

More on Sorting: Quick Sort and Heap Sort

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March 14, 2024

- Another divide-and-conquer sorting algorithm
- The *heap*
- Heap sort

Sorting Algorithms Seen So Far

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Algorithm	Complexity			In place?
	<i>worst</i>	<i>average</i>	<i>best</i>	
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- *Basic step*: partition A in three parts based on a *chosen value* $v \in A$
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- *Can we partition A **in place**?*

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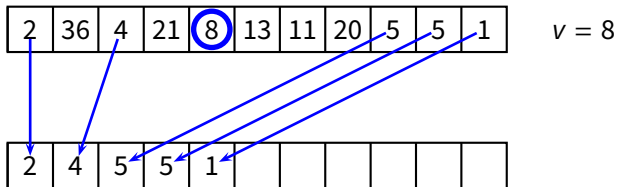
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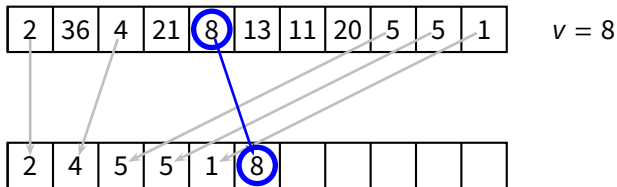


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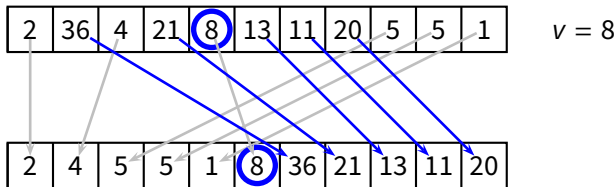


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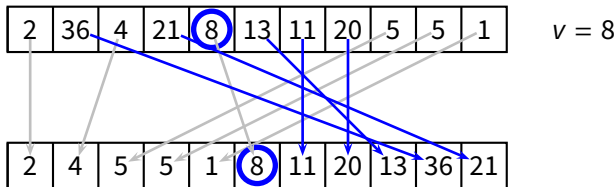


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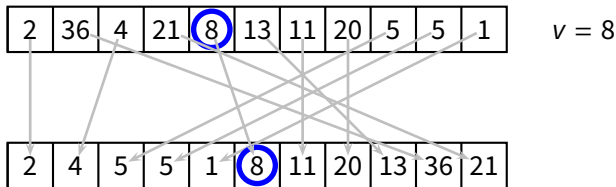


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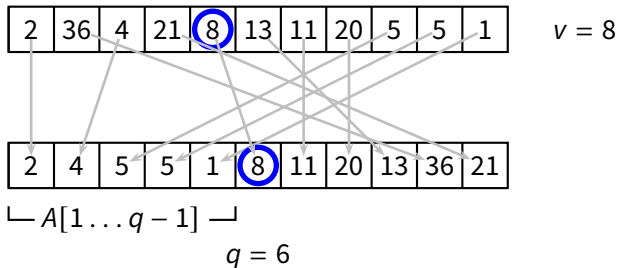
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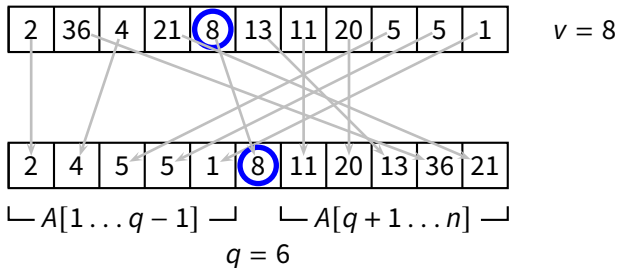


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```
QUICKSORT( $A, begin, end$ )
```

