Algorithms and Data Structures

Course Introduction

Antonio Carzaniga

Faculty of Informatics
Università della Svizzera italiana

February 16, 2026

General Information

- On-line course information
  - on iCorsi: ***INF.B.ALGO.S.2026***
  - and on my web page: ***https://www.inf.usi.ch/carzaniga/edu/algo/***
  - previous edition also on-line: ***https://www.inf.usi.ch/carzaniga/edu/algo25s/***

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- Announcements
  - ***you are responsible for reading the announcements (posted through iCorsi)***

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- Announcements
  - ***you are responsible for reading the announcements (posted through iCorsi)***
- Personal consultations: ***by appointment***
  - Antonio Carzaniga (yours, truly)
  - Koppány Encz
  - Stefano Huber
  - Lidia Anna Maria Losavio
  - Ramesh Aditya

Lecture Time Slot

Official lecture slots:
Monday, 11:00–12:30
Thursday, 9:00–10:30
no breaks during class

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Proposed lecture slots:

Monday, ***10:50***–12:30

Thursday, 9:00–***10:40***

with a 10-minute break in the middle of the lecture

Assessment

Assessment

- +40% midterm exam — ***15 April, 9:00-11:00***, Aula Polivalente
- +60% final exam — during the Summer exam session
- ±10% instructor's discretionary evaluation
  - participation
  - extra credits
  - trajectory
  - …

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- −100% plagiarism penalties

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- "material" means ideas, words, code, suggestions, corrections on one's work, etc.
- Using someone else's material may be appropriate
  - e.g., software libraries
  - ***always clearly identify the external material, and acknowledge its source!*** Failing to do so means committing plagiarism.
  - the work will be evaluated based on its *added value*

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  - the work will be evaluated based on its *added value*
- Plagiarism or cheating on an assignment or an exam may result in
  - failing that assignment or that exam
  - losing one or more points *in the final note!*
- Penalties may be escalated in accordance with the regulations

What's the Point?

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you are here *to learn!*

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- understanding algorithmic problems
- solving algorithmic problems
- analyzing algorithms
- understanding and using algorithmic strategies

How does one learn?

How does one learn?

Study, Practice, Study, Practice, . . .

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reading, reflecting

How does one learn?

solving problems

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"slow thinking"

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- ***you*** have to do it
- I will do my best to present ideas, show their beauty, stimulate your interest
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at least 3 hours per week, better 4. . .

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   (the ***textbook*** is by far the best source!)

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   - ask *a person!* Ask your teacher or your TAs. We're very happy to help!

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4. When you feel you understand the main ideas, put those into practice!

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effortful, "slow thinking"
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   - read the problem specification carefully
   - develop a high-level strategy
   - write the algorithm in pseudo-code
   - write the algorithm in Python (we’ll get to that)
   - test your Python implementation (we’ll get to that, too)

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4. Repeat from step 1

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The point is ***NOT*** to obtain solutions!

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Would that work for other skills?

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YOU *have to practice*

“The first principle is that you must not fool yourself
and you are the easiest person to fool.”

—Richard P. Feynman

Textbook

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Thomas H. Cormen
Charles E. Leiserson
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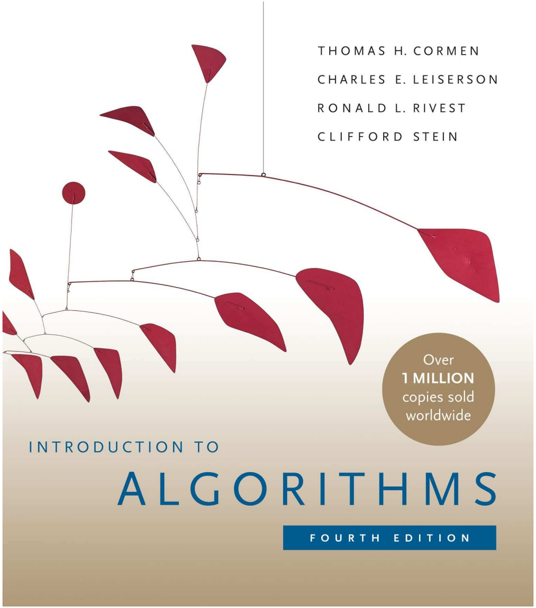
Clifford Stein

