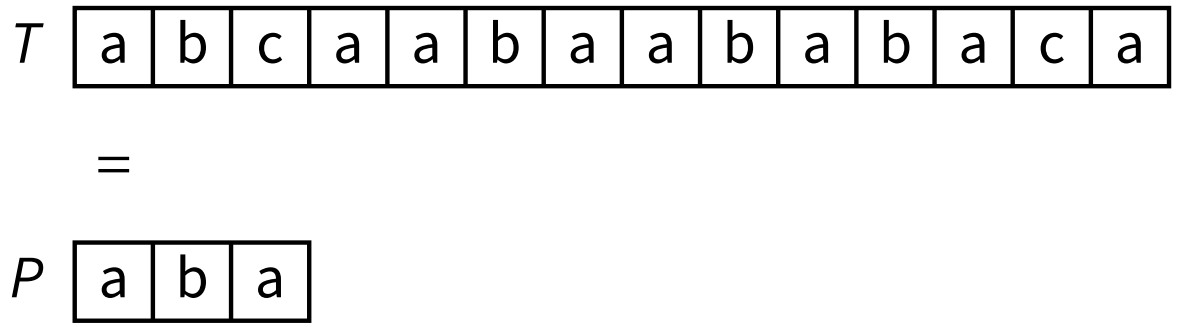

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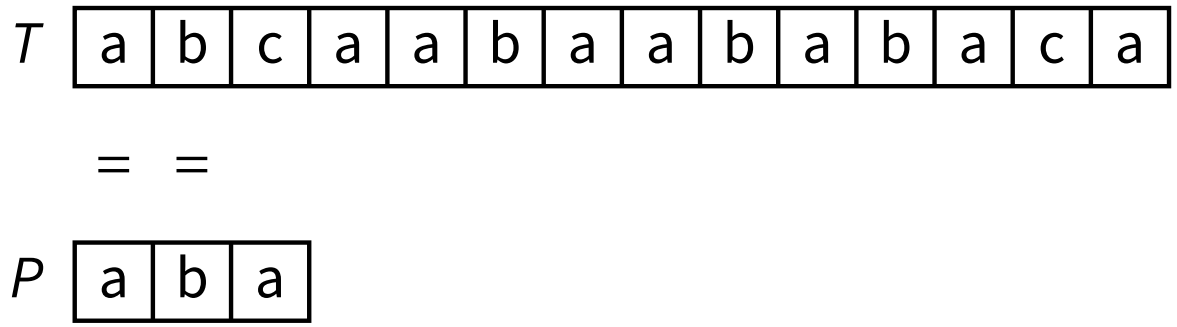

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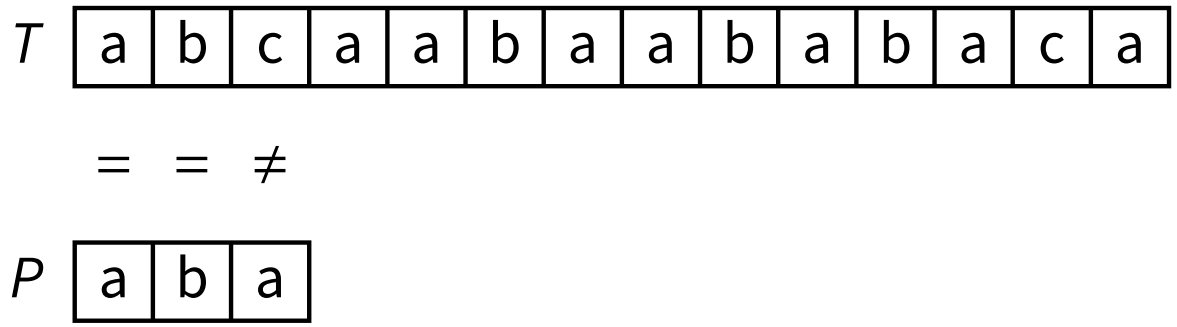

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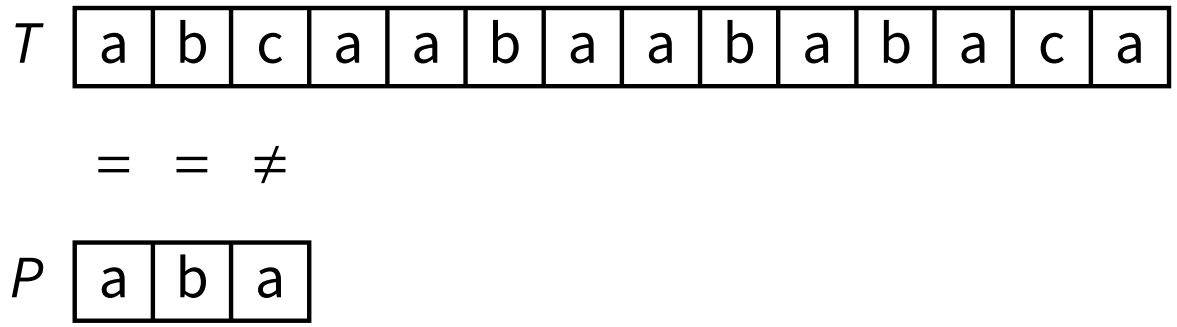

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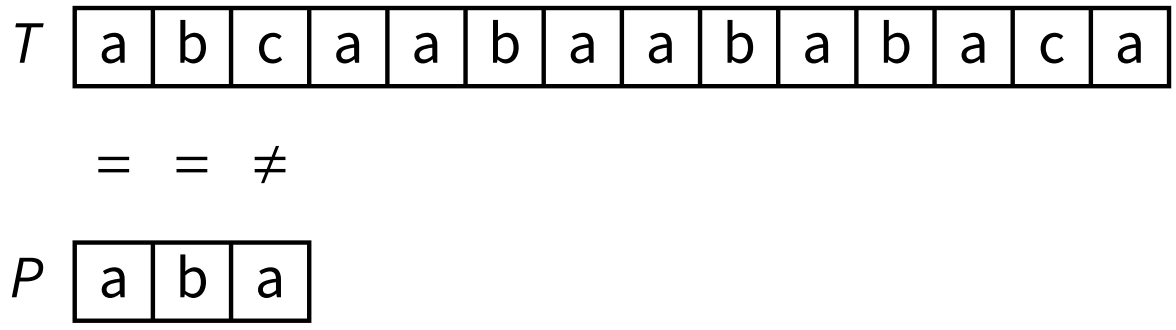

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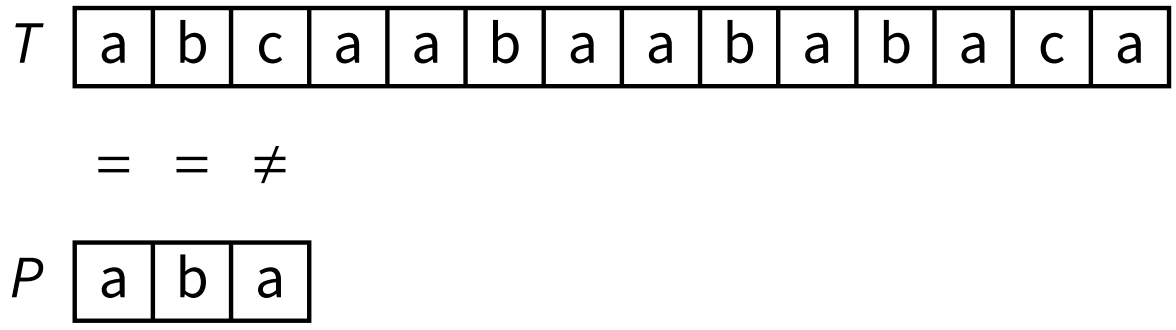

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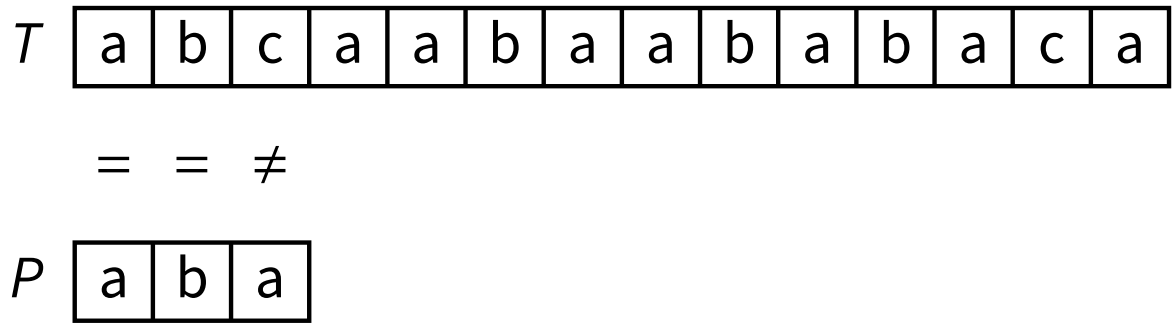

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  - the naïve algorithm ***goes back to the second position in*** *T* and ***starts from the beginning of*** *P*
  - can't we simply move along through *T*?
  - why?

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WRONG-STRING-MATCHING(T, P)
 1  n = length(T)
 2  m = length(P)
 3  q = 0            // number of characters matched in P
 4  s = 1
 5  while s ≤ n
 6        s = s + 1
 7        if T[s] == P[q + 1]
 8              q = q + 1
 9              if q == m
10                    OUTPUT(s − m)
11                    q = 0
12        else q = 0
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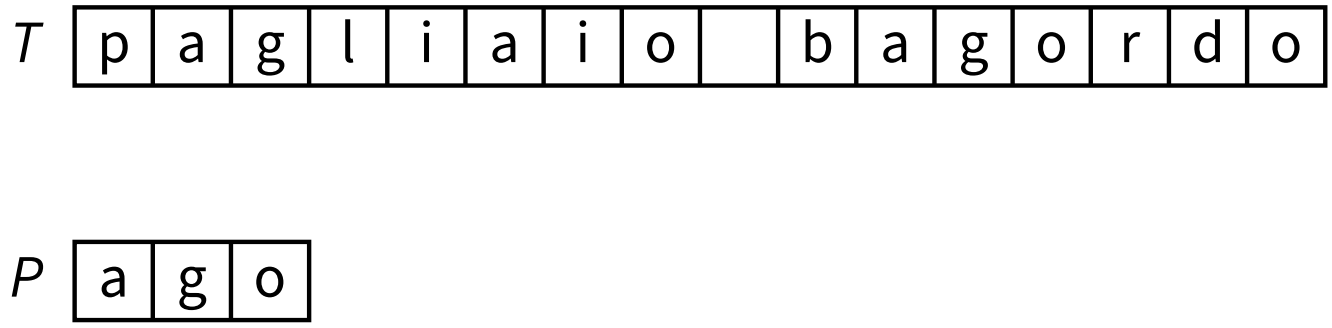

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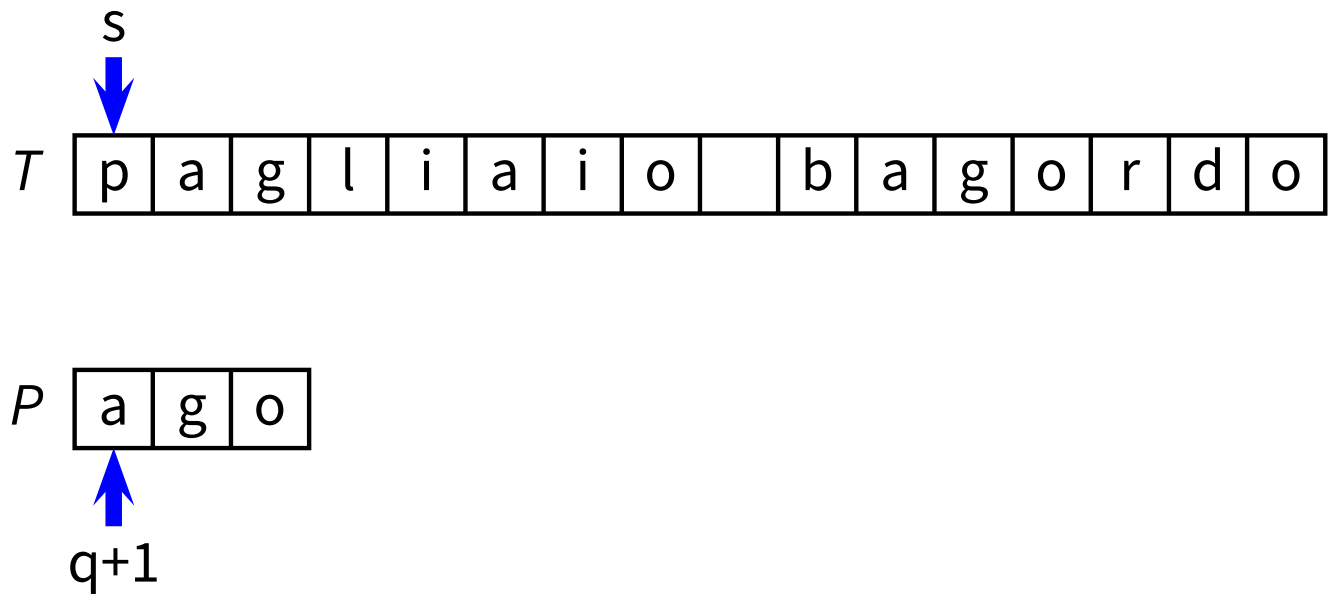

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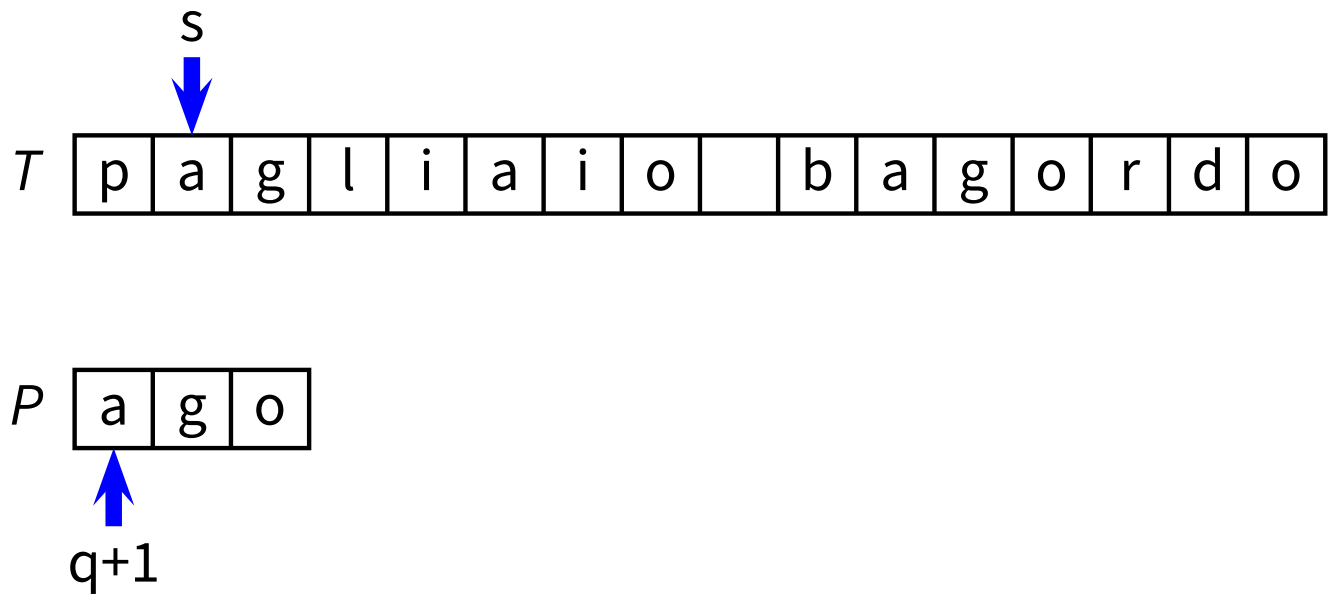

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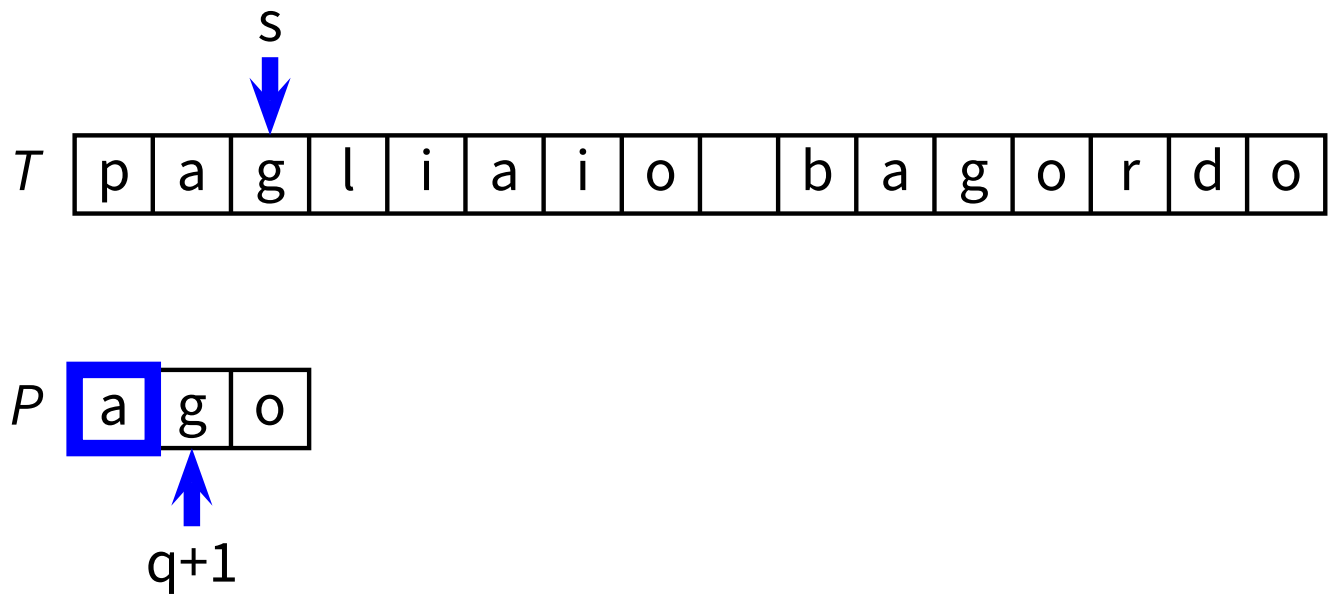

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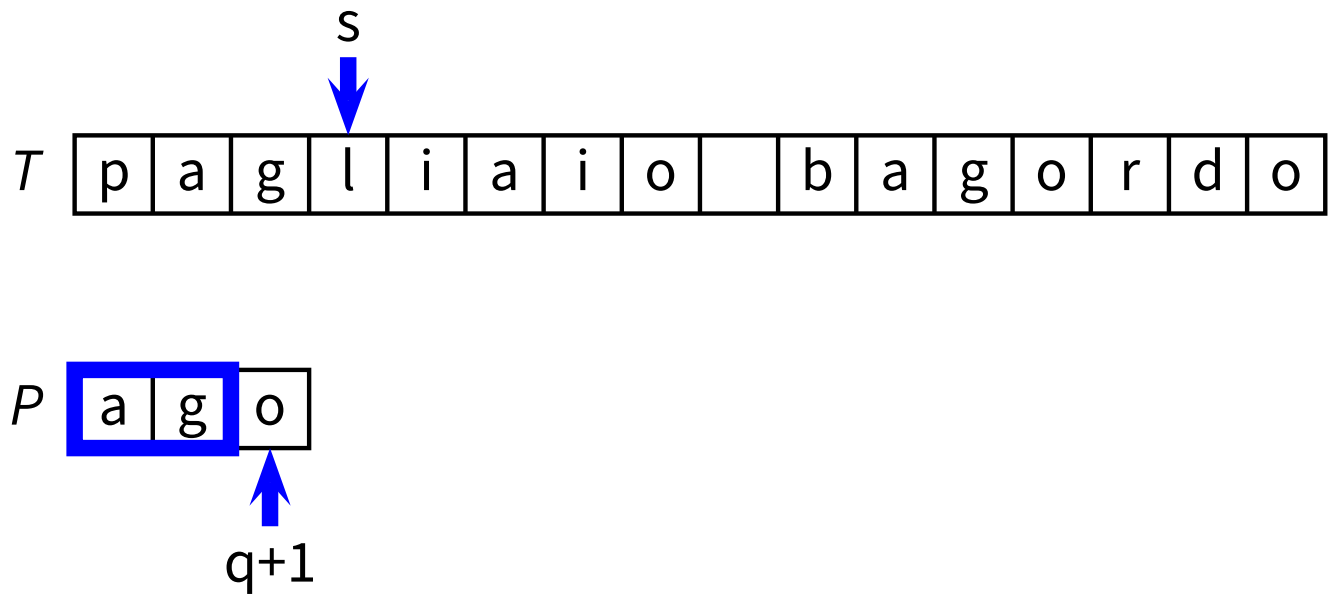

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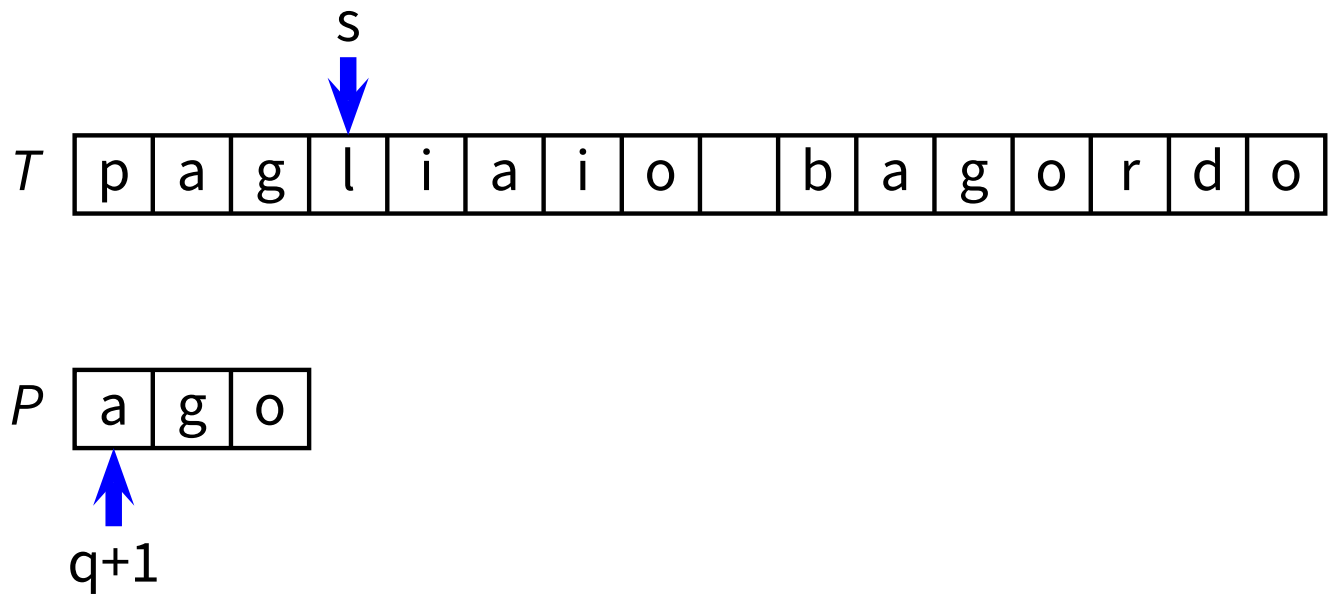