```
1  if  $begin < end$ 
```

```
2       $q = \mathbf{PARTITION}(A, begin, end)$ 
```

```
3      QUICKSORT( $A, begin, q - 1$ )
```

```
4      QUICKSORT( $A, q + 1, end$ )
```


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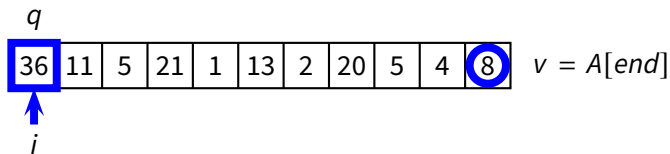
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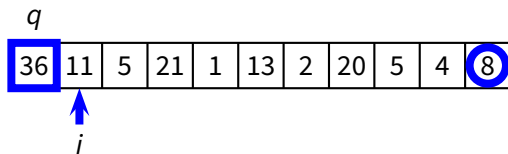
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$v = A[end]$

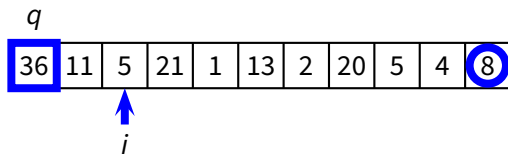
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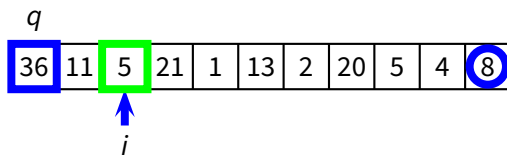
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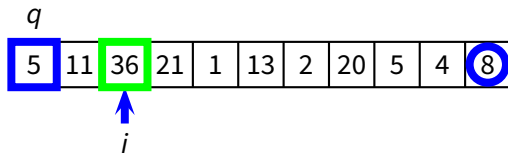
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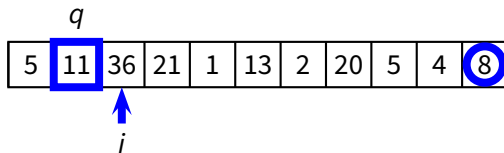
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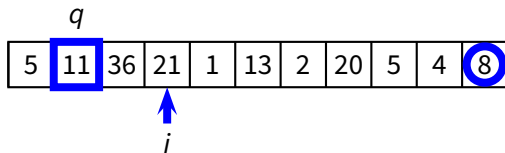
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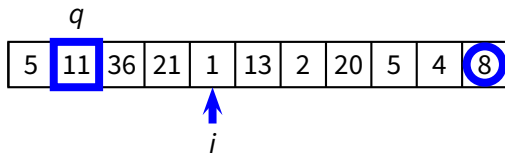
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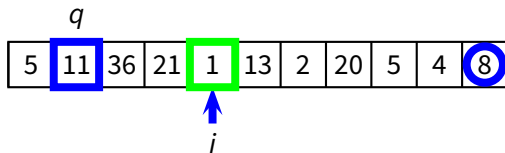
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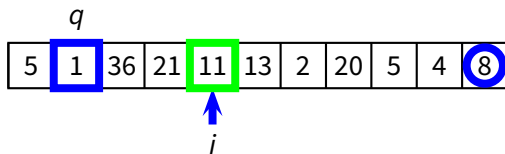
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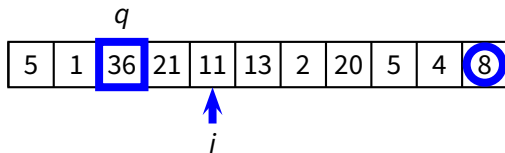
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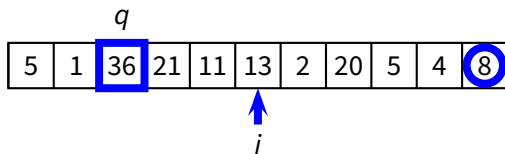
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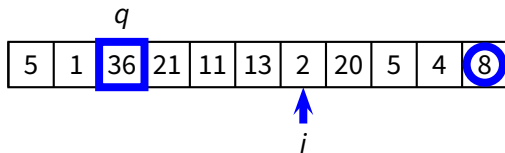
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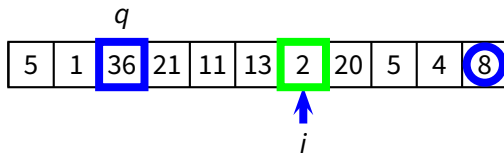