The MIT Press



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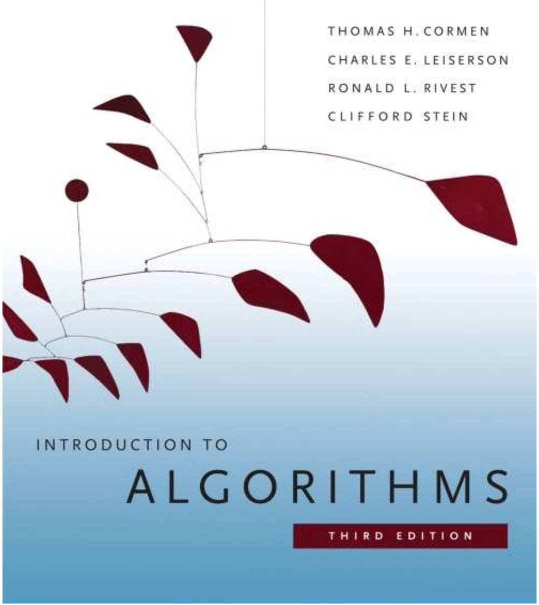

Other Learning Material

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- *Notes on Elementary Algorithmic Programming in Python*

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- ***Lecture slides***
- ***Lecture recordings (audio/video)***

Our Time and Energy Beyond Lectures

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- Personal meetings
  - extemporaneous, *any time I have time!*
  - individually or in small groups
  - questions, exercises, discussions, ...

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- ***Exercise sessions***
  - every *Wednesday 15:30–17:00 in C1.04*
  - supervised exercises, analysis of solutions, discussions

an introductory example. . .

Fundamental Ideas

Fundamental Ideas

Johannes Gutenberg invents movable type and the printing press in Mainz, circa 1450
(already known in China and Korea, circa 1200 CE)

Maybe More Fundamental Ideas

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  - *they were **algorithms!***

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Example: Poetic Rhythms

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We have $n = 4$ total beats. How many different rhythms can we have?

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1–1–2	*Ta–Ta–Ta-a–*
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Example:

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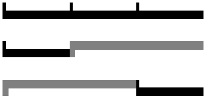

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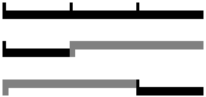

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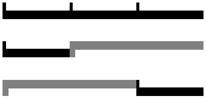

$$P(8) = ?$$

We want a general *algorithm* to compute $P(n)$

$n = 5$:

A First Algorithm

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$\textsc{Pingala}(5) = 8$

A First Algorithm

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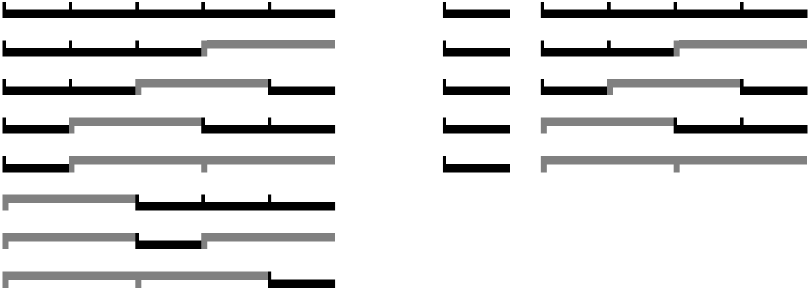