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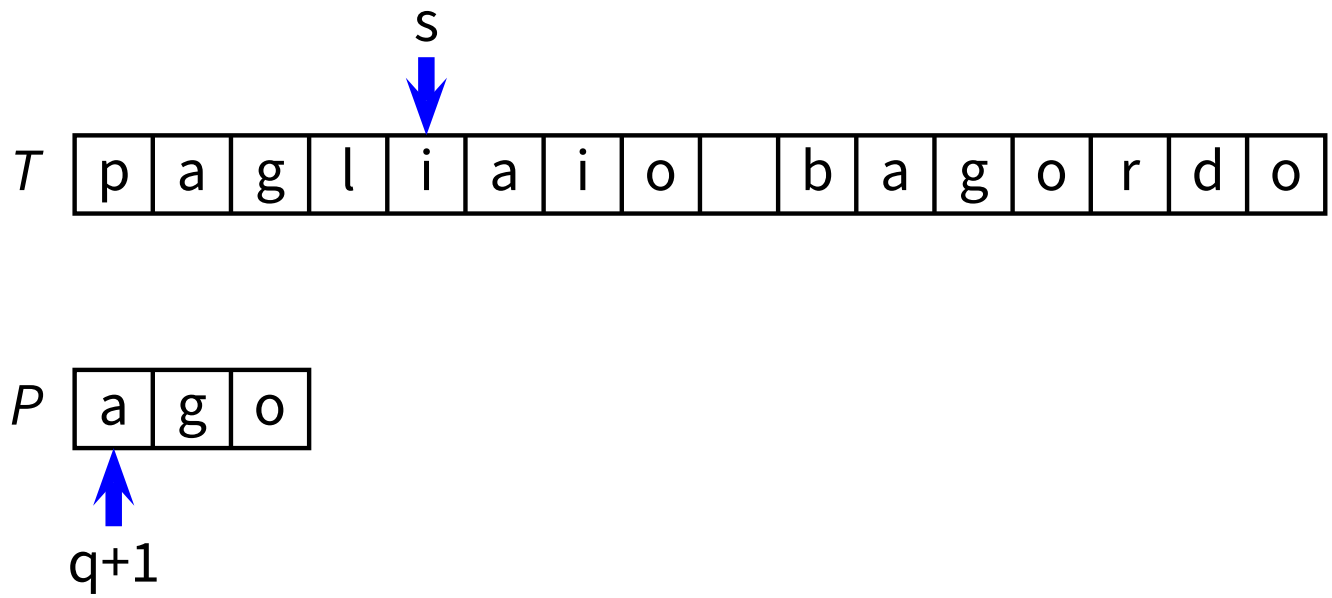

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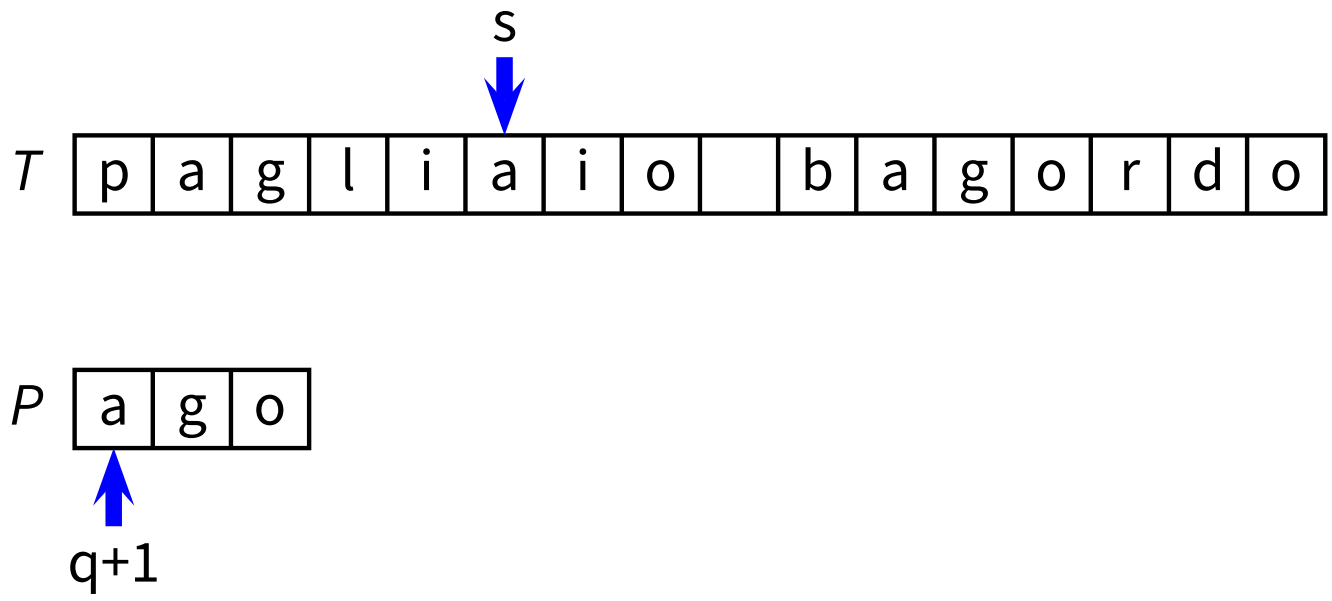

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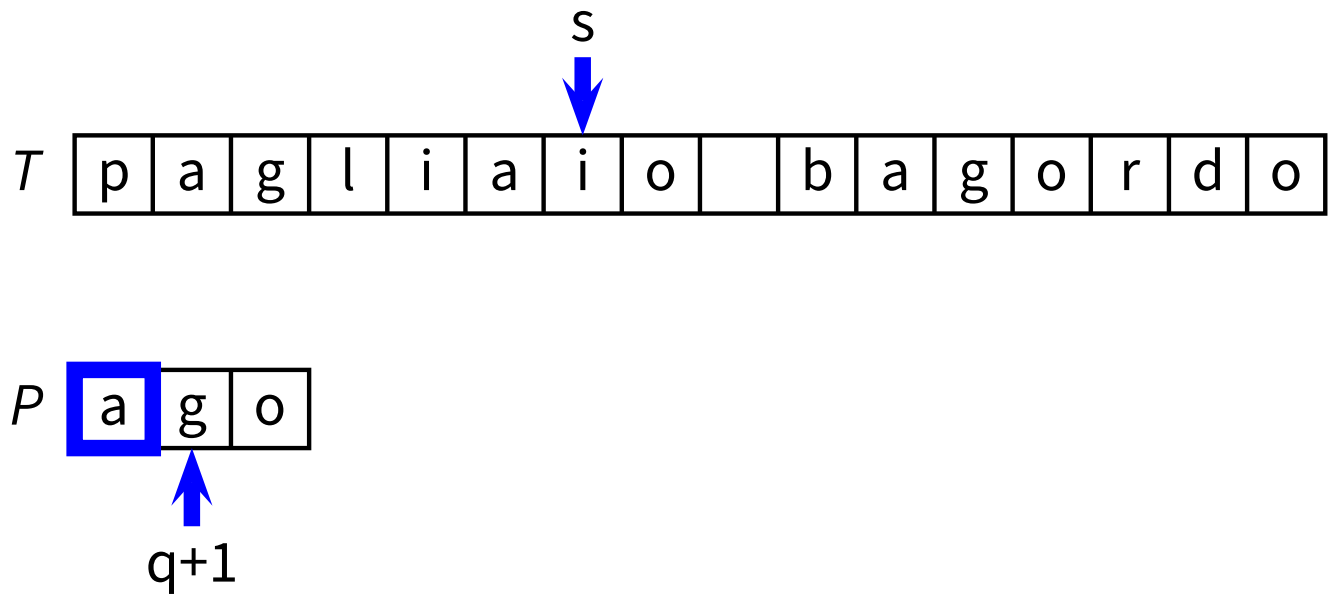

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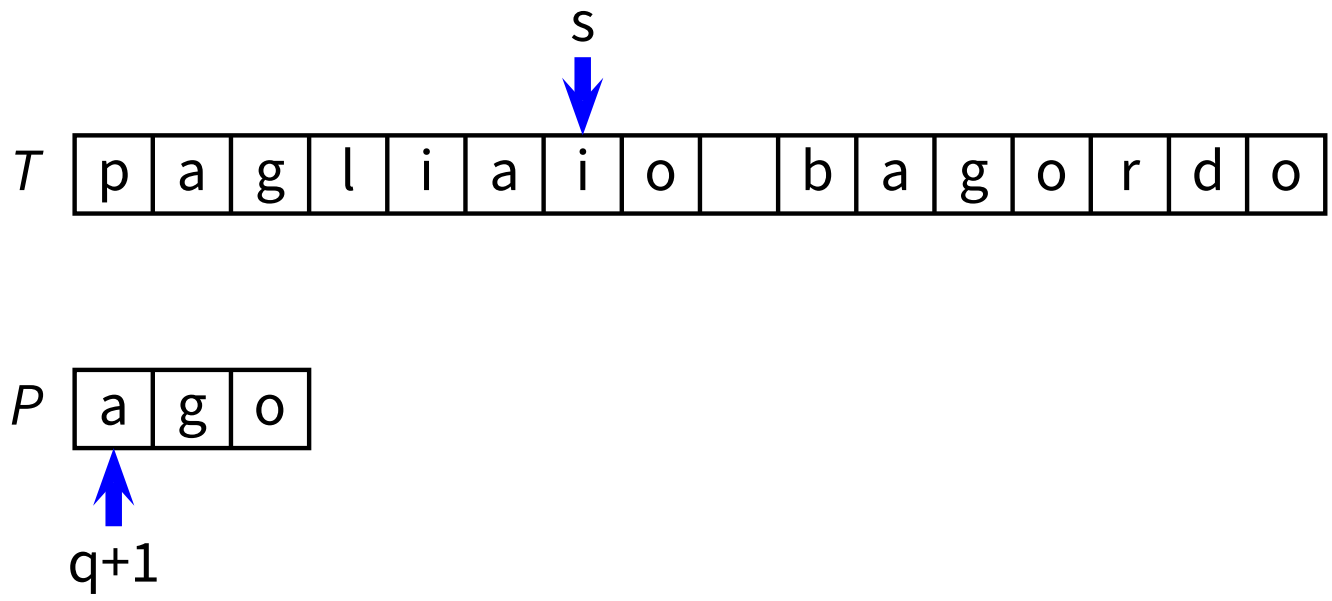

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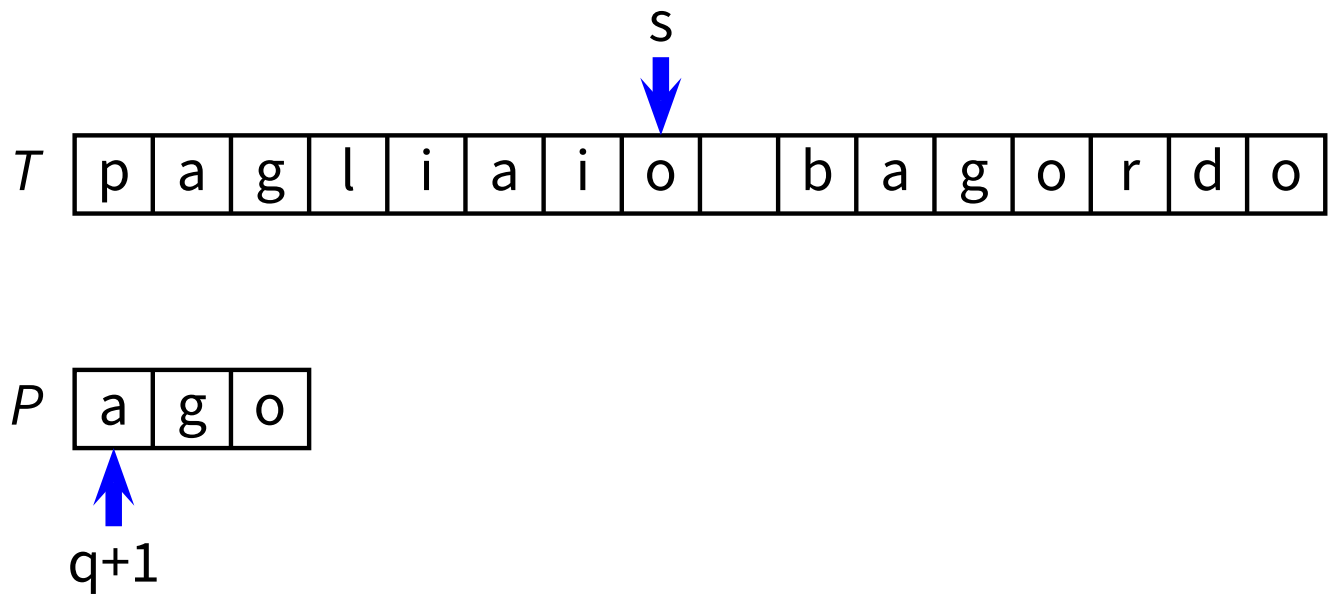

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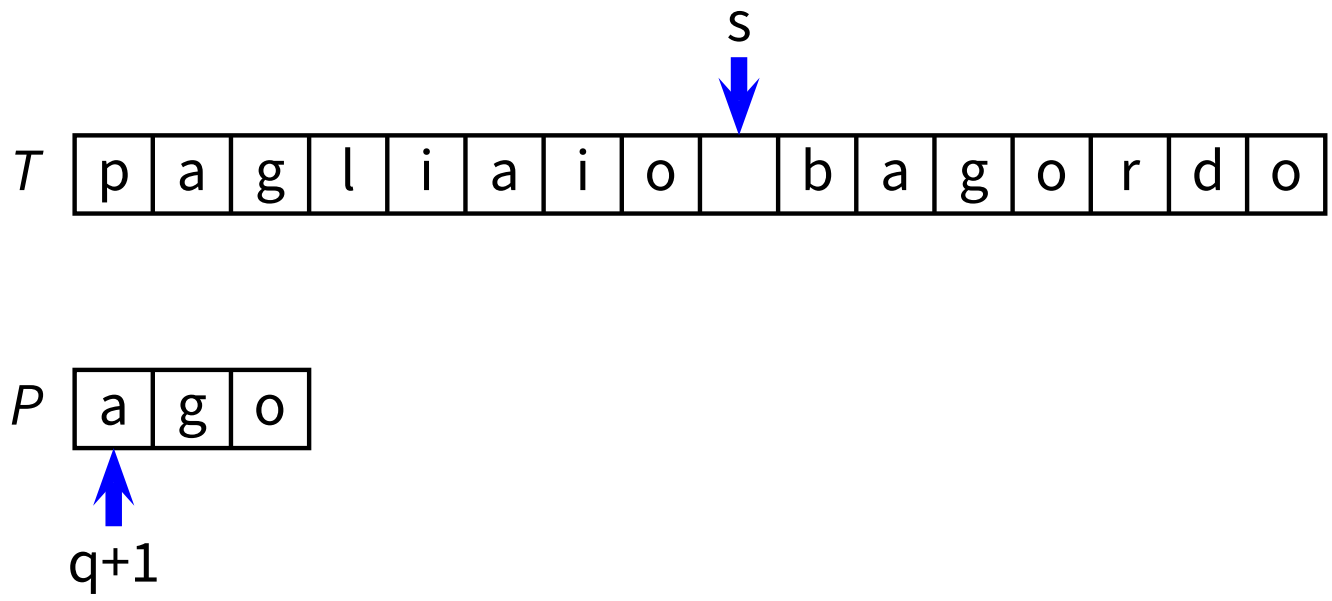

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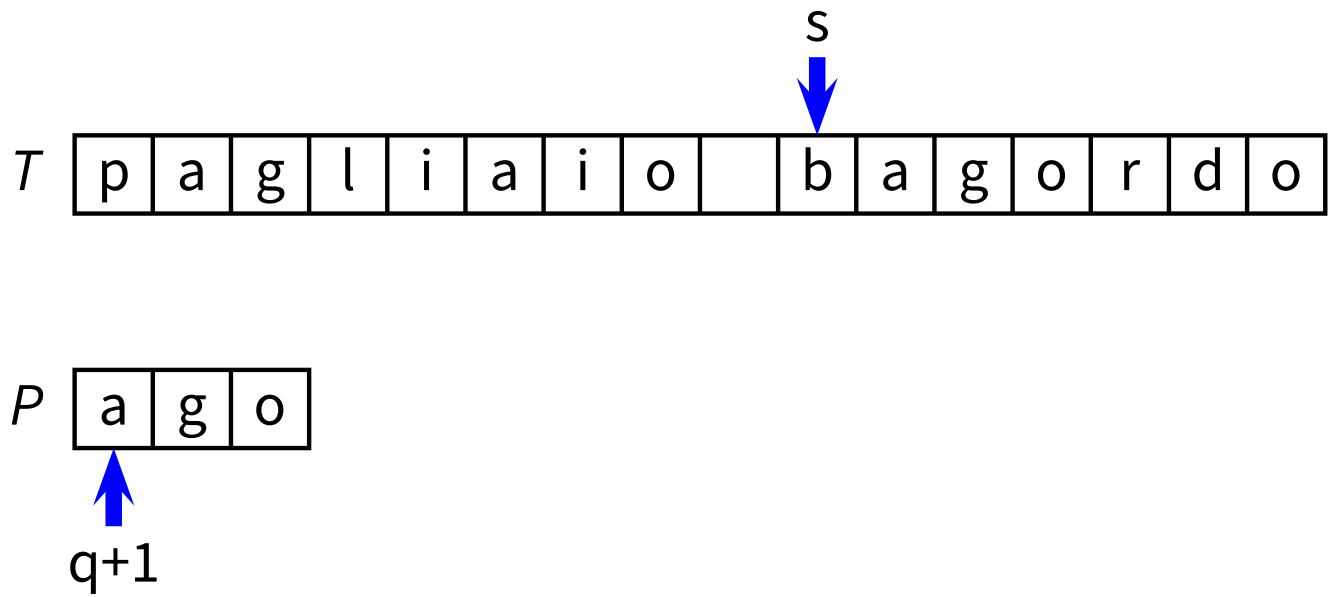

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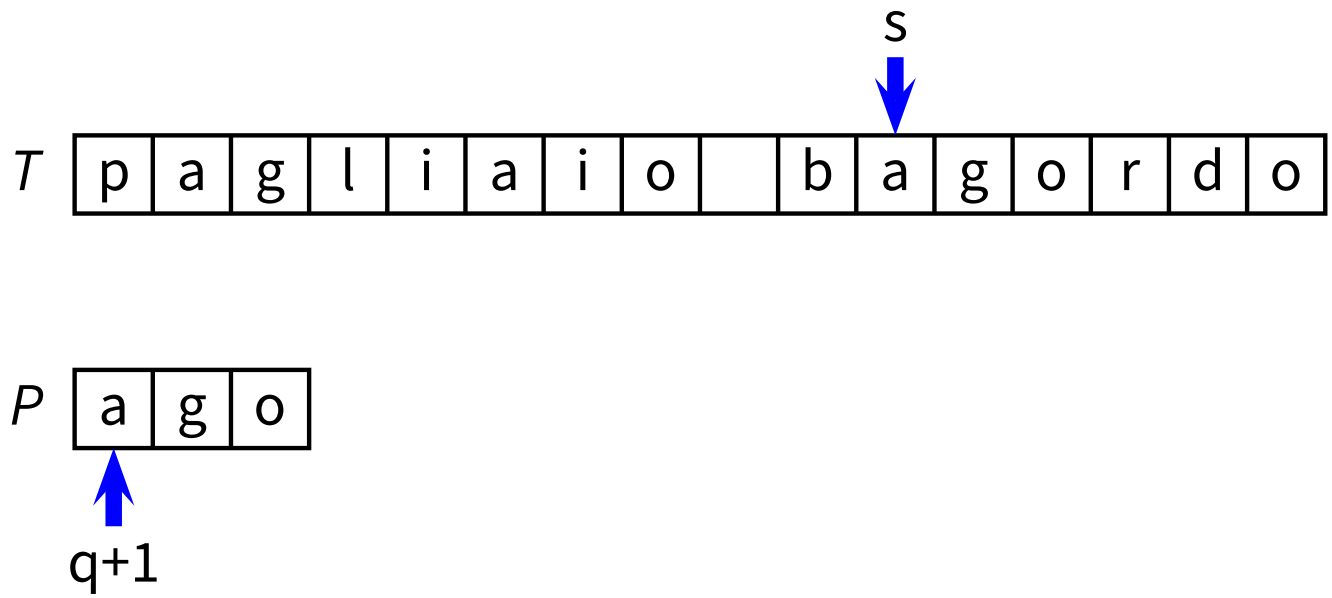

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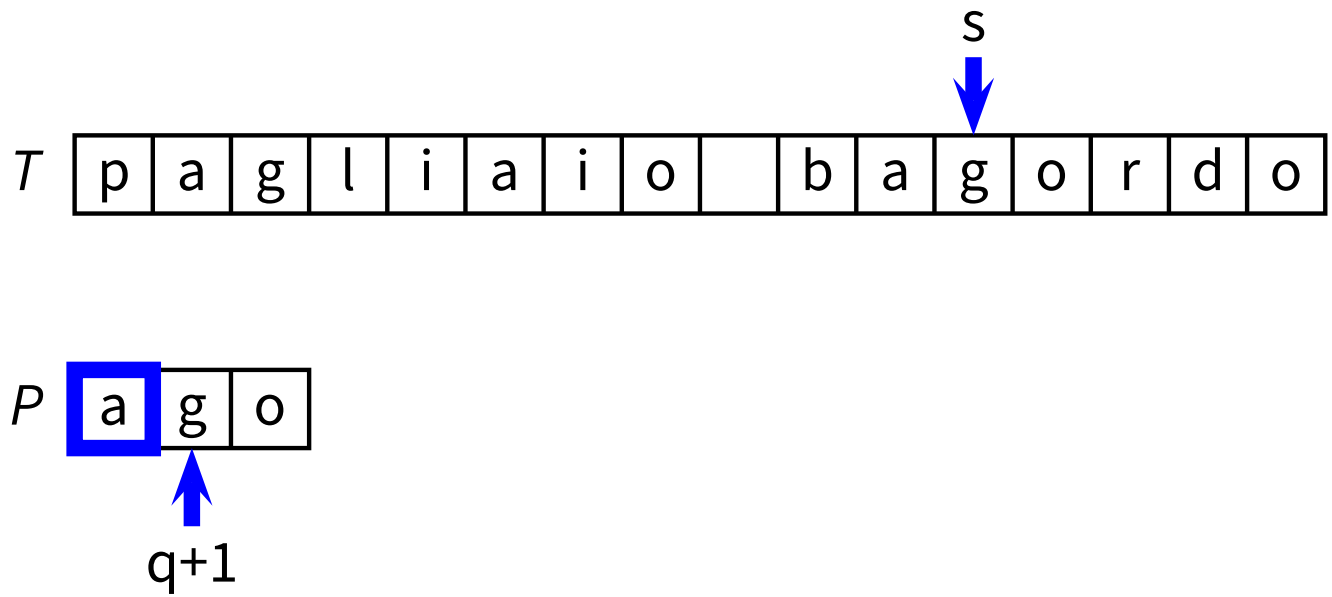