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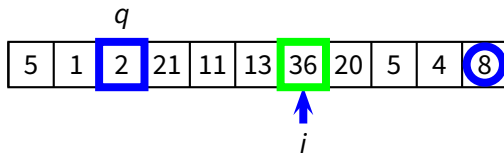
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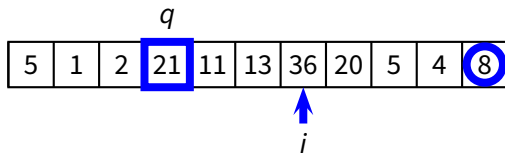
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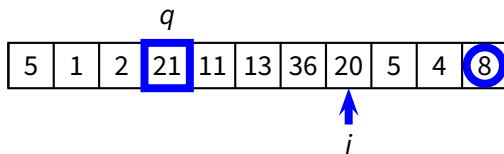
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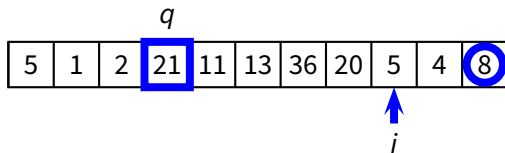
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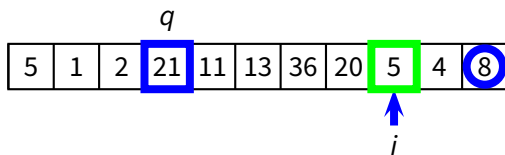
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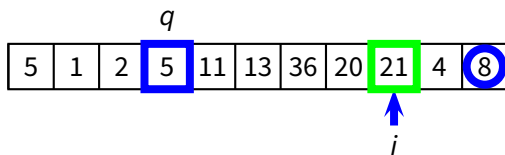
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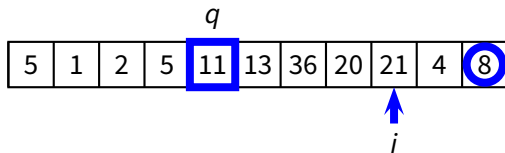
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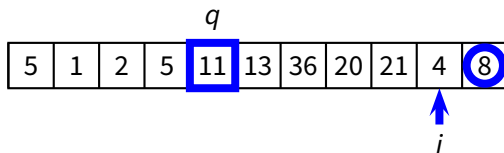
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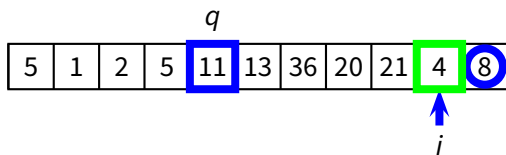
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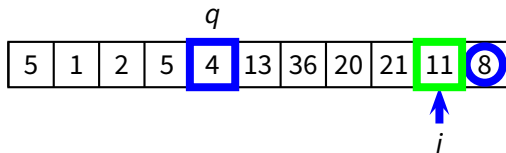
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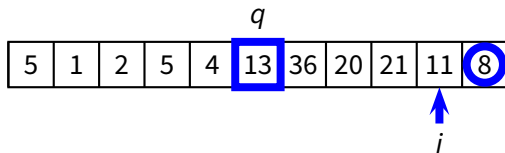
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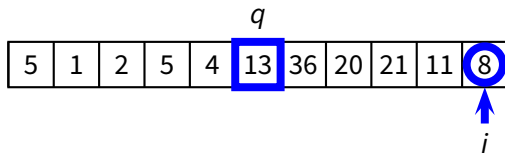
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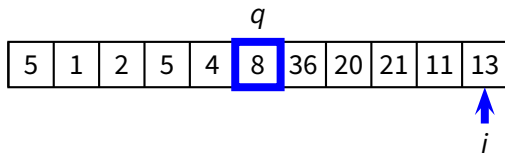
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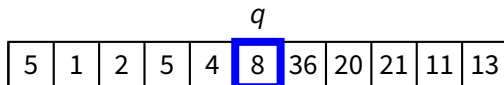
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Complete QUICKSORT Algorithm

PARTITION($A, begin, end$)

```
1   $q = begin$ 
2   $v = A[end]$ 
3  for  $i = begin$  to  $end$ 
4      if  $A[i] \leq v$ 
5          swap  $A[i]$  and  $A[q]$ 
6           $q = q + 1$ 
7  return  $q - 1$ 
```

QUICKSORT($A, begin, end$)

```
1  if  $begin < end$ 
2       $q = \mathbf{PARTITION}(A, begin, end)$ 
3      QUICKSORT( $A, begin, q - 1$ )
4      QUICKSORT( $A, q + 1, end$ )
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$$T(n) = \Theta(n)$$

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■ Worst case

- ▶ $q = begin$ or $q = end$

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QUICKSORT(A, begin, end)
```

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Complexity of QUICKSORT (2)

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- ▶ $q = \lceil n/2 \rceil$
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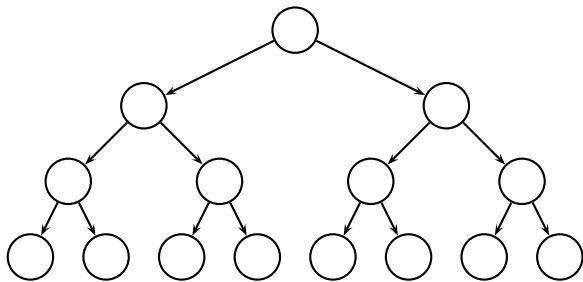
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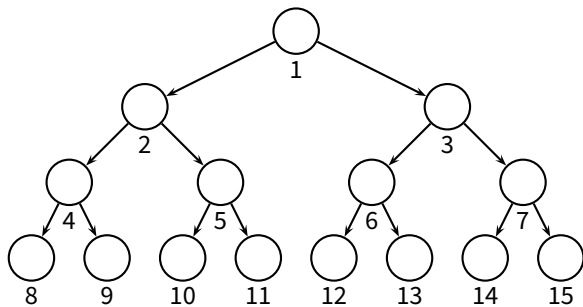
Binary Heap: Structure

- Conceptually a full binary tree