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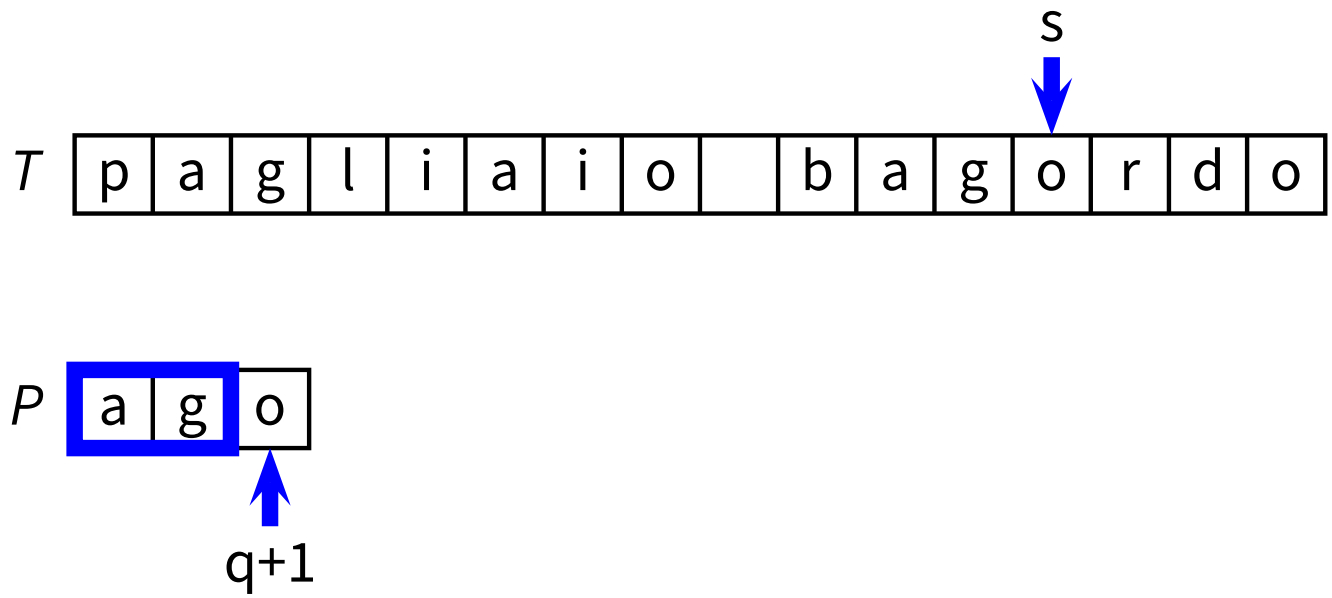

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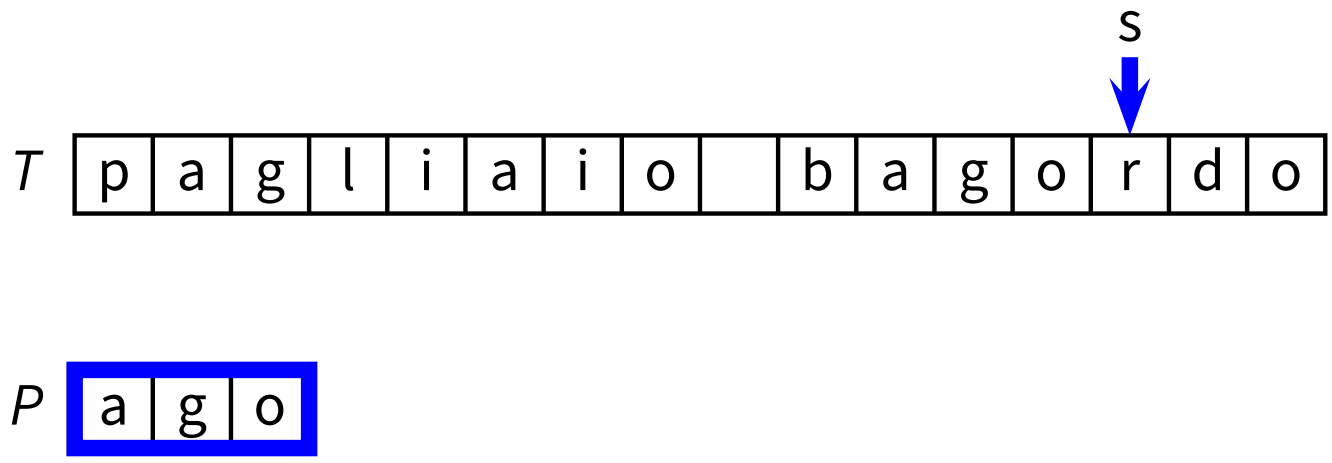

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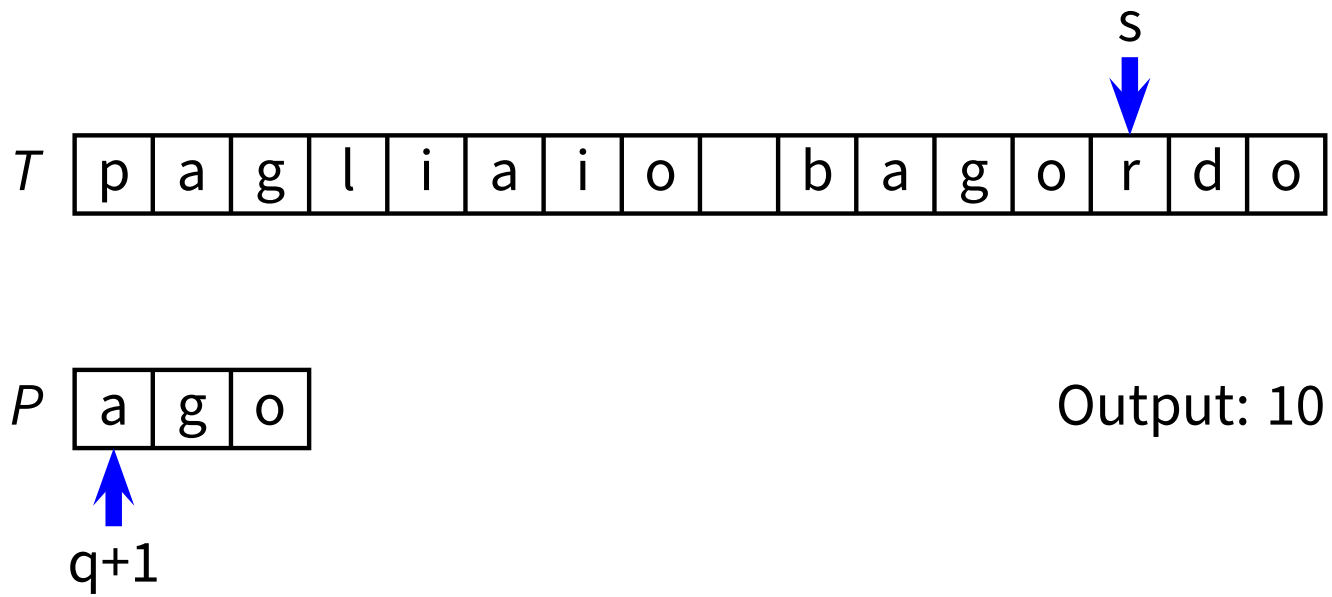

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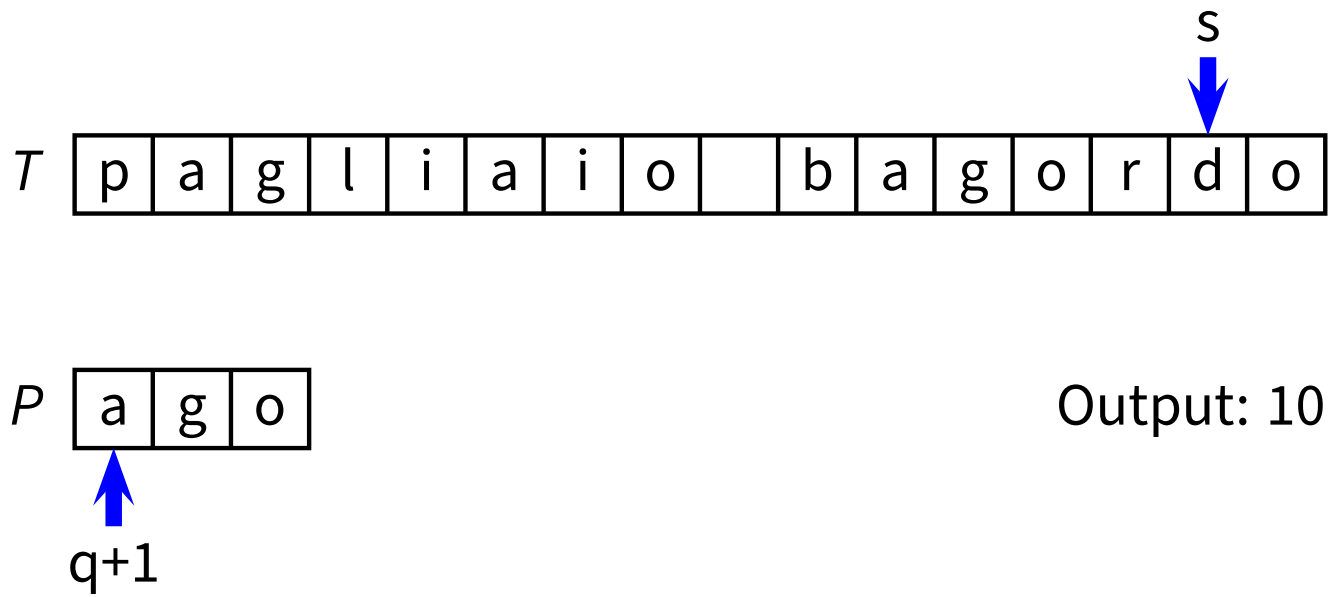

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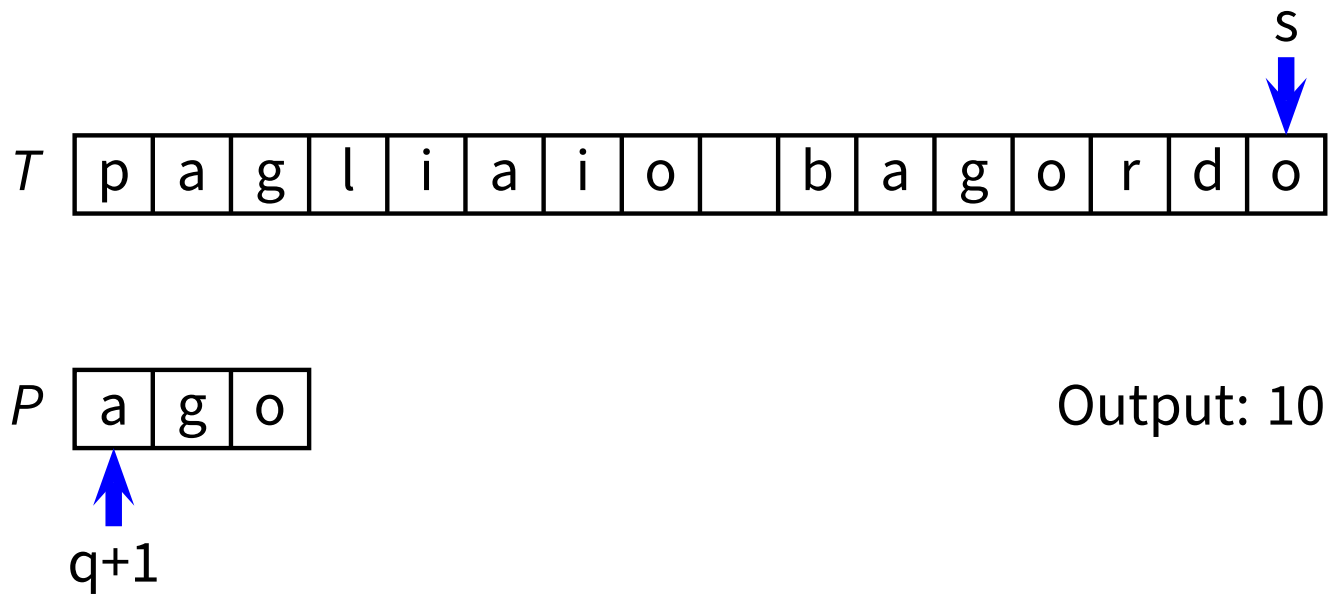

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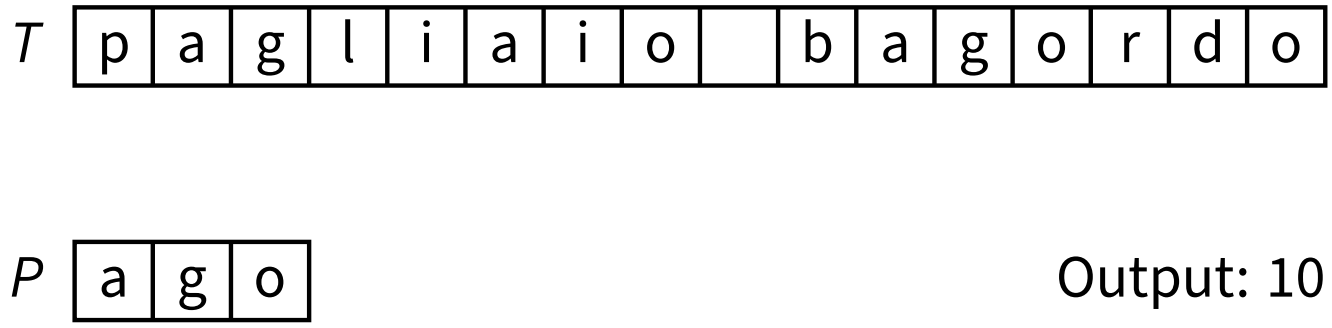

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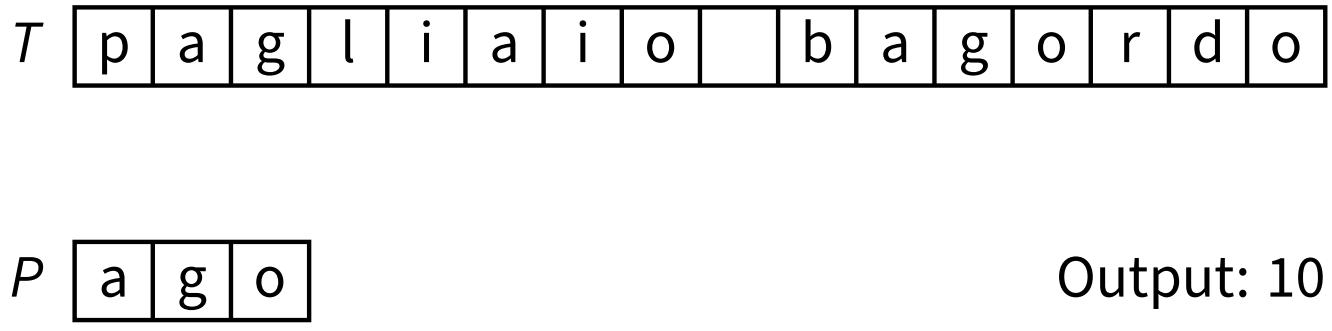

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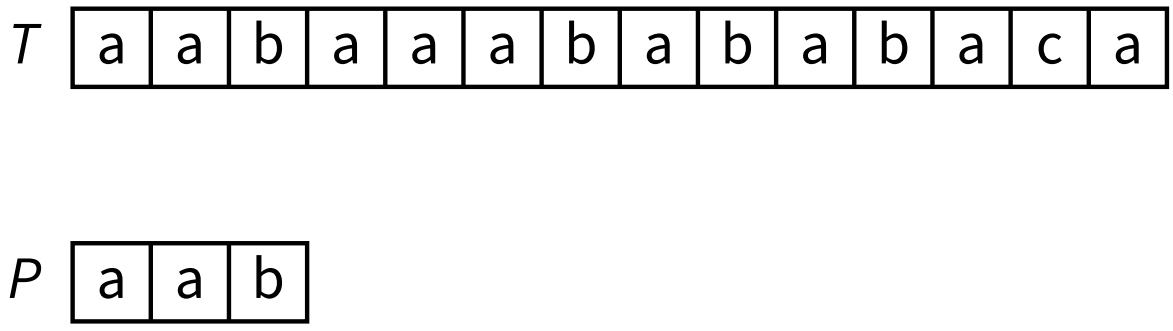

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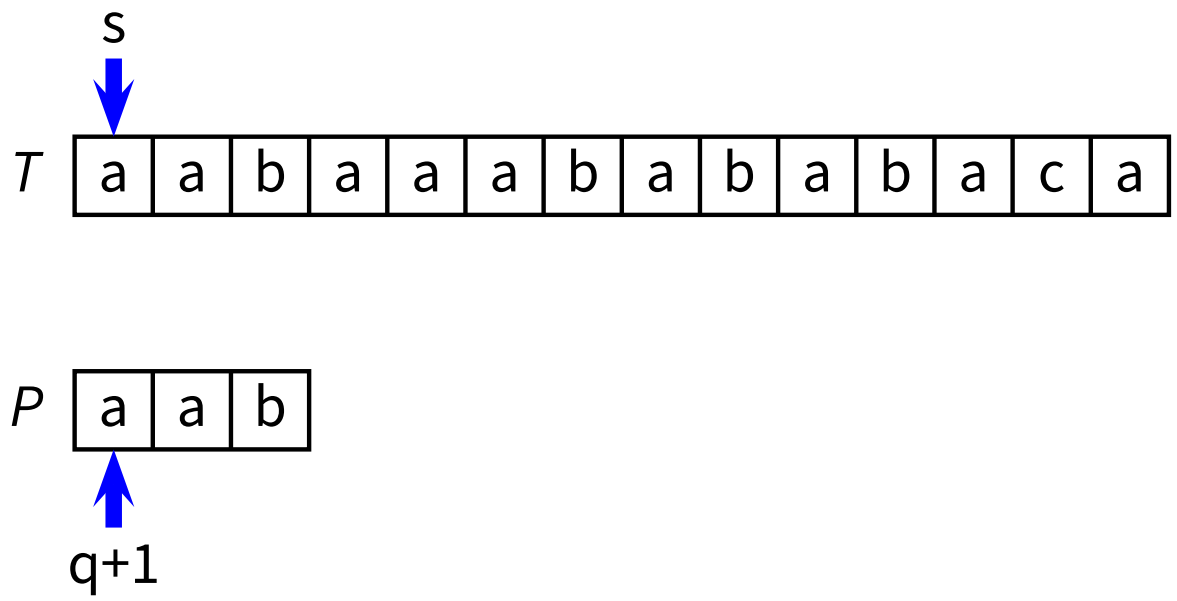

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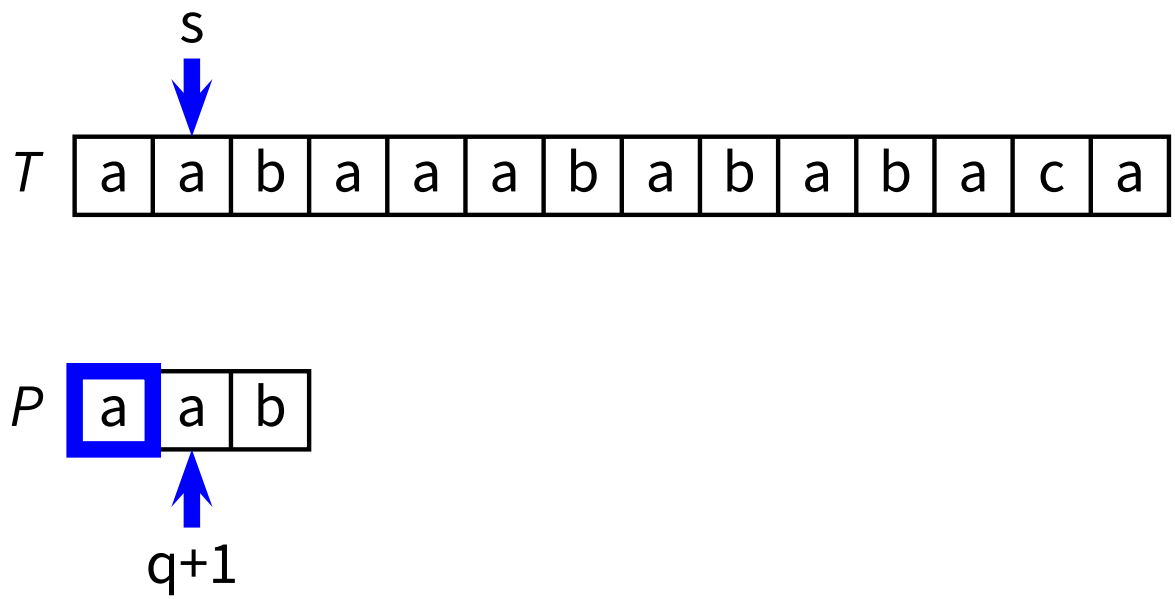

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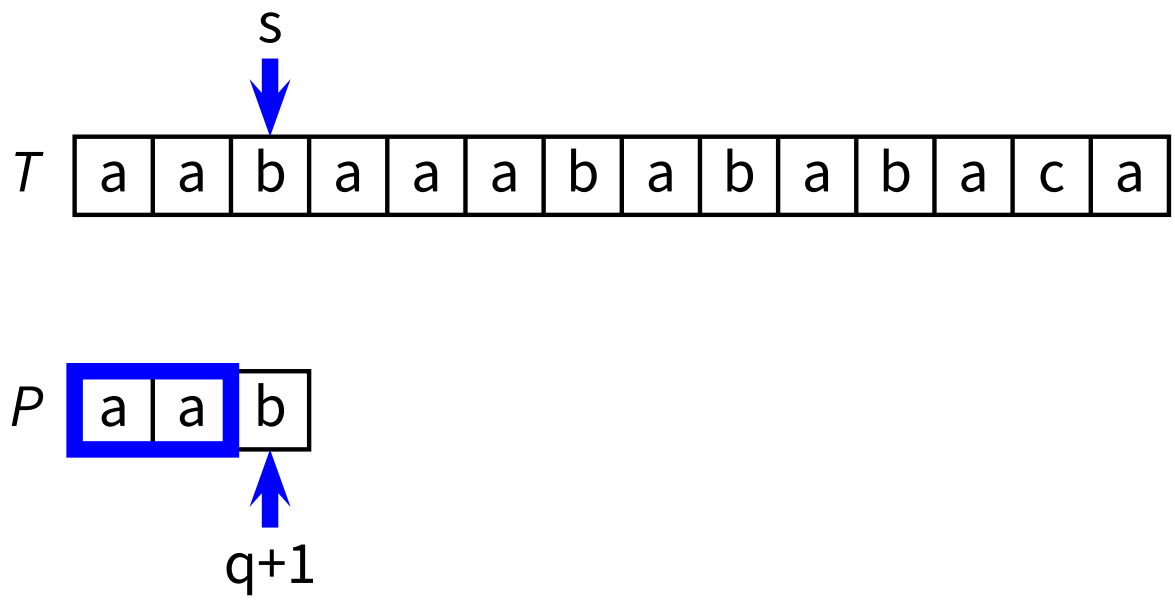

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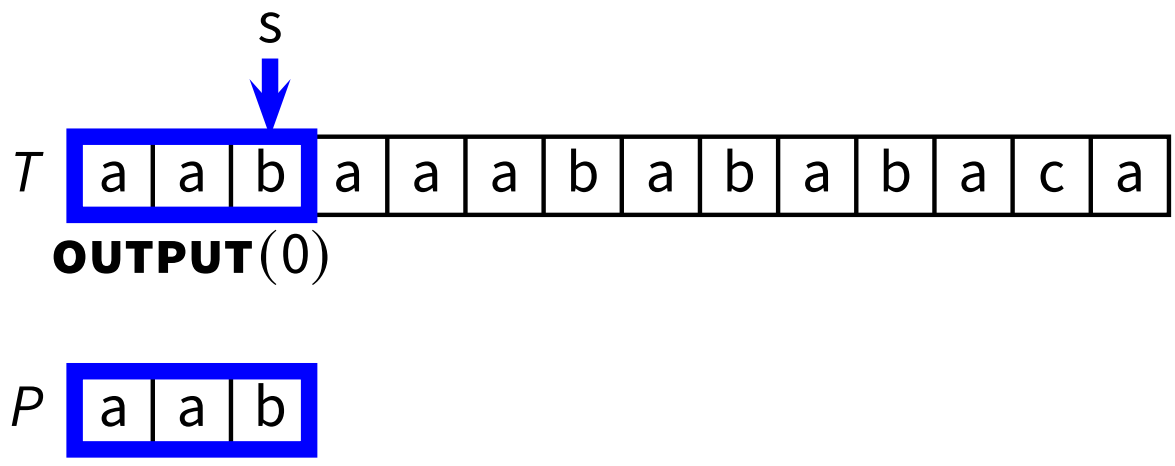