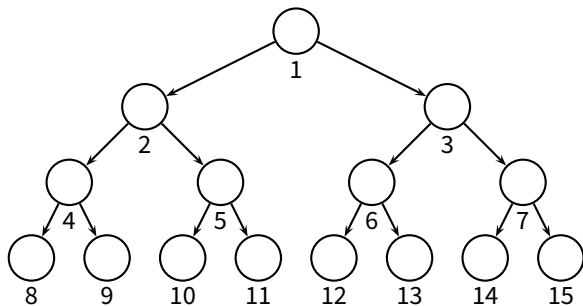
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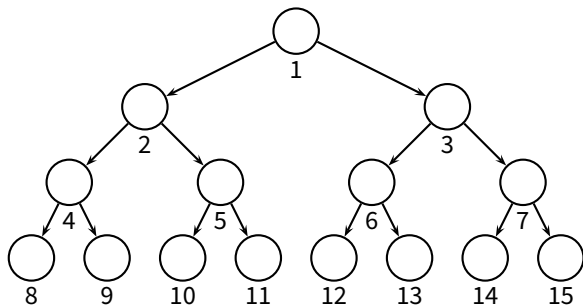


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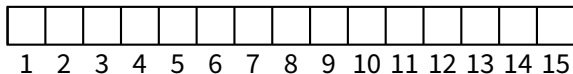


- Implemented as an array

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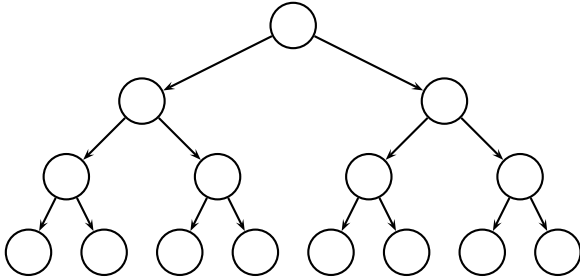


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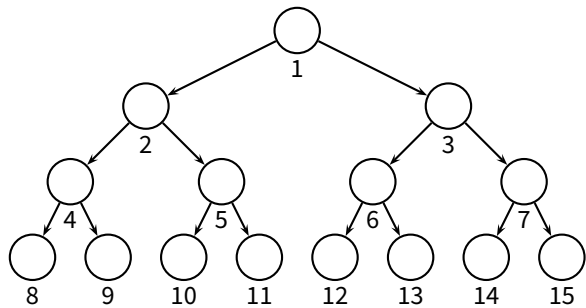


Binary Heap: Properties

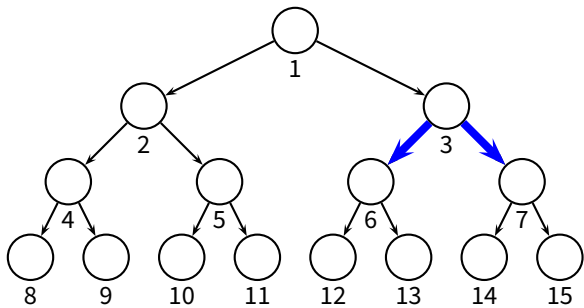
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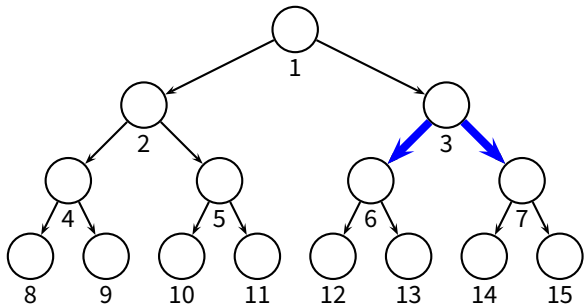
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PARENT(i)

return $\lfloor i/2 \rfloor$

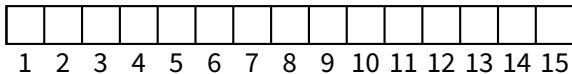
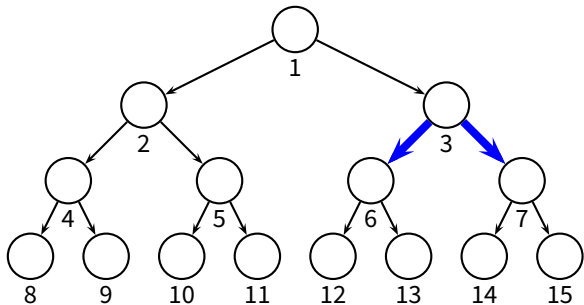
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Binary Heap: Properties



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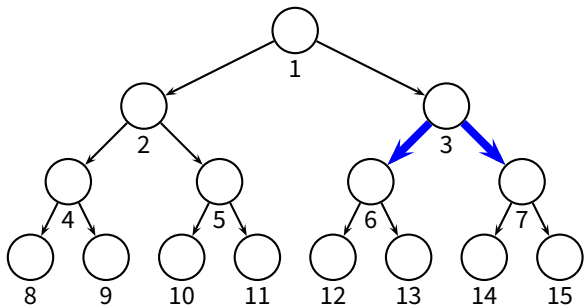
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Binary Heap: Properties