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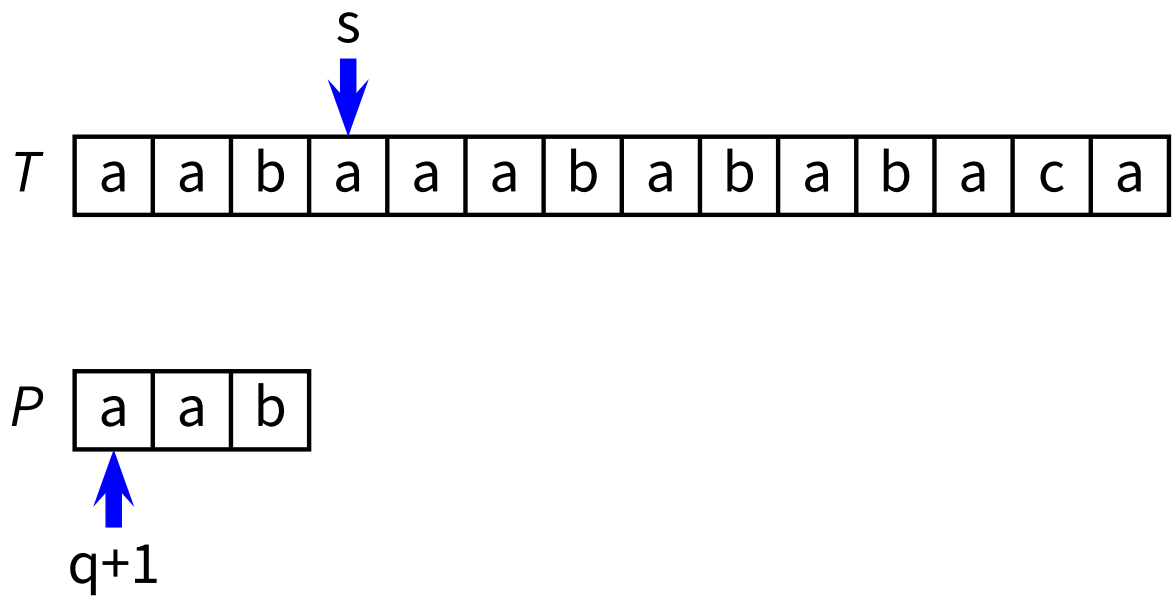

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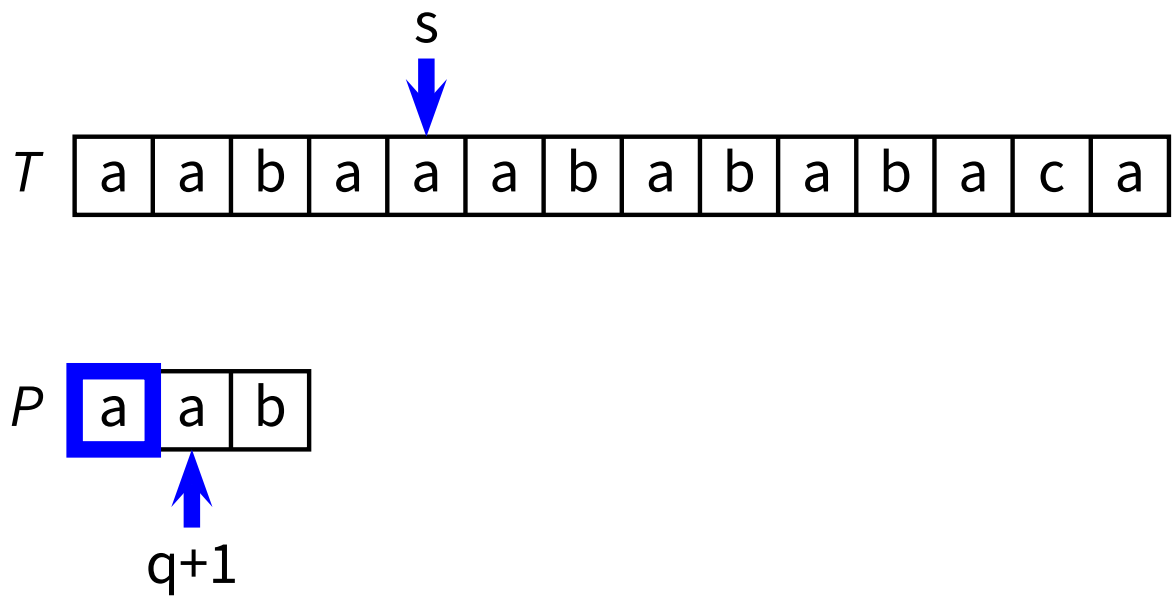

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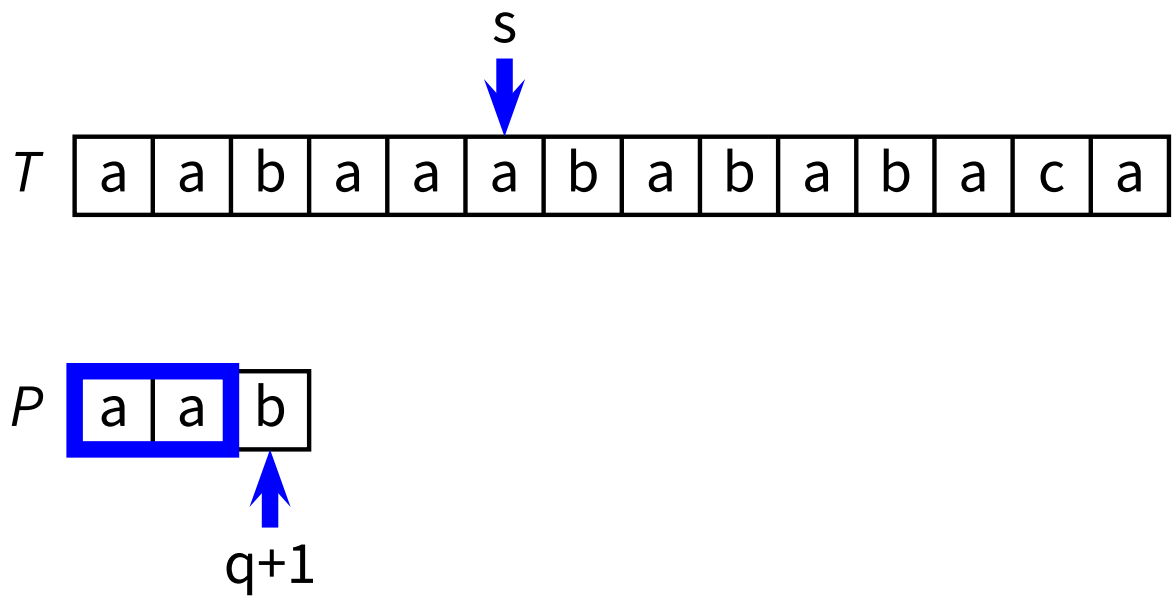

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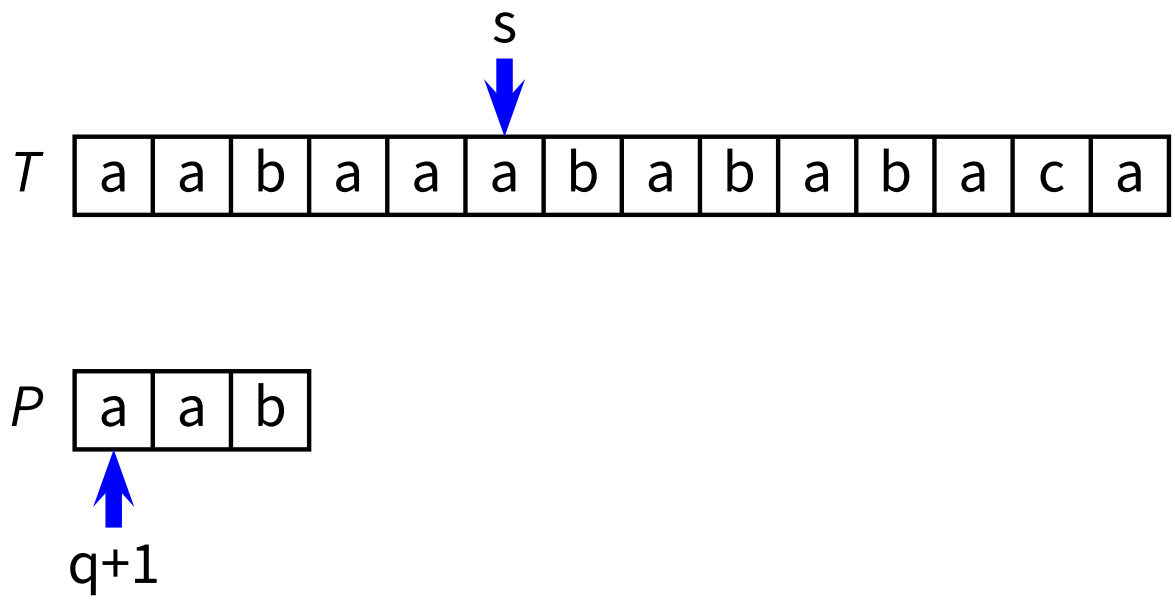

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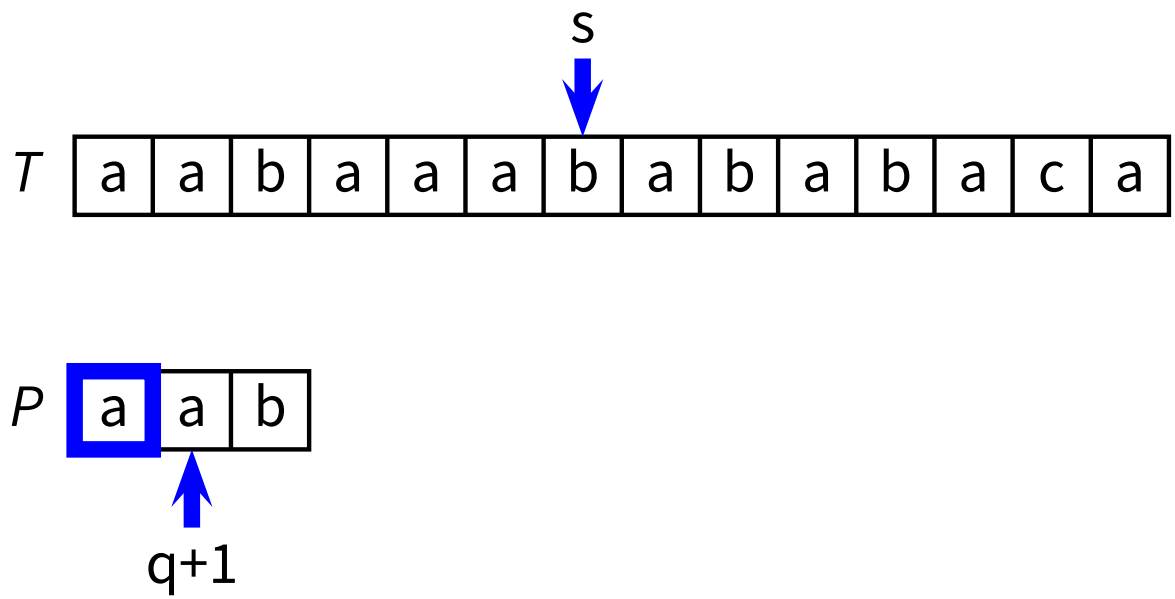

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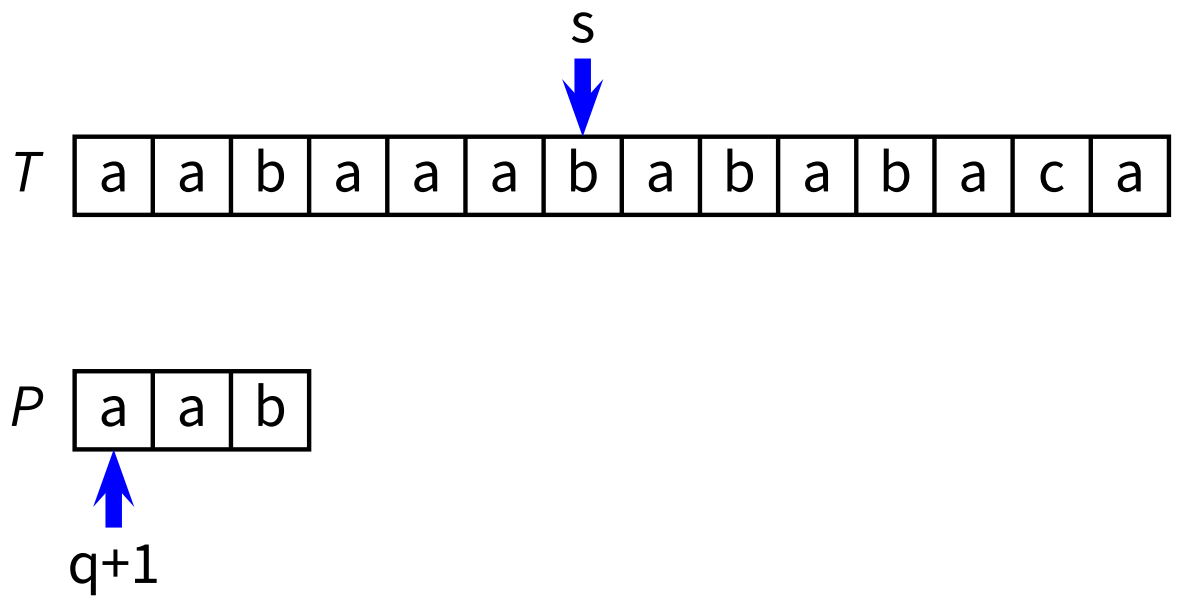

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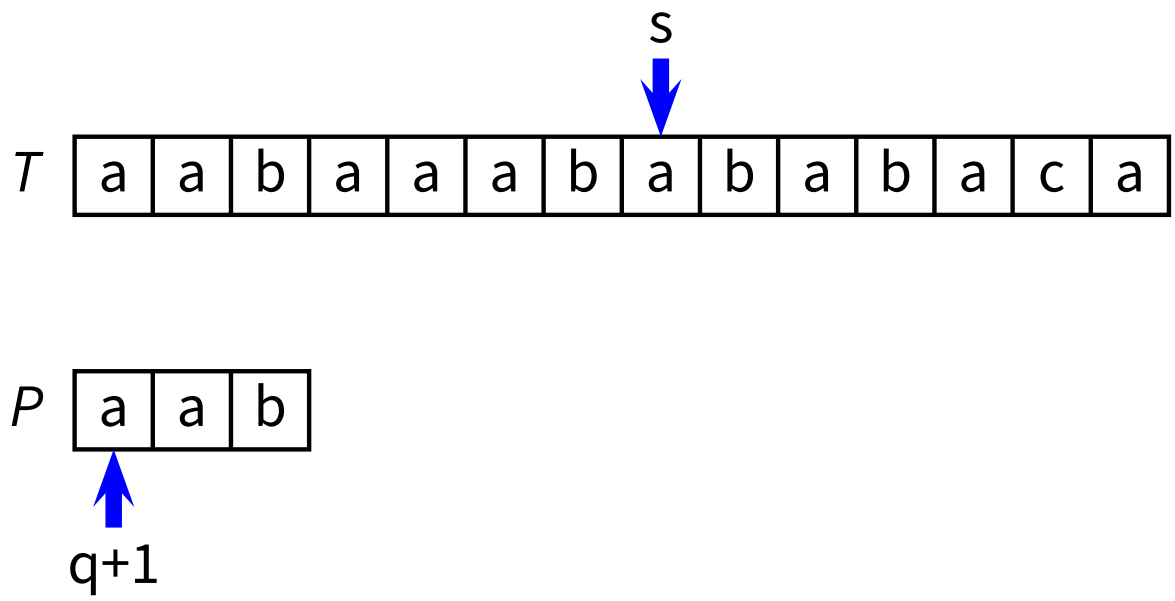

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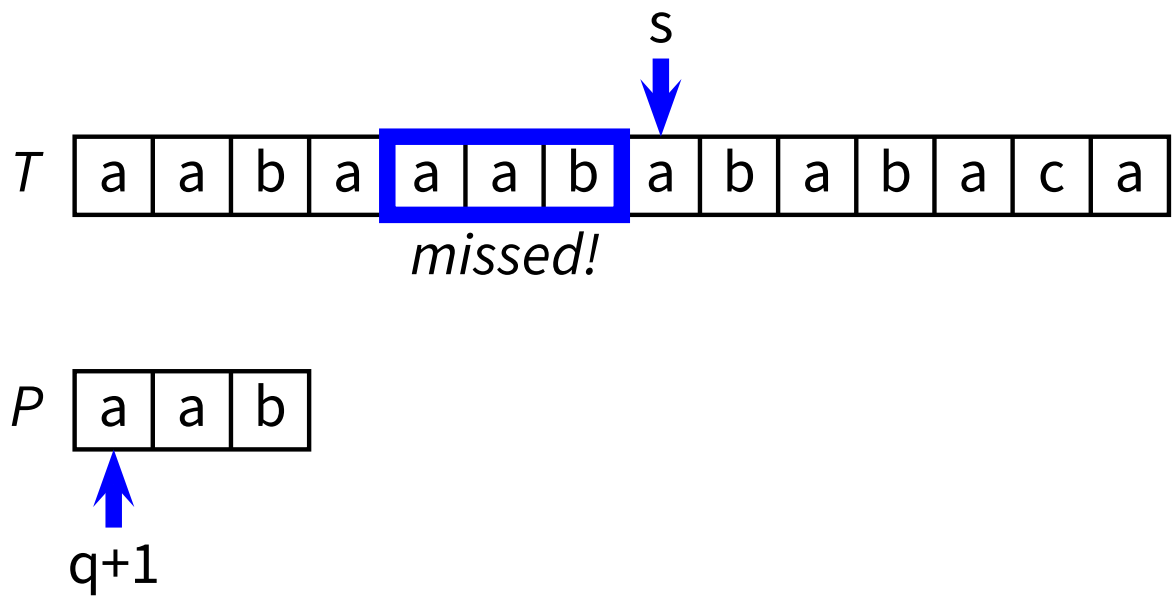

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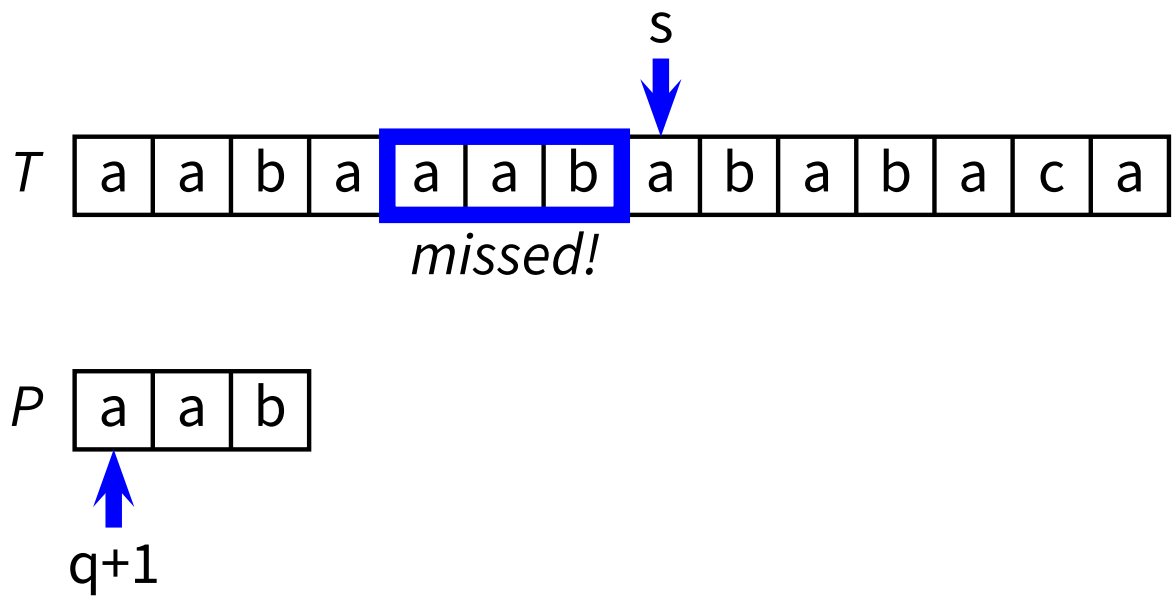

- So **WRONG-STRING-MATCHING** doesn't work, but it tells us something useful

Improvement Strategy (5)

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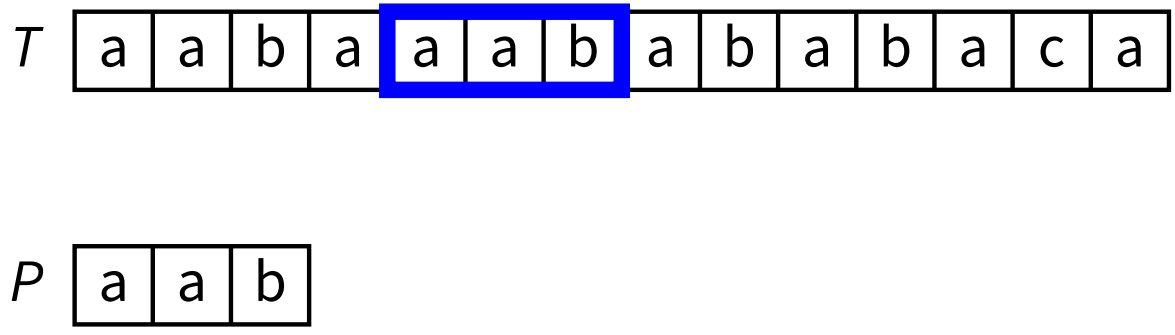

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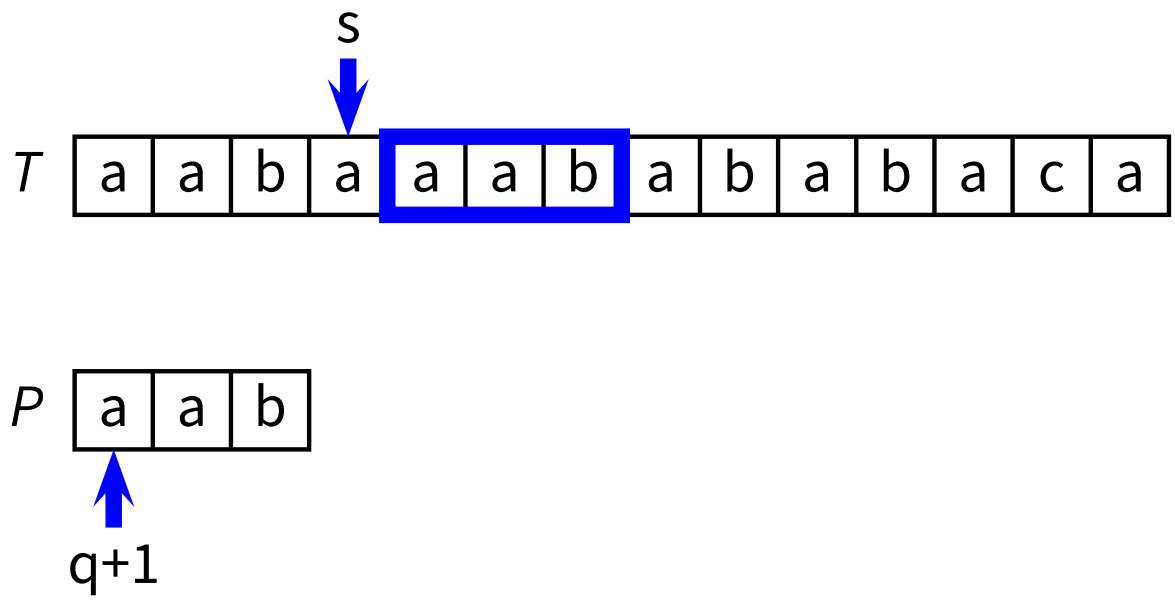

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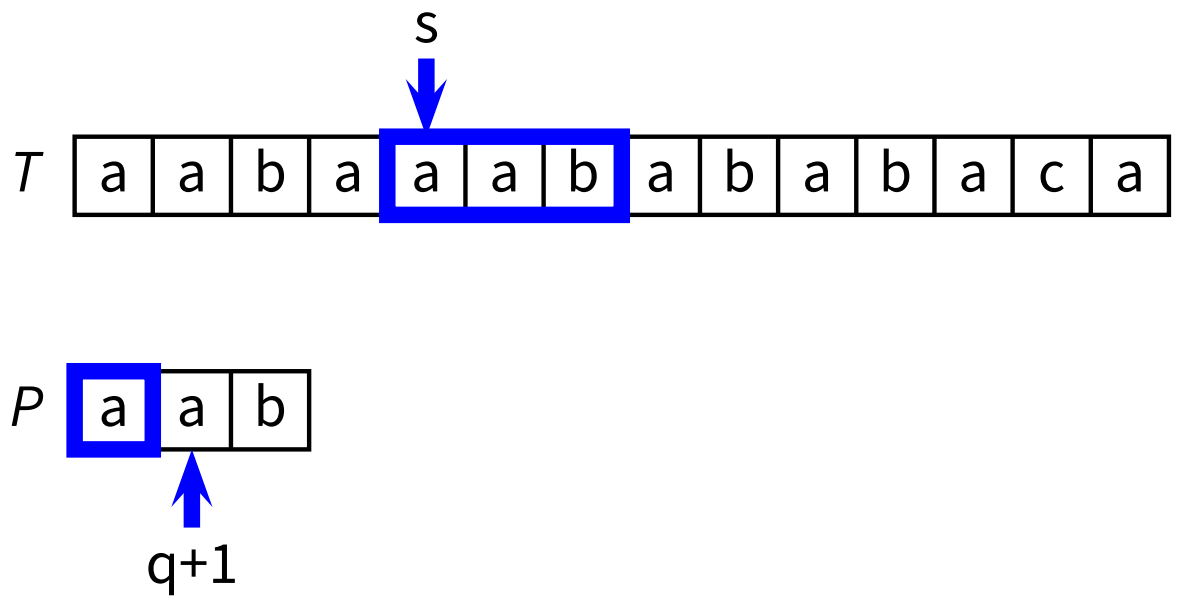

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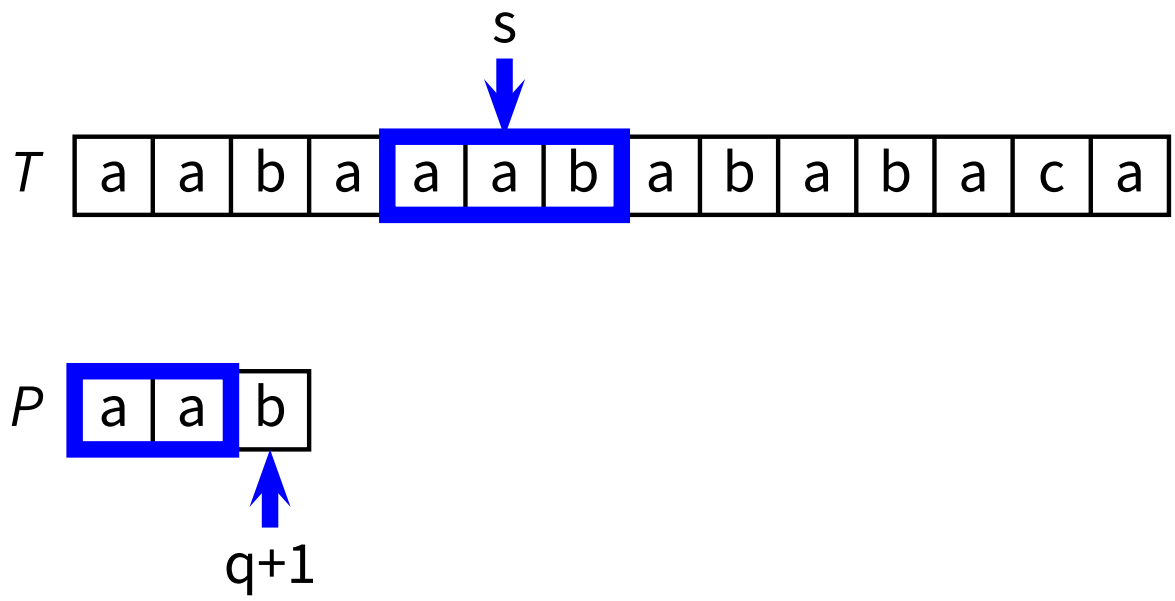

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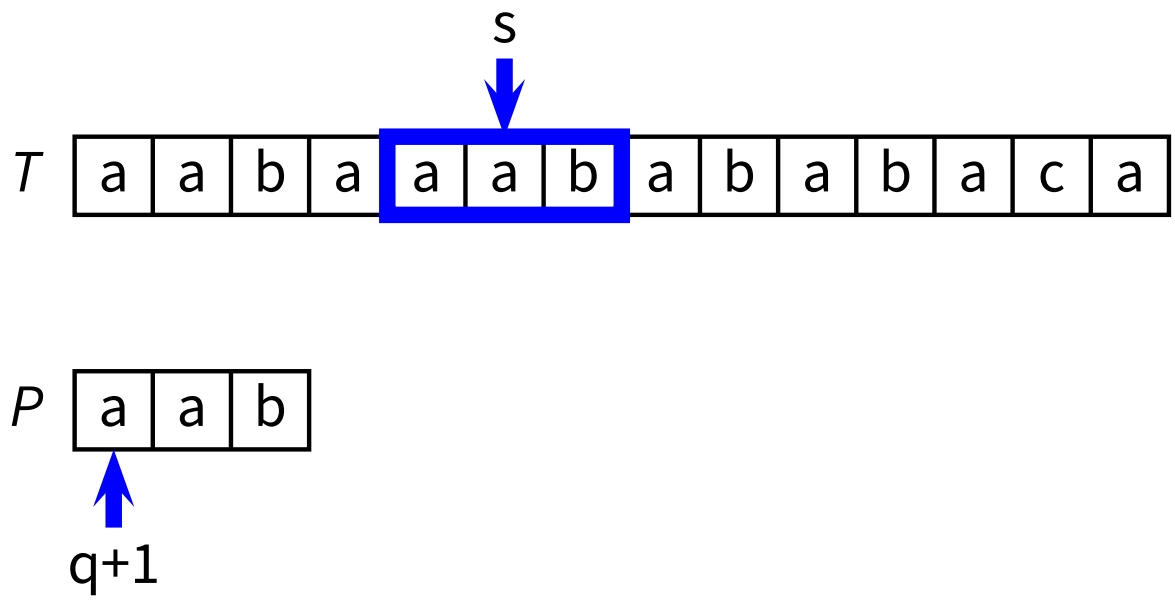

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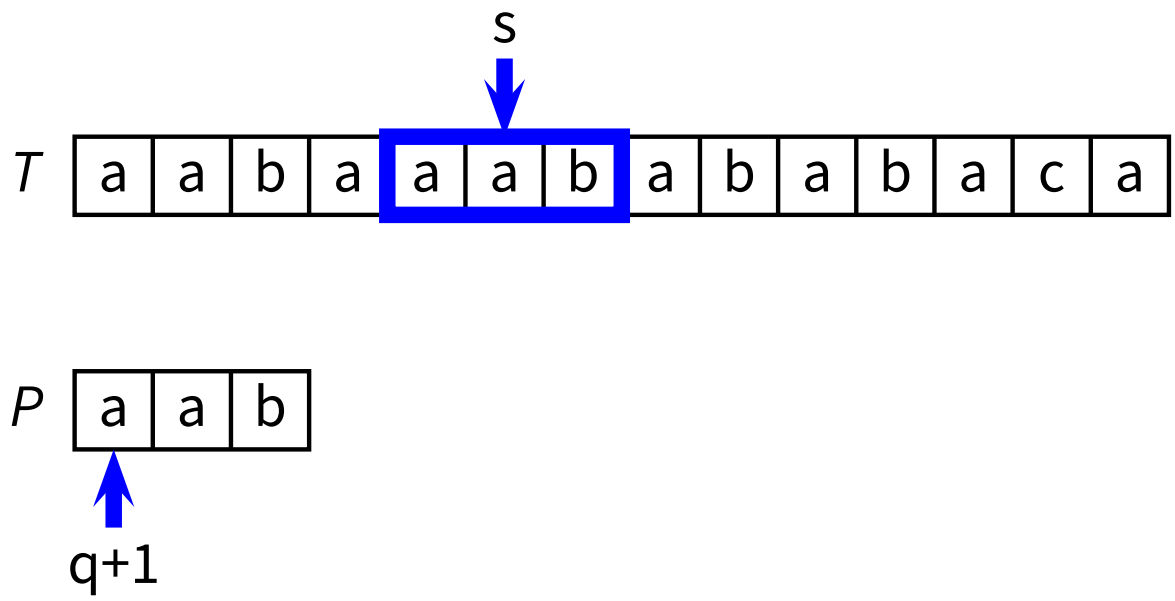

- Wrong: by going all the way back to $q = 0$ we throw away a good prefix of P that we already matched

Improvement Strategy (6)

- Another example

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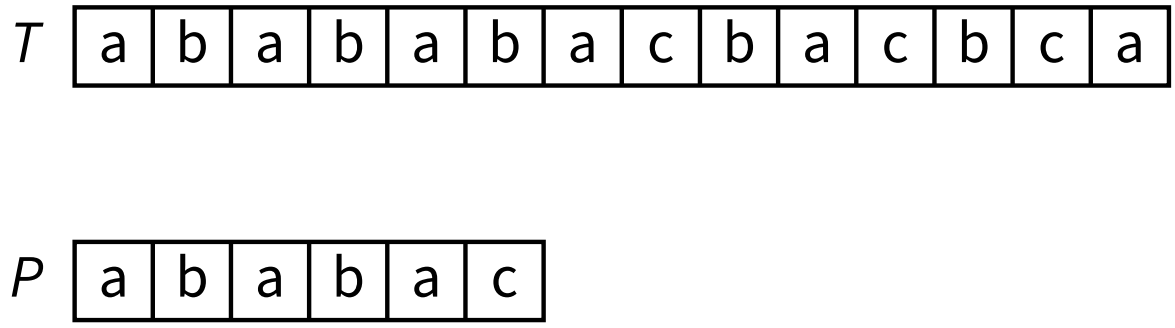

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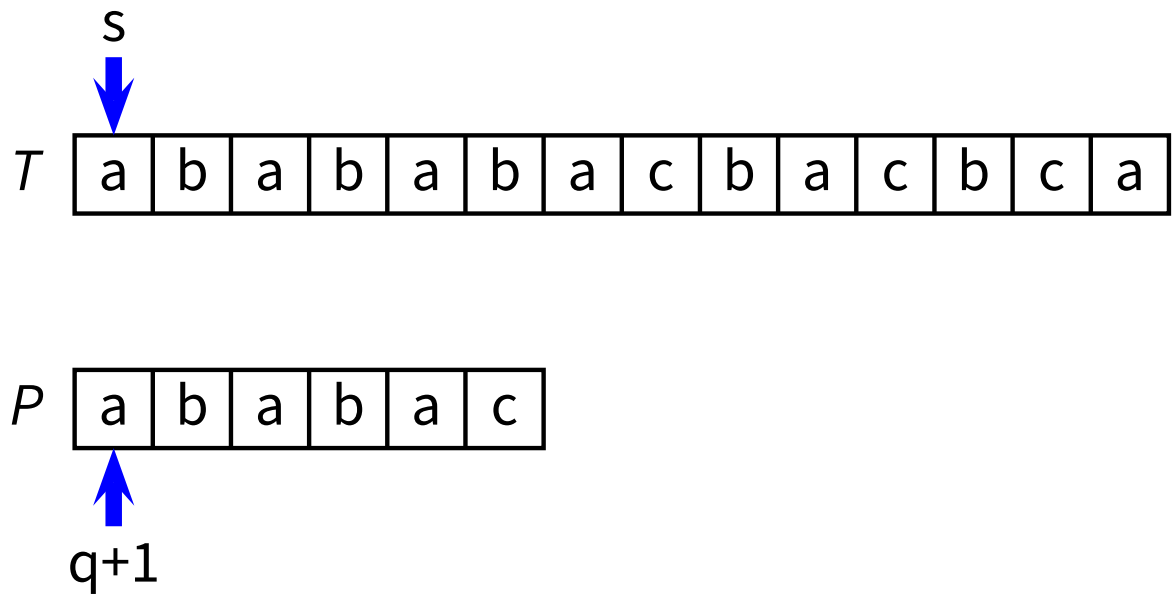

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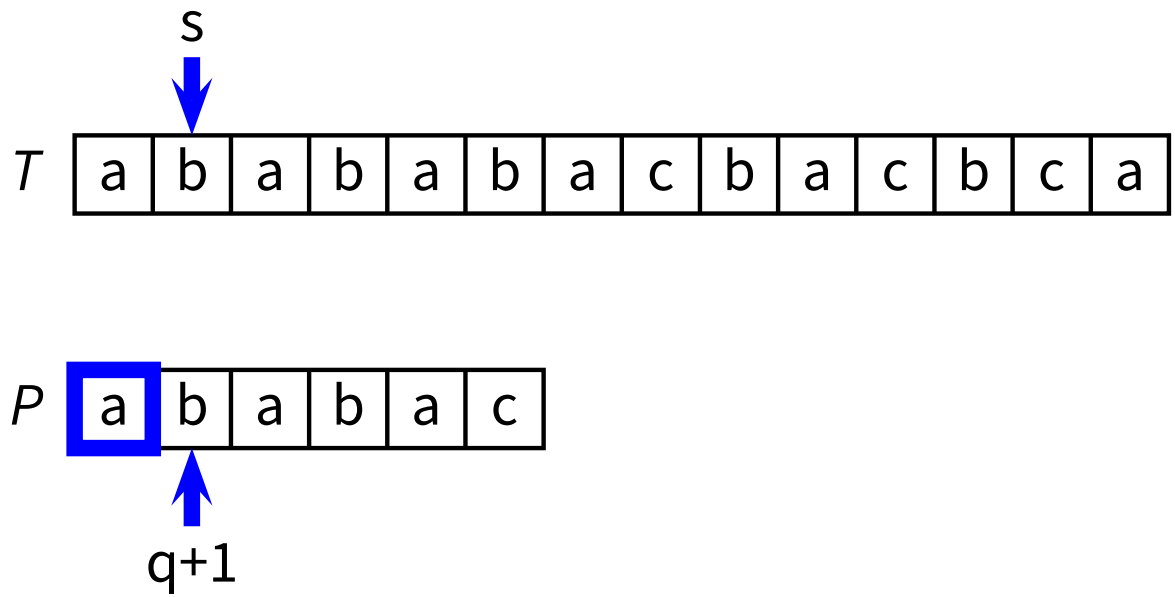

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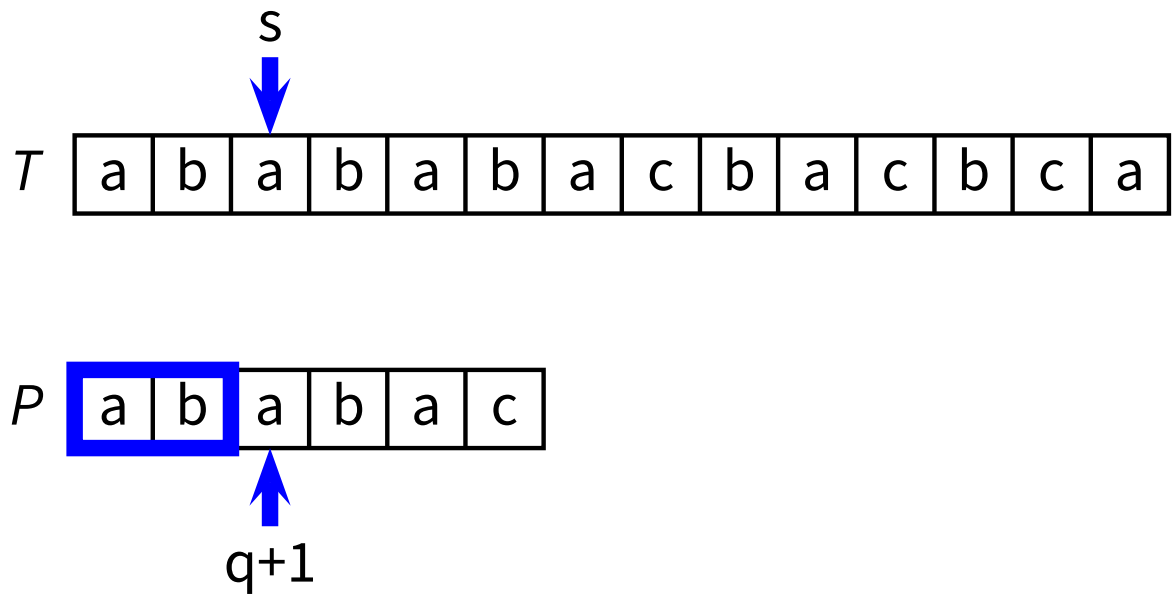

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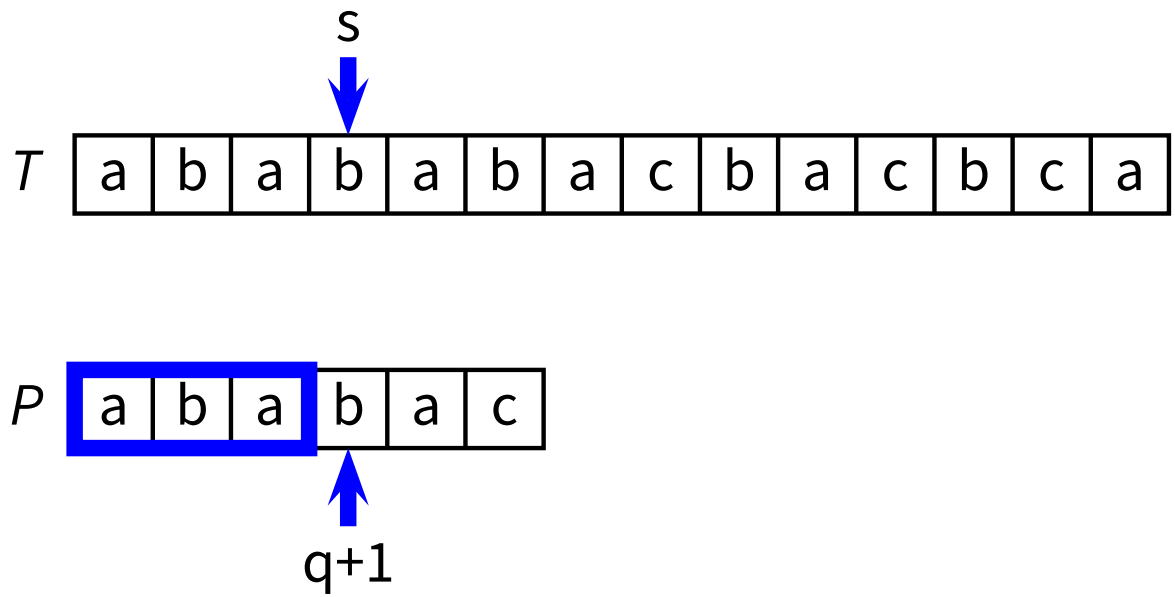

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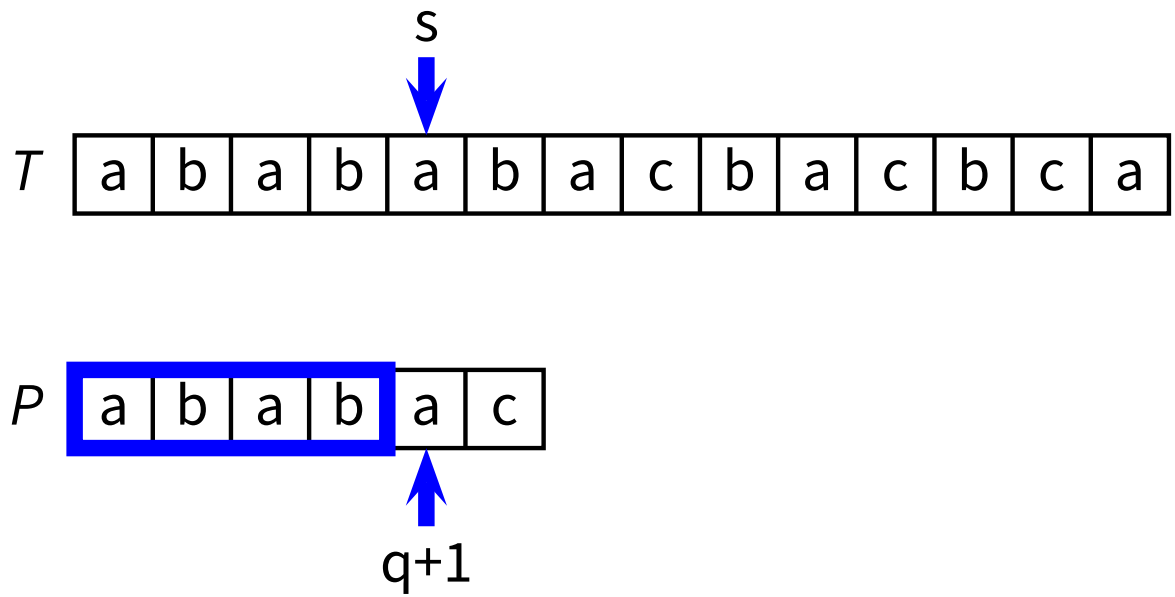