PARENT(i)

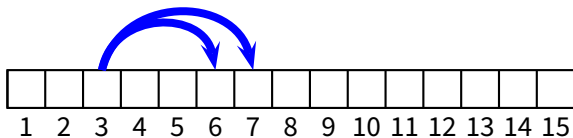
return $\lfloor i/2 \rfloor$

LEFT(i)

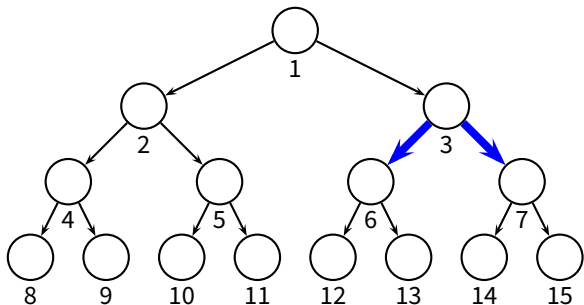
return $2i$

RIGHT(i)

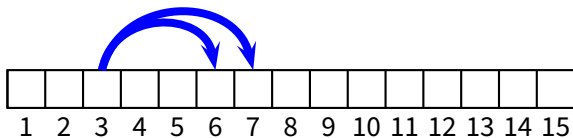
return $2i + 1$



Binary Heap: Properties



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 return $\lfloor i/2 \rfloor$
LEFT(i)
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RIGHT(i)
 return $2i + 1$

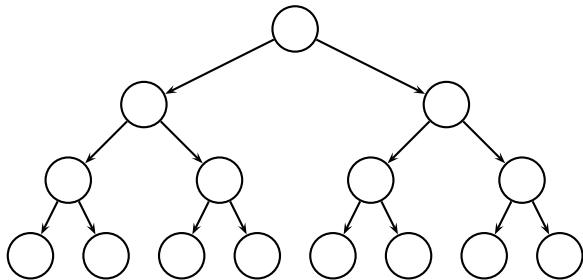


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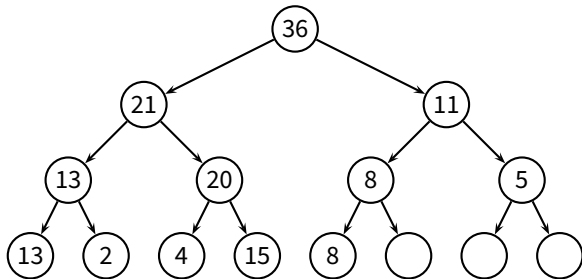
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E.g.,



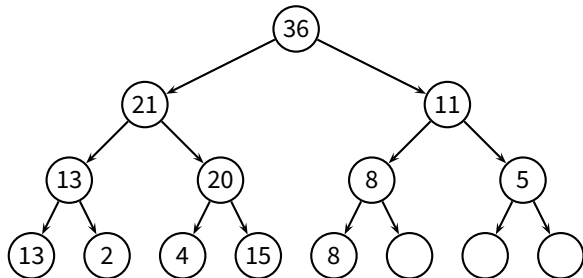
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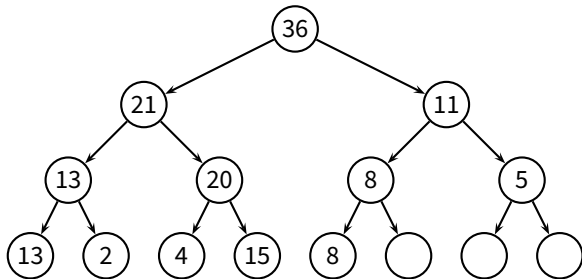
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- Where is the max element?

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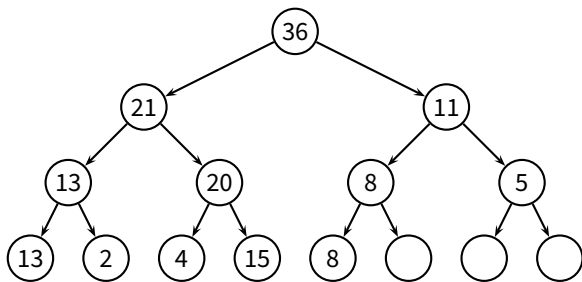
- Where is the max element?
- How can we implement **HEAP-EXTRACT-MAX**?

■ **HEAP-EXTRACT-MAX** procedure