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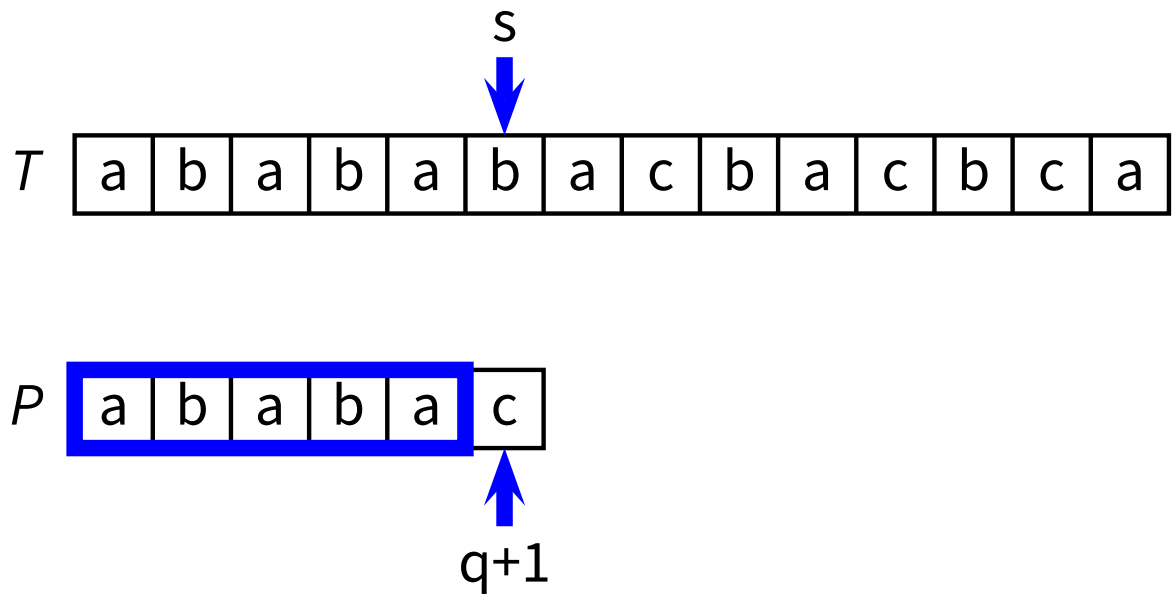

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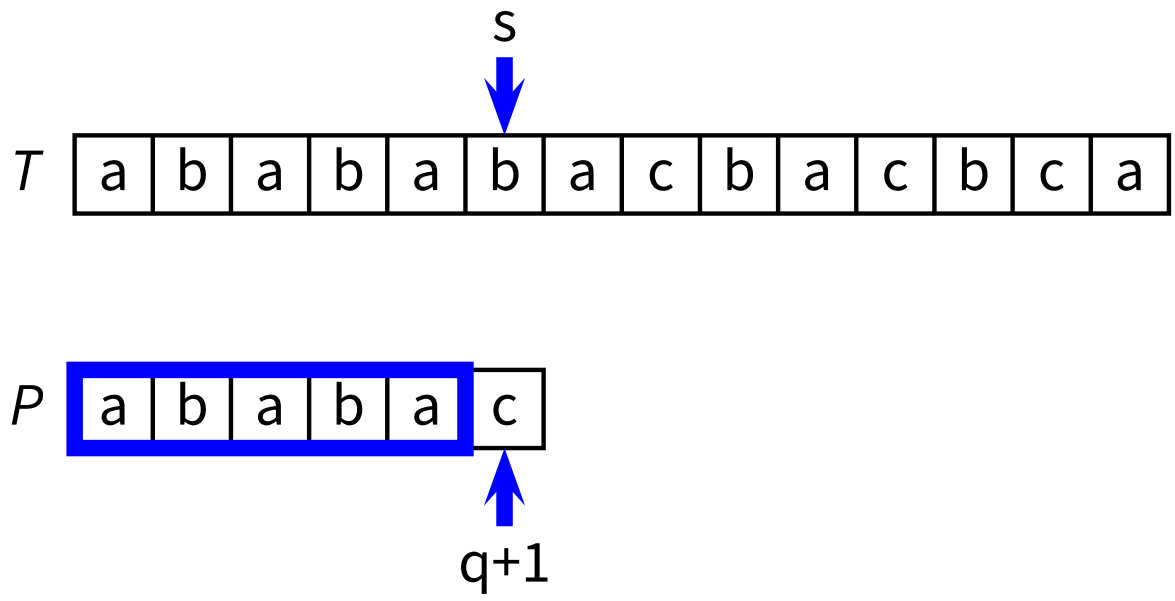

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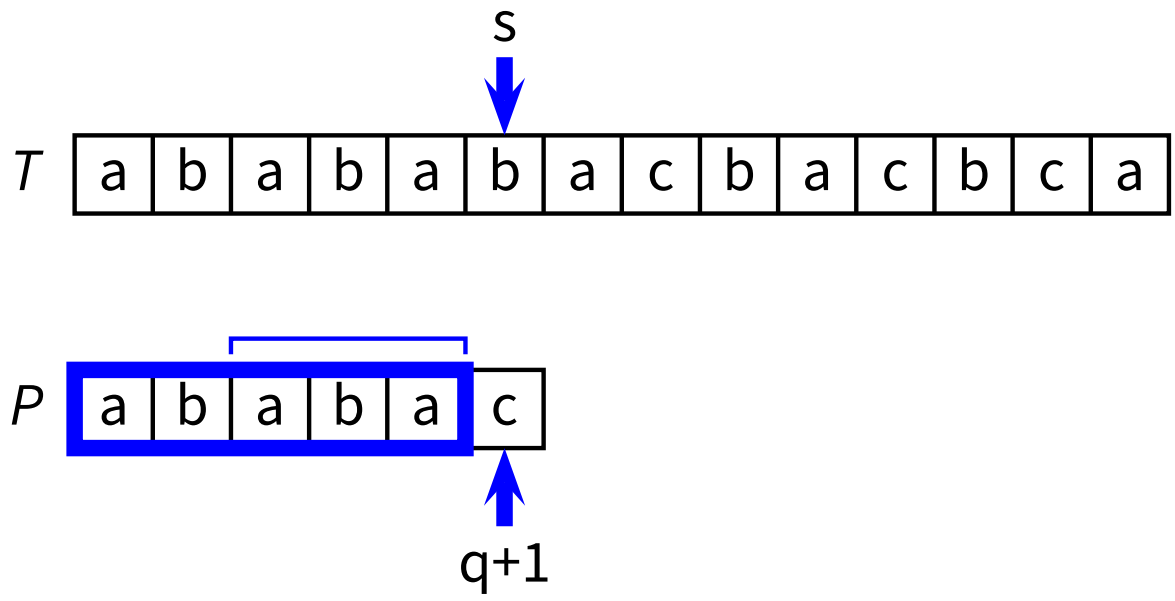

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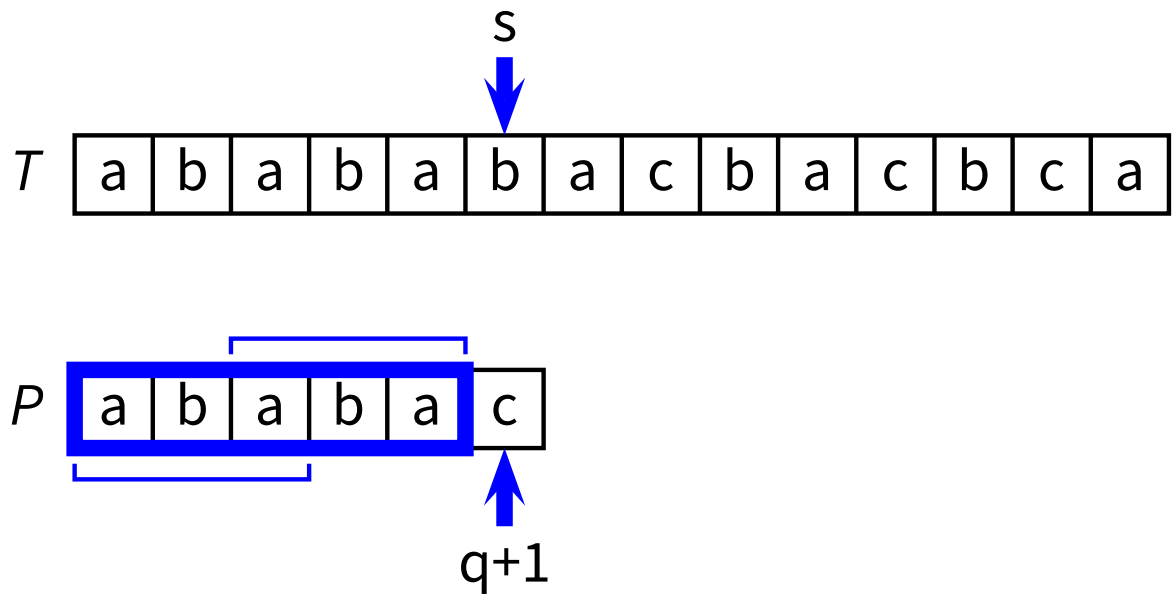

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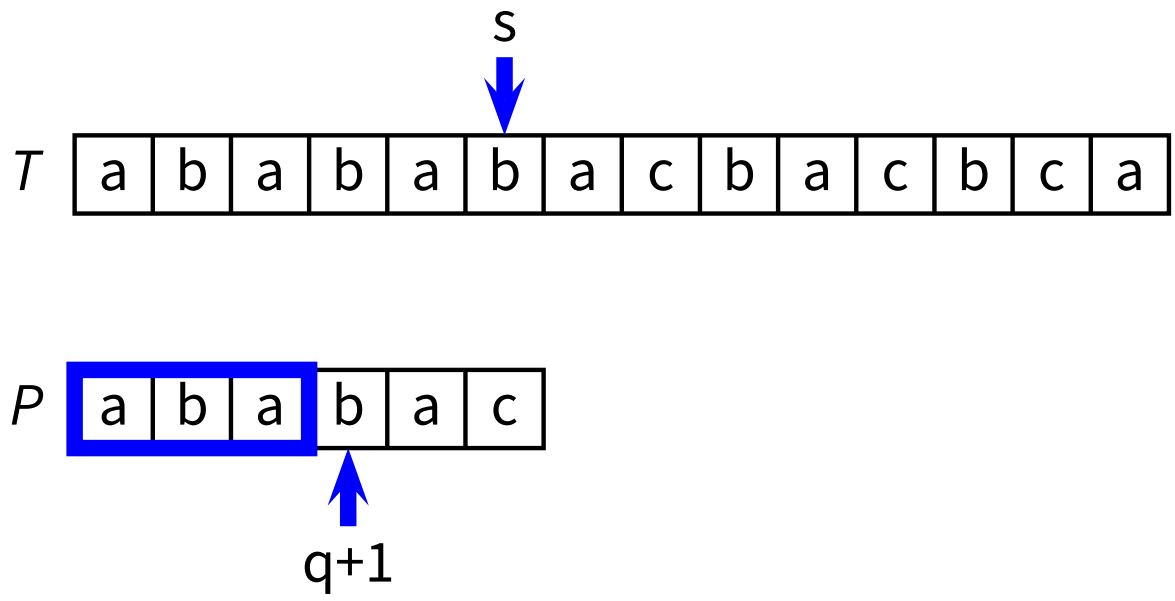

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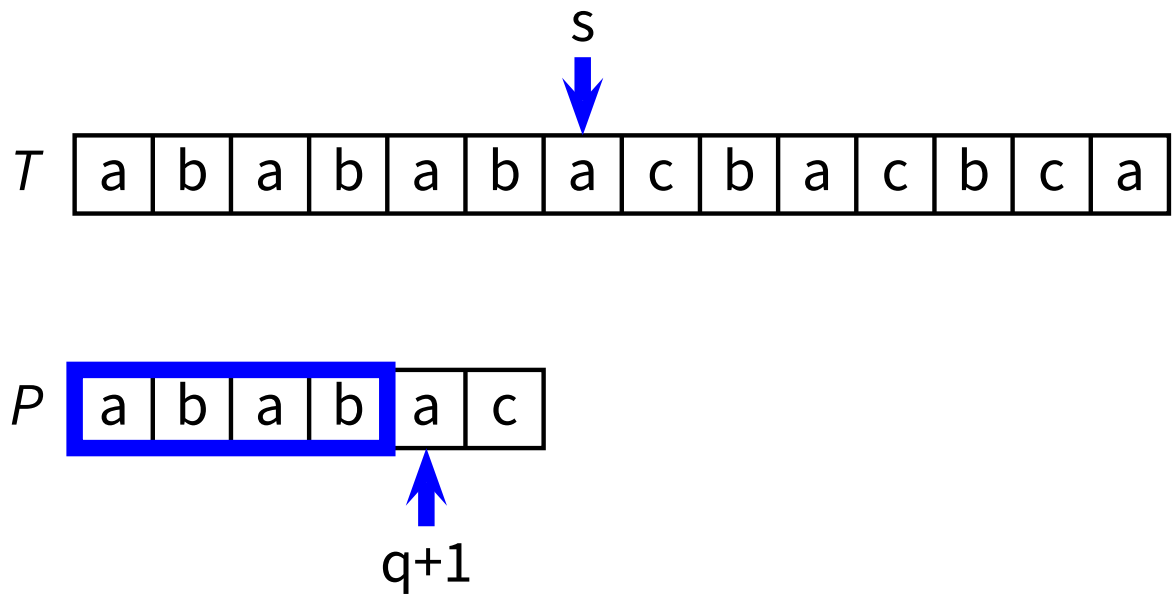

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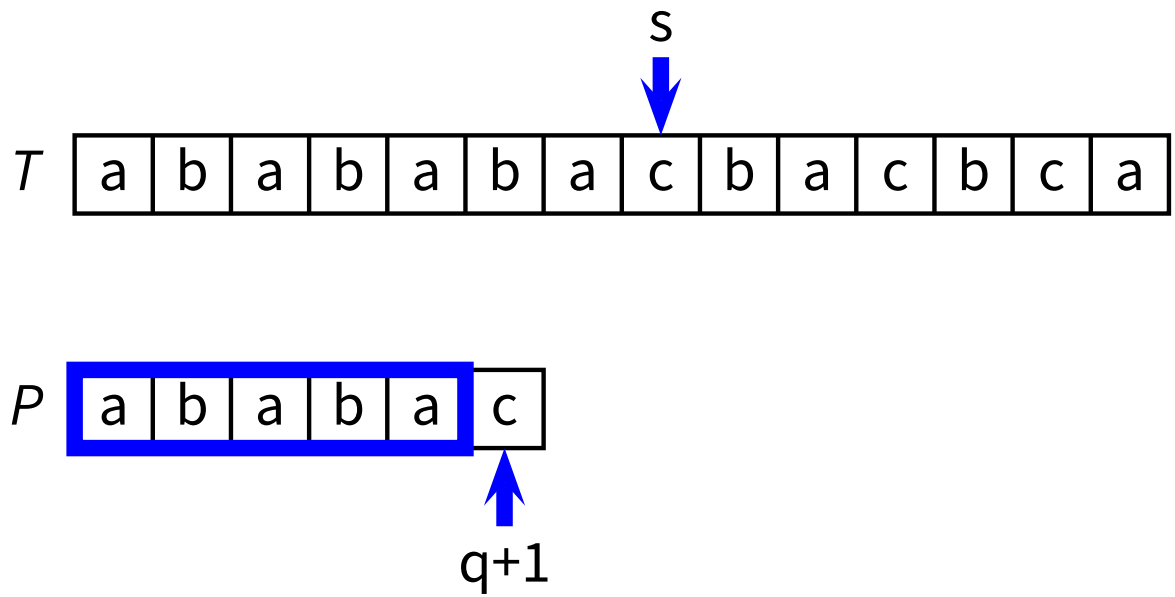

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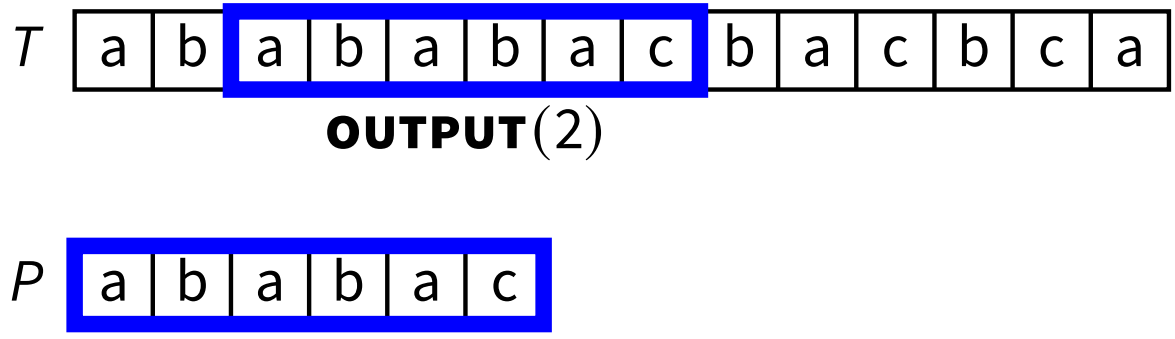

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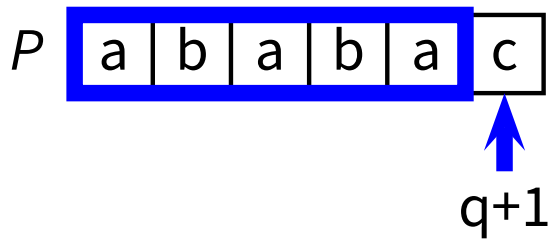

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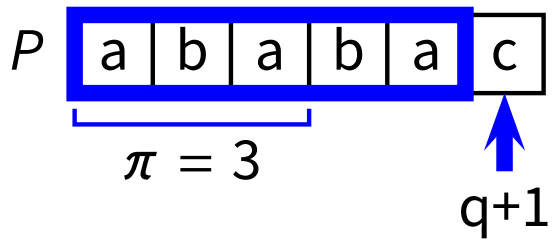

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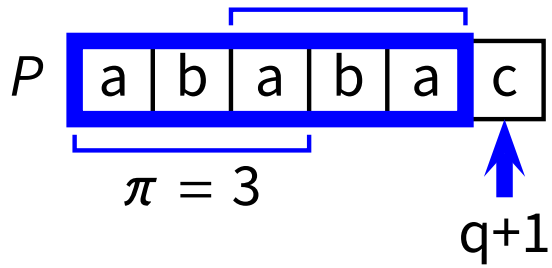

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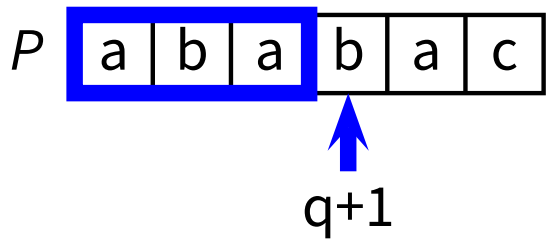

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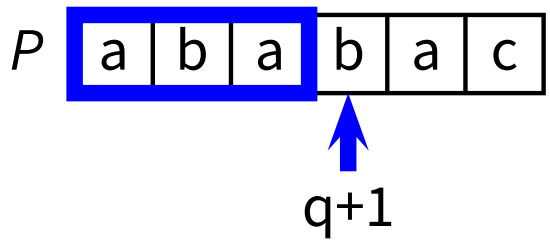

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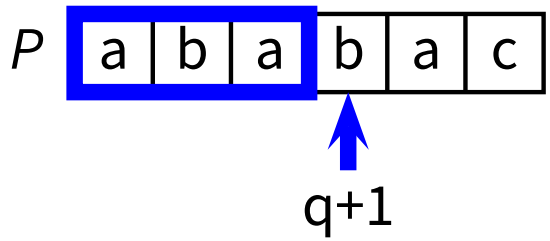

- Restart from $q = \pi$
- Iterate as usual
- In essence, this is the Knuth-Morris-Pratt algorithm

The Prefix Function

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π	0	0	1	2	3	0

The Knuth-Morris-Pratt Algorithm

```
KMP-String-Matching(T, P)
 1   n = length(T)
 2   m = length(P)
 3   π = Prefix-Function(P)
 4   q = 0                          // number of character matched
 5   for i = 1 to n                 // scan the text left-to-right
 6       while q > 0 and P[q + 1] ≠ T[i]
 7           q = π[q]               // no match: go back using π
 8       if P[q + 1] == T[i]
 9           q = q + 1
10       if q == m
11           output(i − m)
12           q = π[q]               // go back for the next match
```

Prefix Function Algorithm

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- In fact, **PREFIX-FUNCTION** is remarkably similar to **KMP-STRING-MATCHING**

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PREFIX-FUNCTION(P)
1   m = length(P)
2   π[1] = 0
3   k = 0
4   for q = 2 to m
5       while k > 0 and P[k + 1] ≠ P[q]
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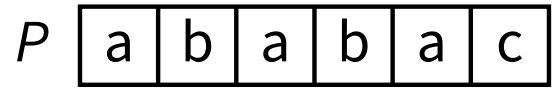

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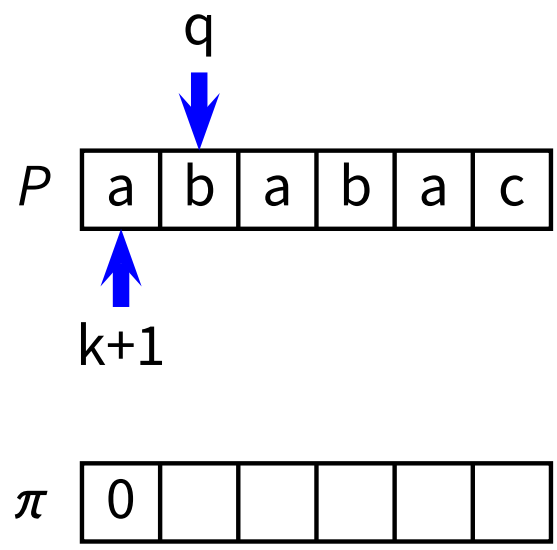

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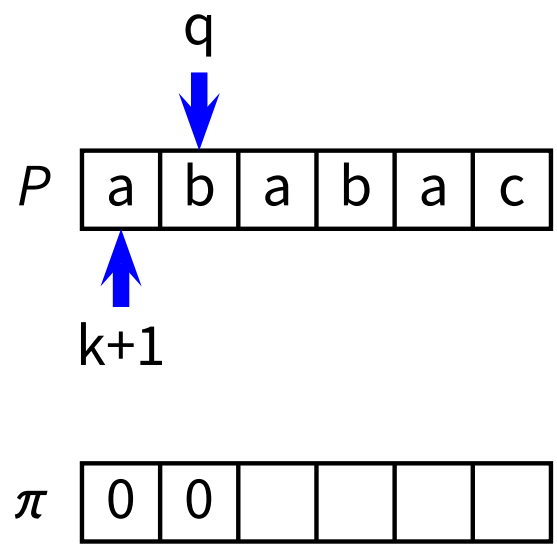

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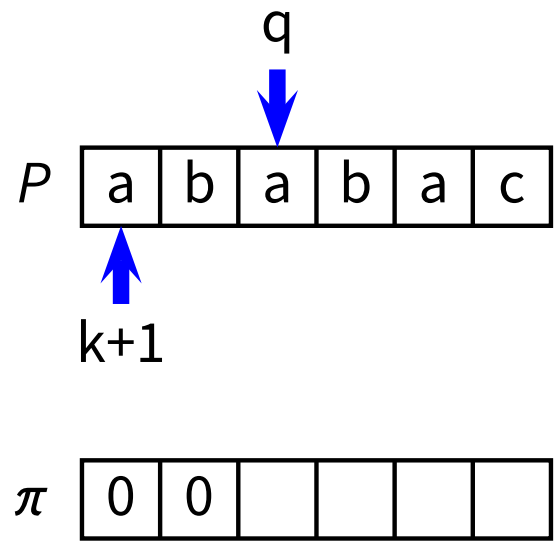

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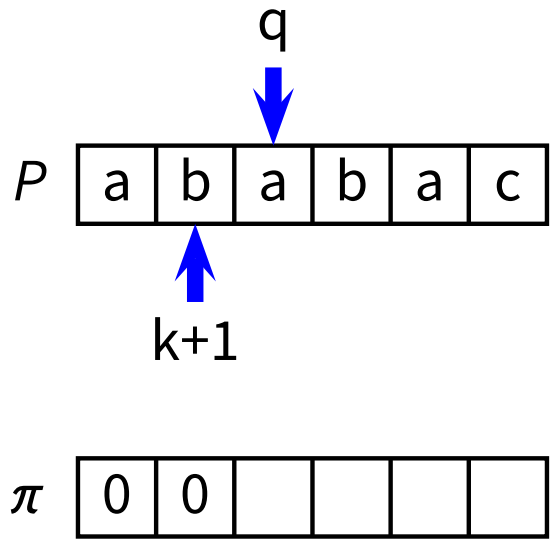

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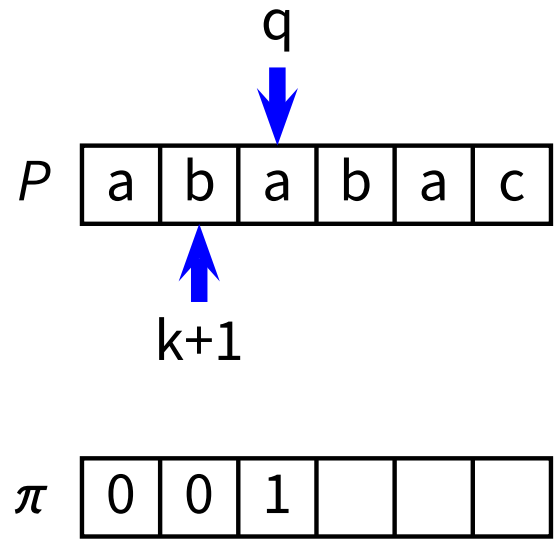

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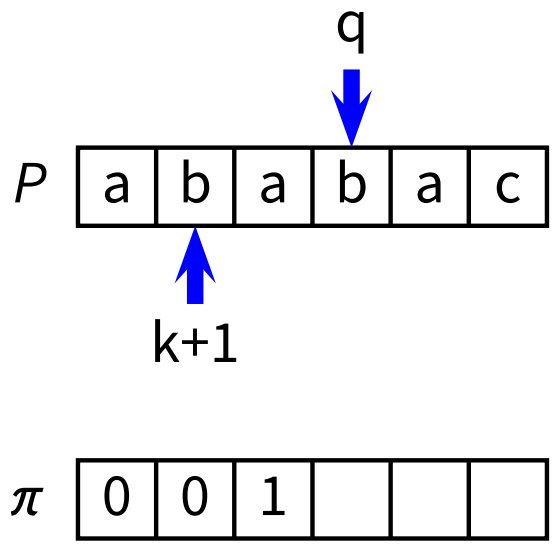

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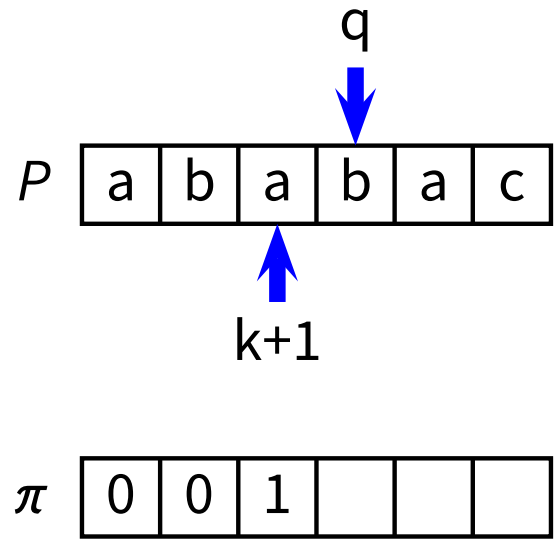

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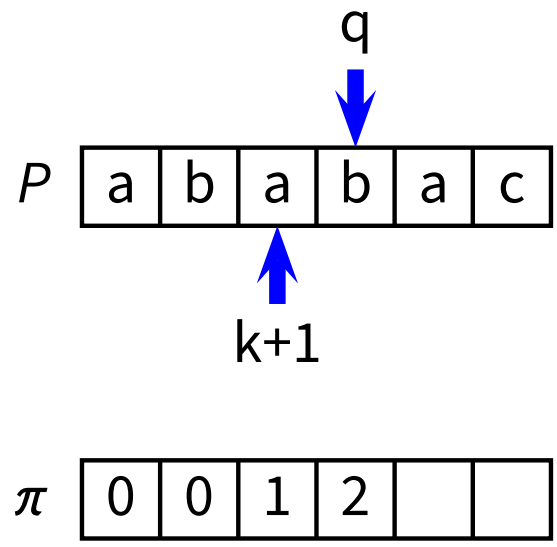

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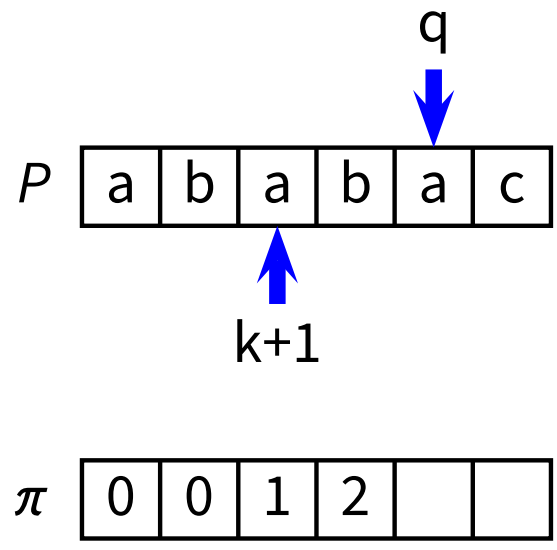

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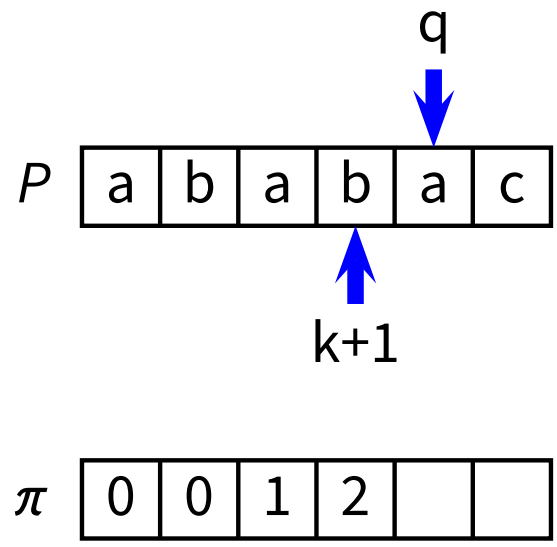

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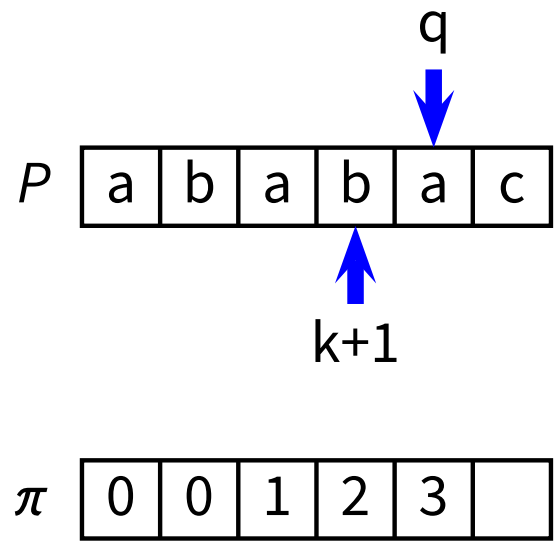

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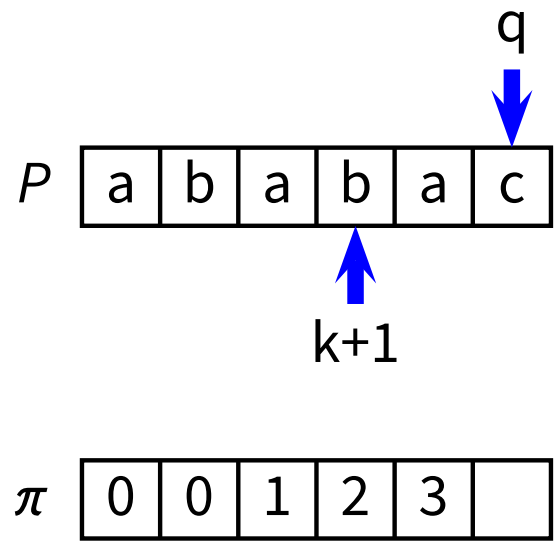

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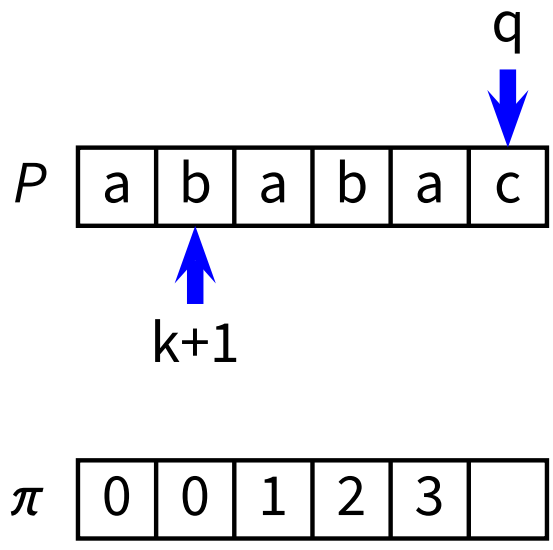

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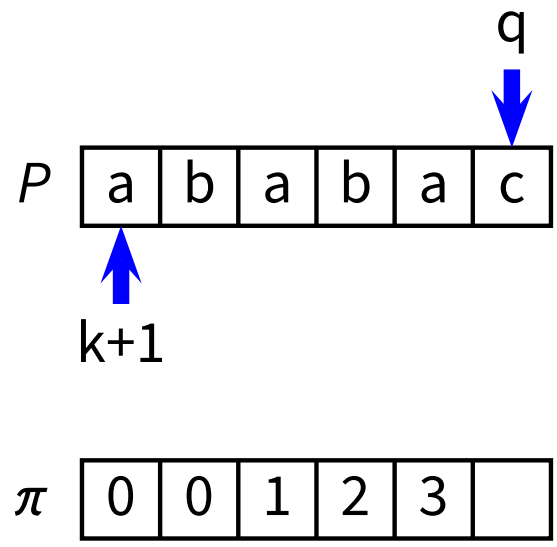

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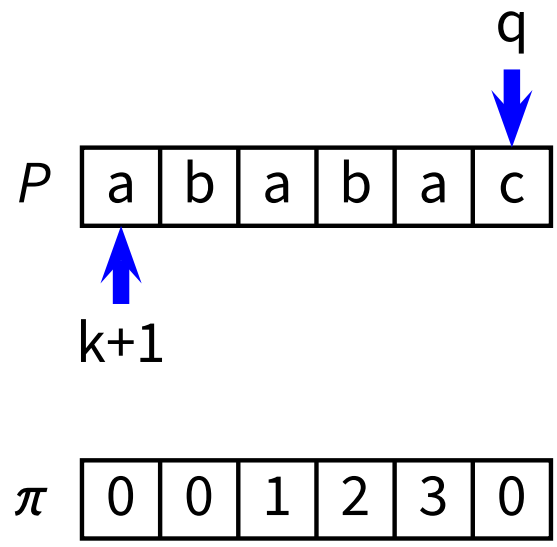

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- Can we do better?

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  - KMP will *always* go through *at least n* character comparisons
  - it fixes our "wrong" algorithm in the case of *periodic* patterns and texts
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  - e.g., in the absence of periodic patterns

A New Strategy

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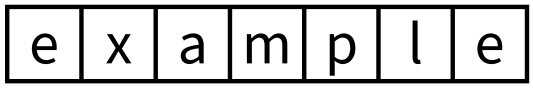

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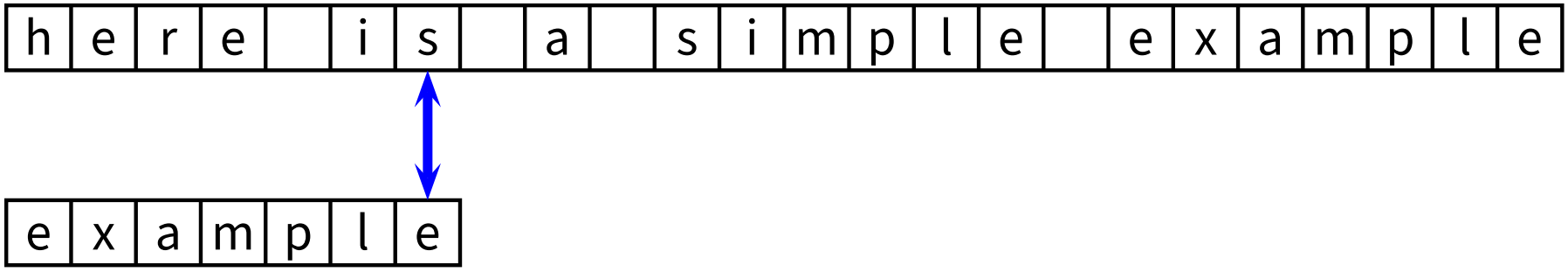

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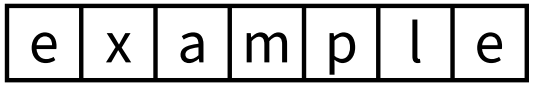

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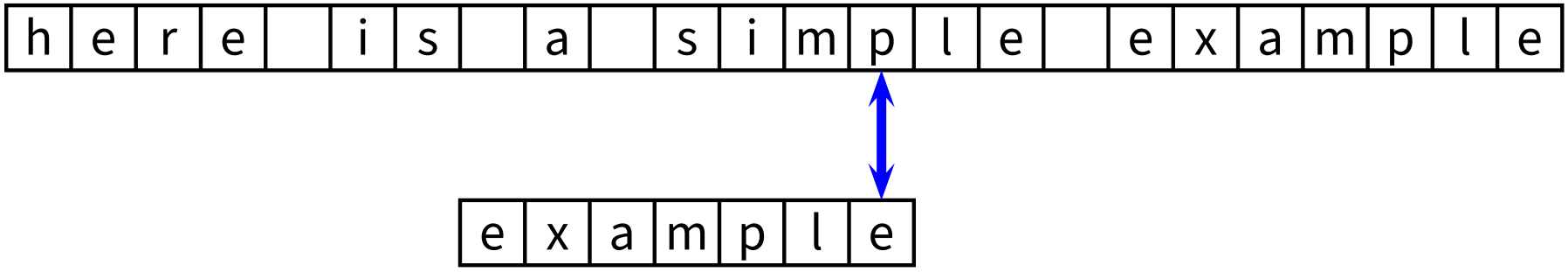

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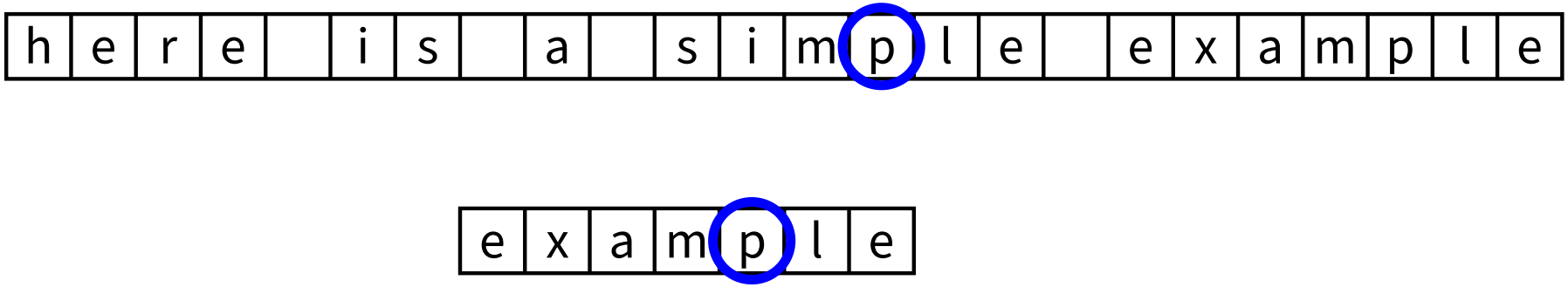

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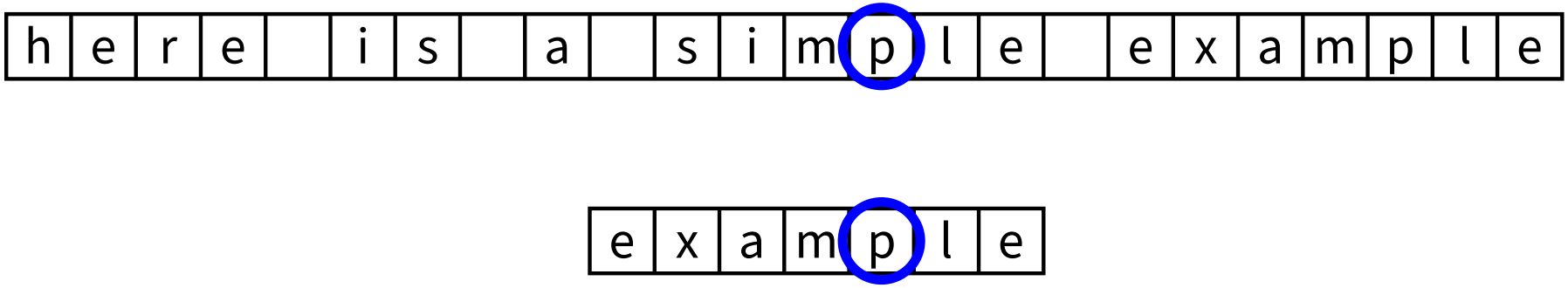

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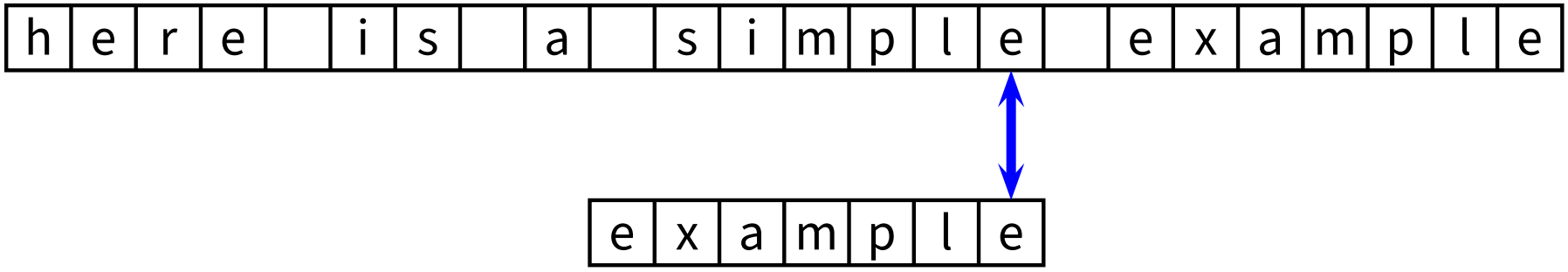

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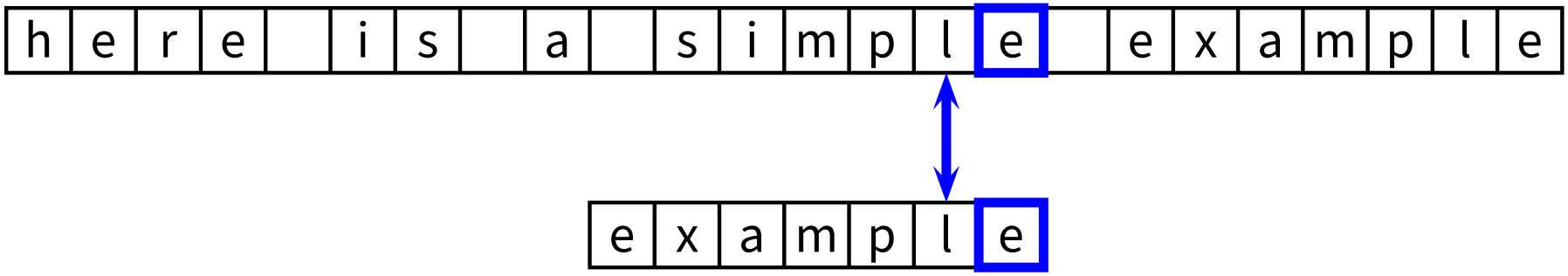

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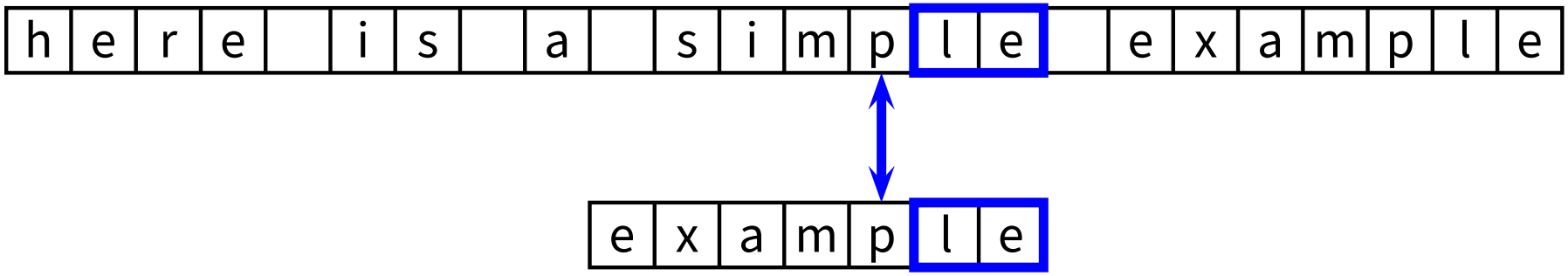

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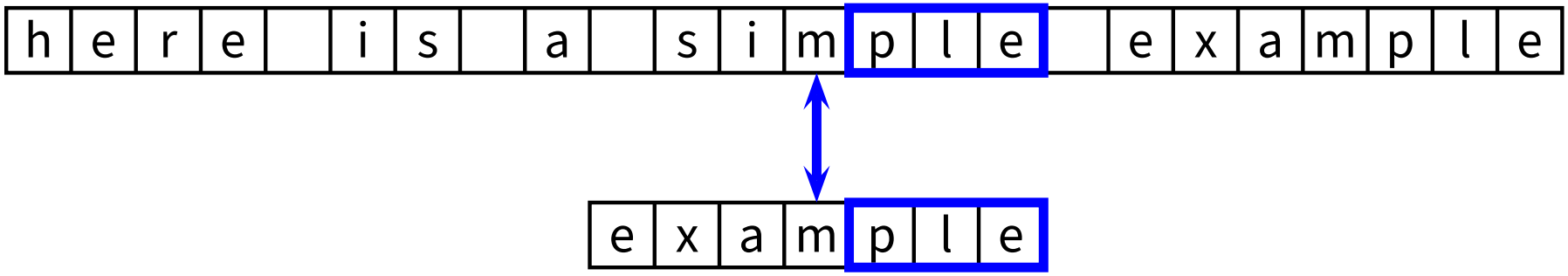