- ▶ extract the max key
- ▶ rearrange the heap to maintain the *max-heap property*

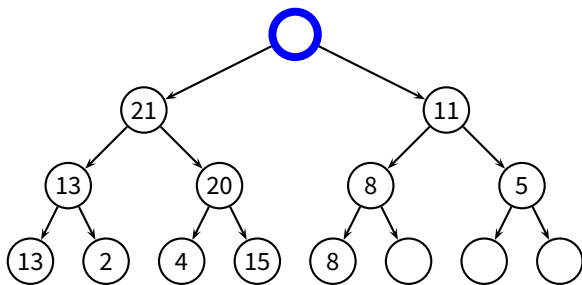
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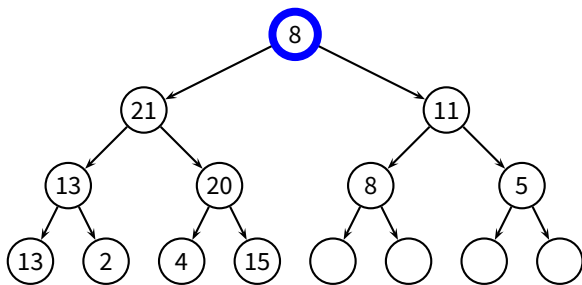
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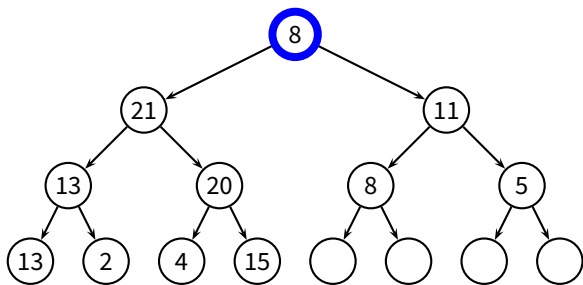
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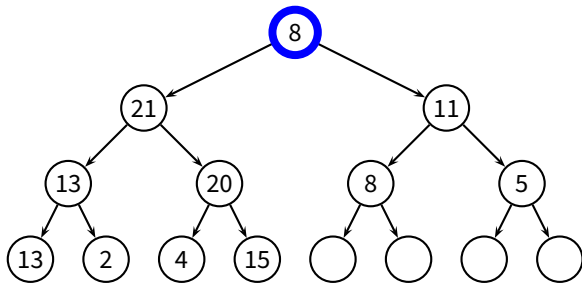
- Now we have two subtrees where the *max-heap property* holds

■ **MAX-HEAPIFY**(A, i) procedure

- ▶ *assume*: the *max-heap property* holds in the subtrees of node i
- ▶ *goal*: rearrange the heap to maintain the *max-heap property*

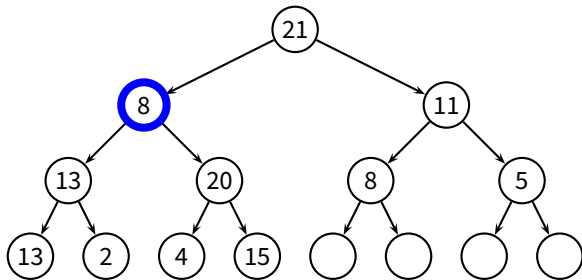
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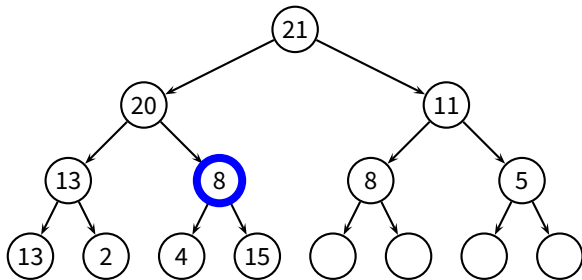
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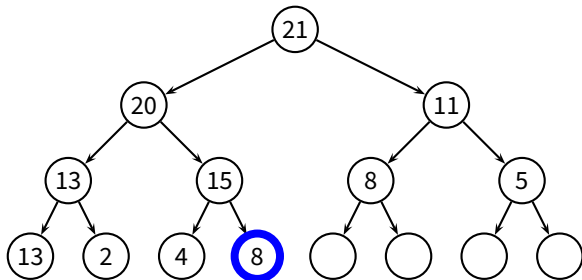
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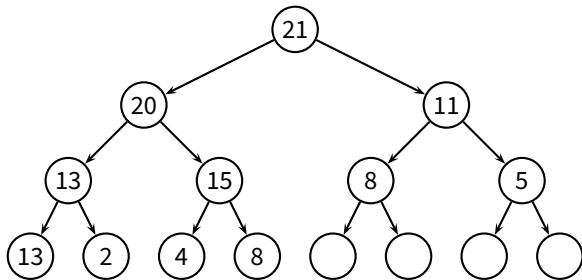
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MAX-HEAPIFY(A, i)