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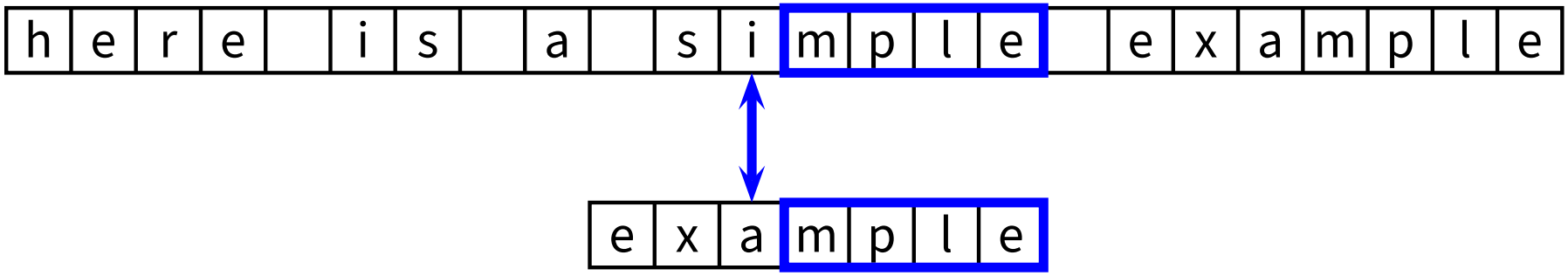

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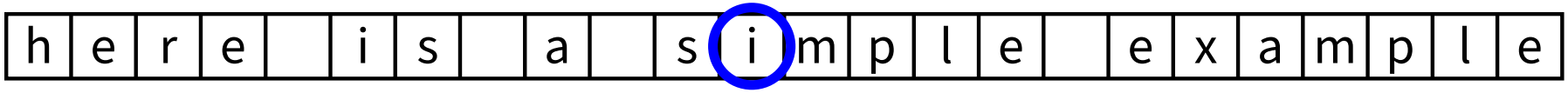

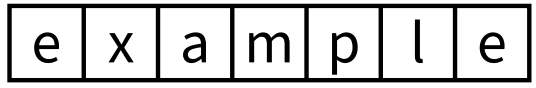

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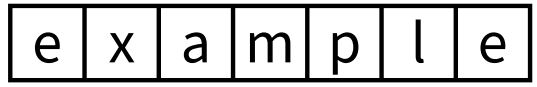

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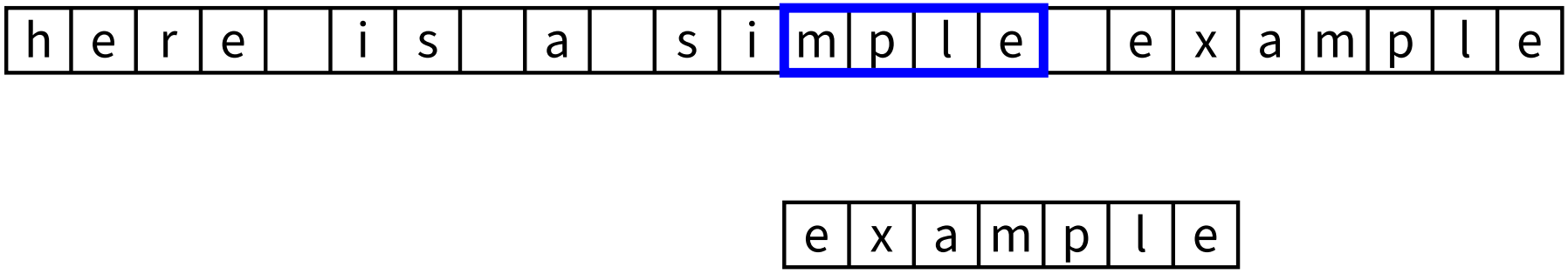

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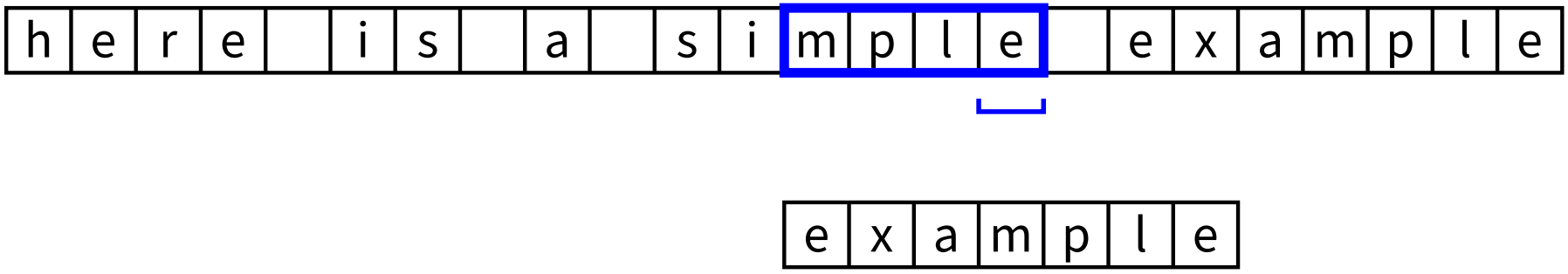

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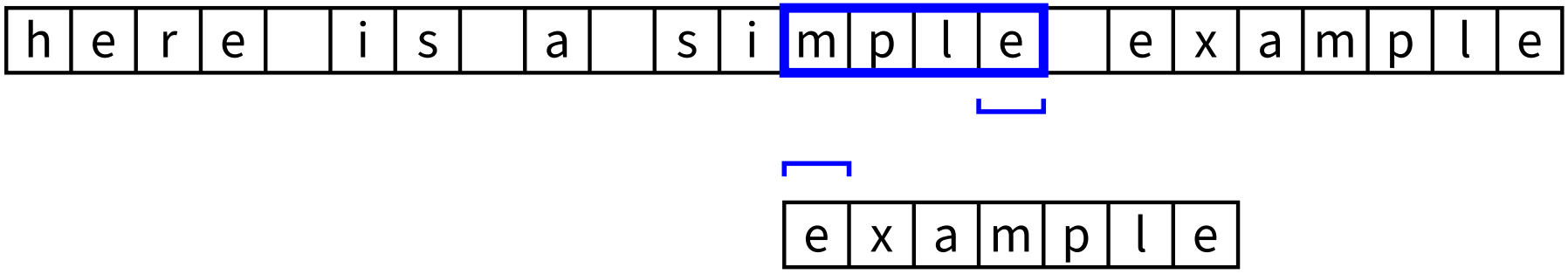

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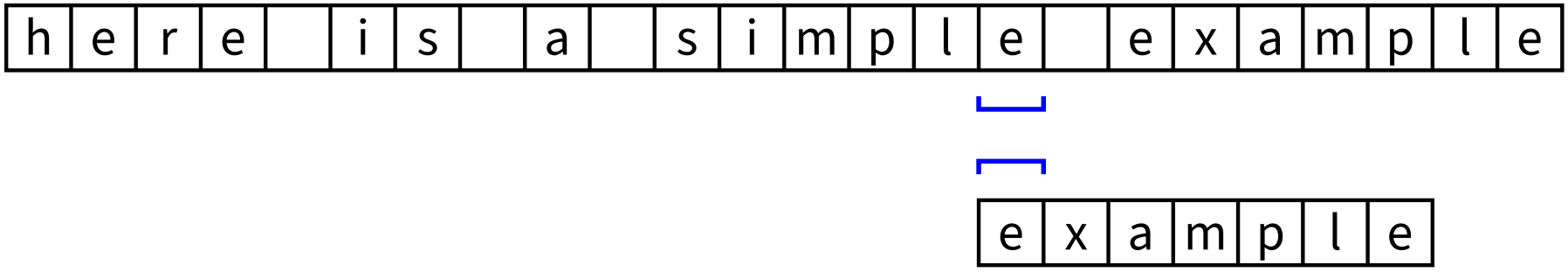

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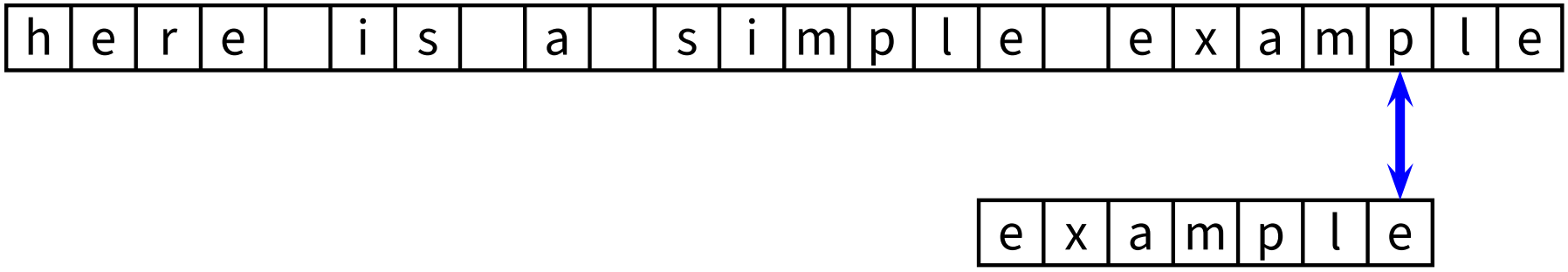

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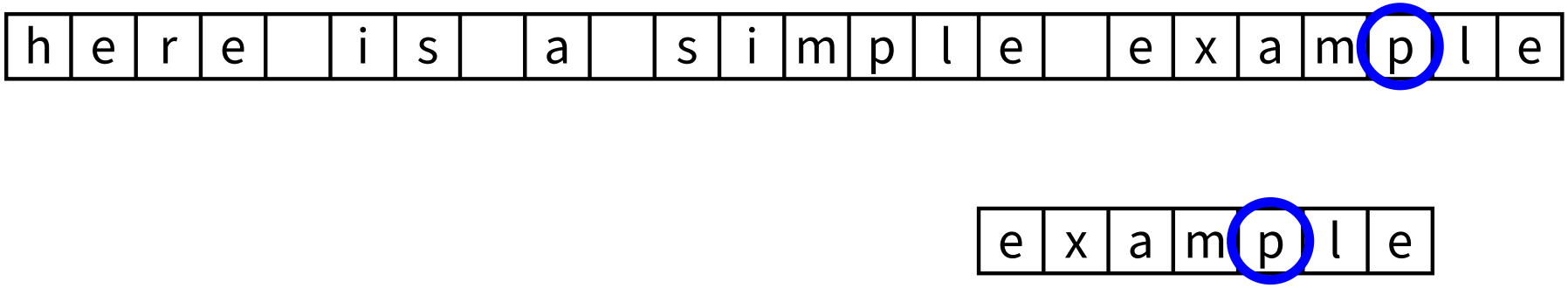

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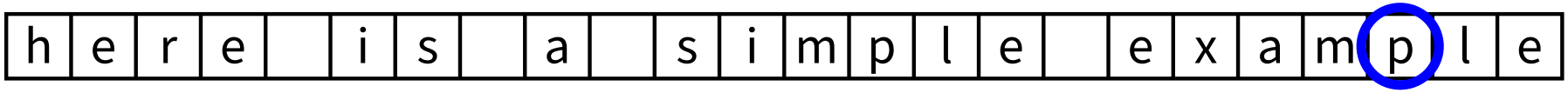

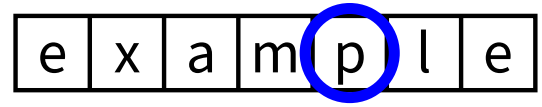

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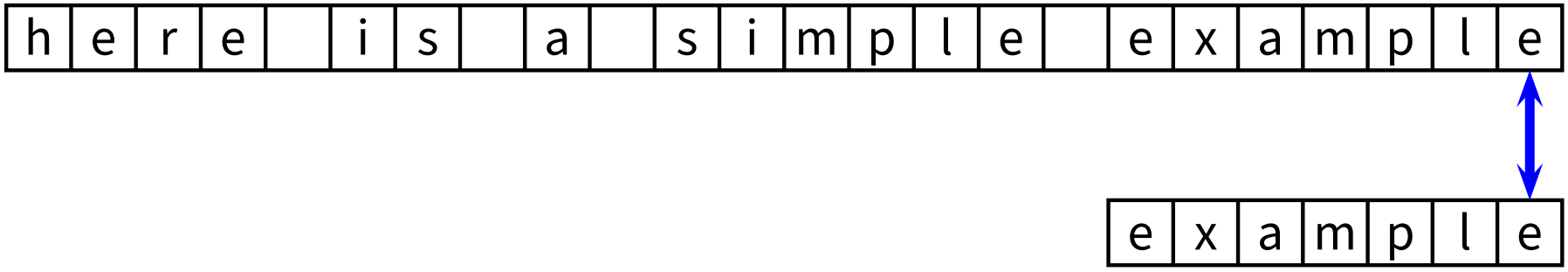

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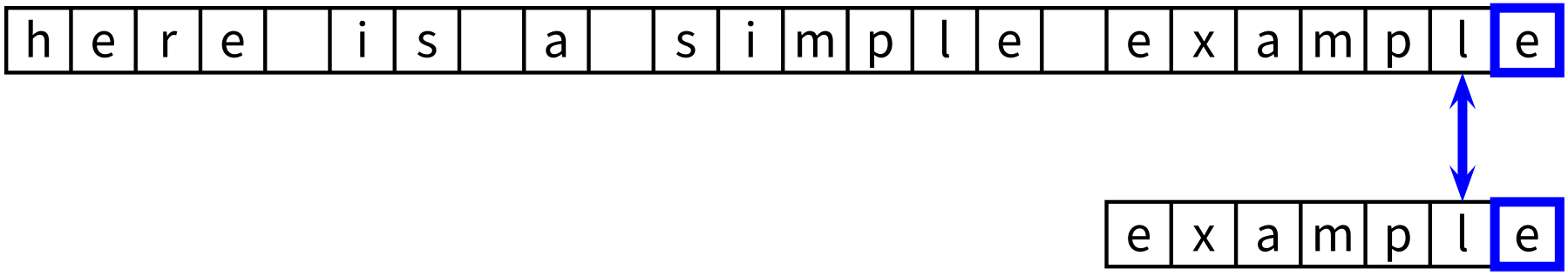

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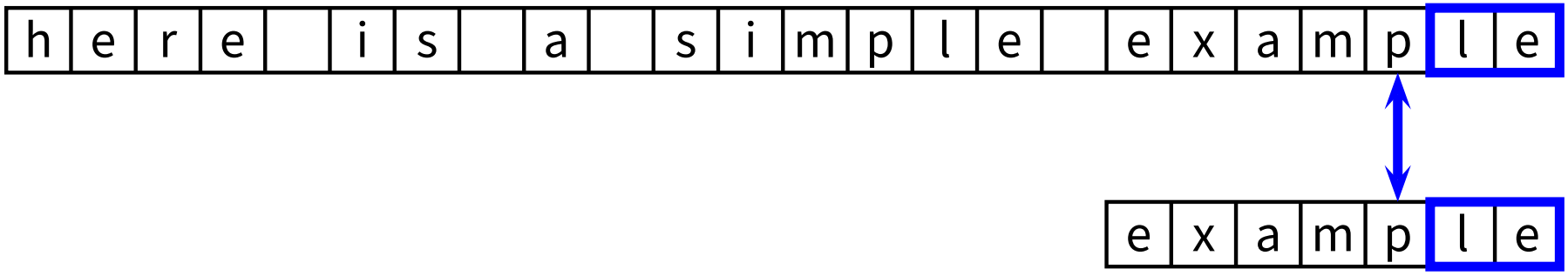

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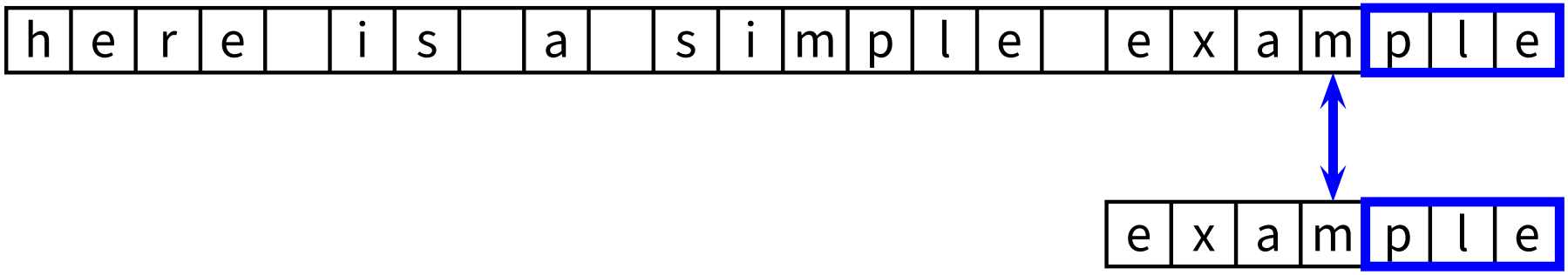

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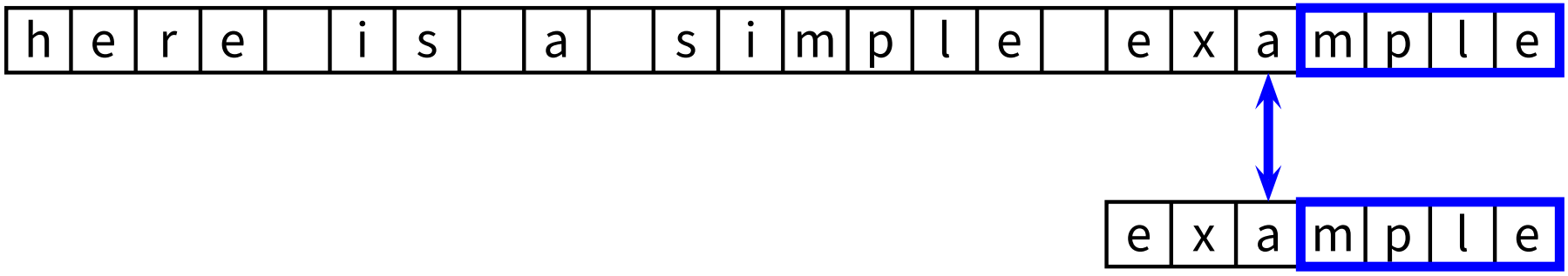

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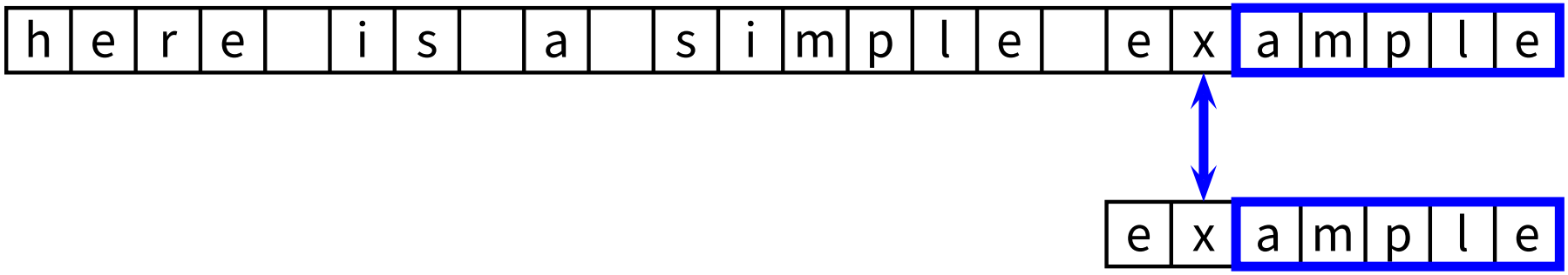

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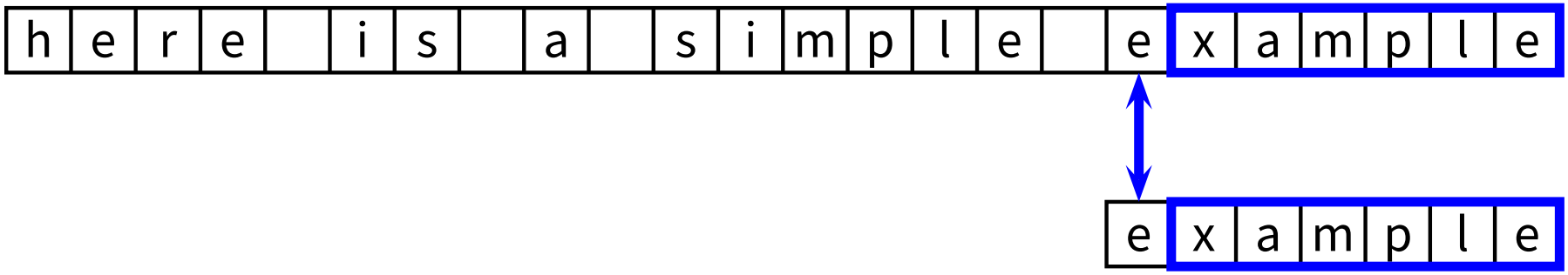

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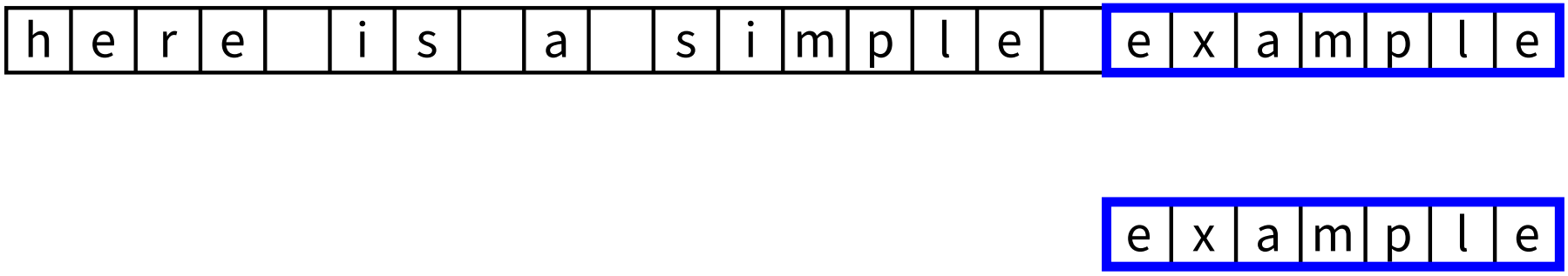

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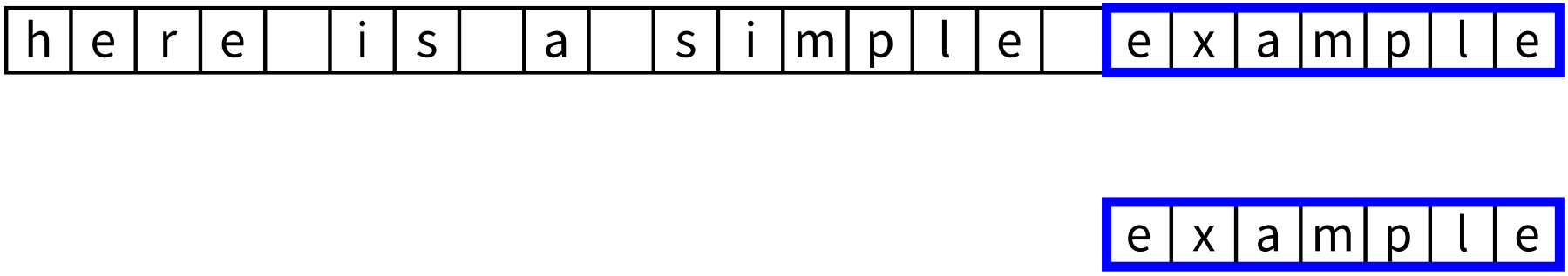

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- In essence, this is the Boyer-Moore algorithm

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- The search phase can be as low as $O(n/m)$ in common cases
- In practice, Boyer-Moore is the fastest string-matching algorithm for most applications