```
1   $l = \text{LEFT}(i)$ 
2   $r = \text{RIGHT}(i)$ 
3  if  $l \leq A.\text{heap-size}$  and  $A[l] > A[i]$ 
4       $largest = l$ 
5  else  $largest = i$ 
6  if  $r \leq A.\text{heap-size}$  and  $A[r] > A[largest]$ 
7       $largest = r$ 
8  if  $largest \neq i$ 
9      swap  $A[i]$  and  $A[largest]$ 
10     MAX-HEAPIFY( $A, largest$ )
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- Complexity of **MAX-HEAPIFY**?

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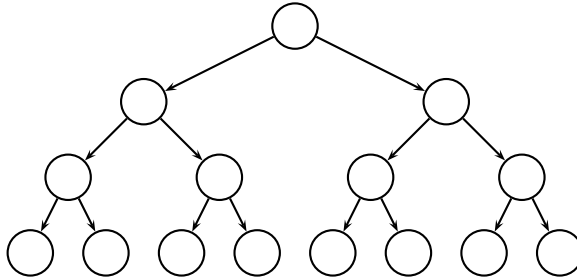
$$T(n) = \Theta(\log n)$$

BUILD-MAX-HEAP(A)

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1  $A.heap\text{-}size = length(A)$   
2 for  $i = \lfloor length(A)/2 \rfloor$  downto 1  
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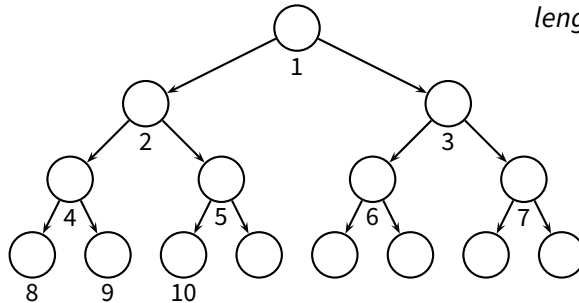
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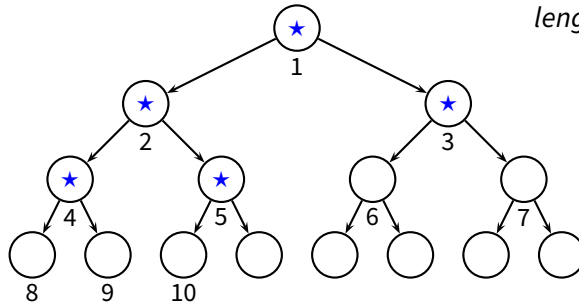
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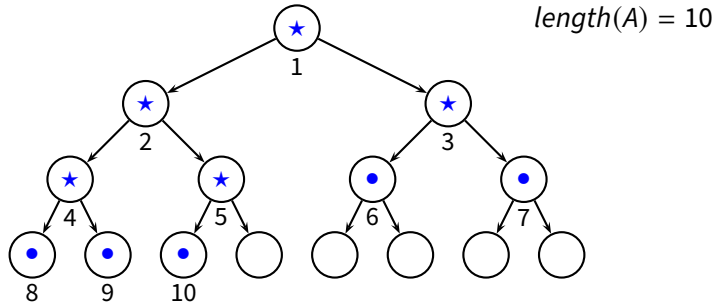
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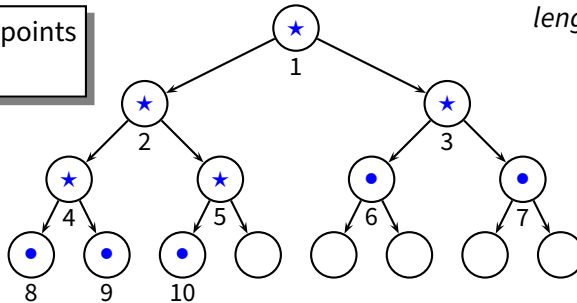


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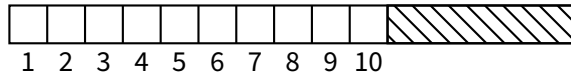
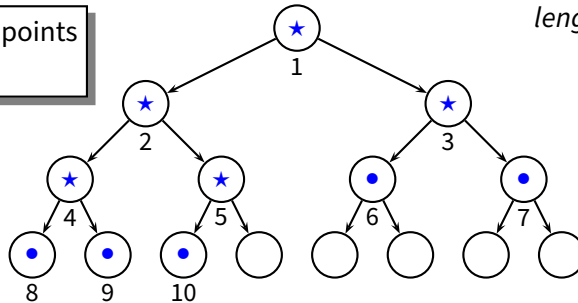
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● leaves



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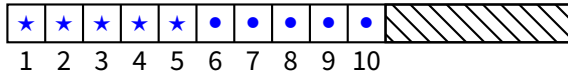
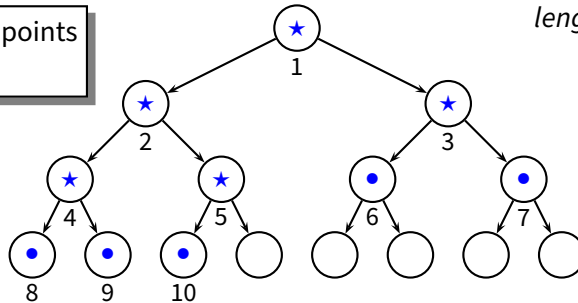
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HEAP-SORT(*A*)

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2  for i = length(A) downto 1
3      swap A[i] and A[1]
4      A.heap-size = A.heap-size - 1
5  MAX-HEAPIFY(A, 1)
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HEAP-SORT(A)

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1  BUILD-MAX-HEAP( $A$ )
2  for  $i = \text{length}(A)$  downto 1
3      swap  $A[i]$  and  $A[1]$ 
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- What is the complexity of **HEAP-SORT**?

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- What is the complexity of **HEAP-SORT**?

$$T(n) = \Theta(n \log n)$$

- Benefits

- ▶ in-place sorting; worst-case is $\Theta(n \log n)$

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Algorithm	Complexity			In place?
	<i>worst</i>	<i>average</i>	<i>best</i>	
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