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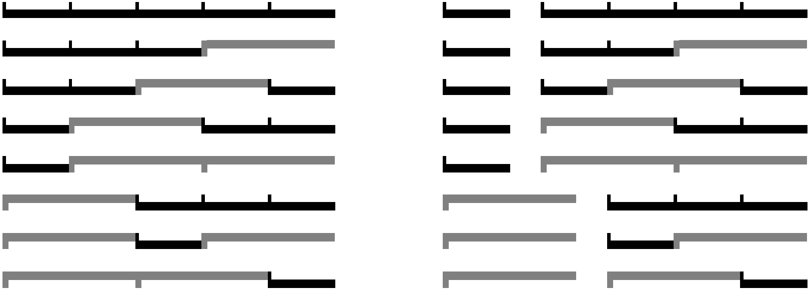

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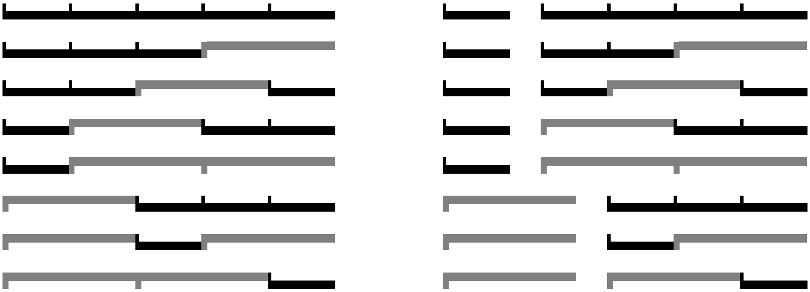

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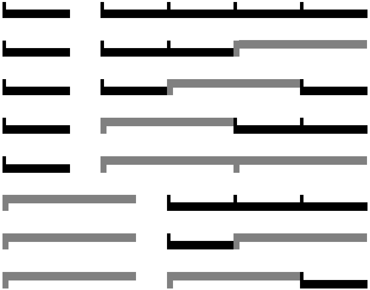

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PINGALA(n)
if n ≤ 2
        return n
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Questions on Our First Algorithm

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3. Can we do better?

Correctness

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Correctness

Pingala(n)

if $n \leq 2$

  return n

return **Pingala**$(n-1)$ + **Pingala**$(n-2)$

- For now we wave our hands…
  - "the algorithm is clearly correct!"
  - assuming $n > 0$

Performance

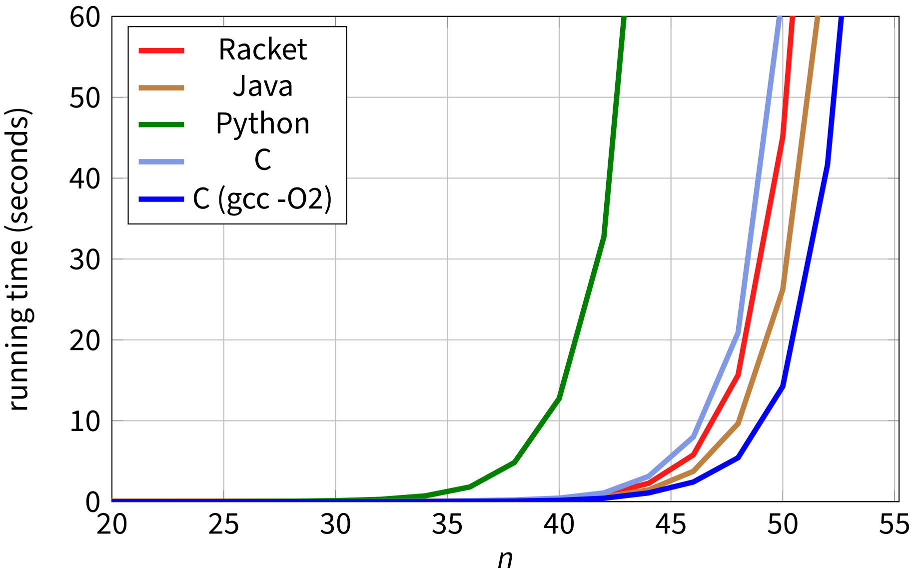
- How long does it take?

Performance

- How long does it take?

  Let's try it out…

Results



Racket
Java
Python
C
C (gcc -O2)
running time (seconds)
60
50
40
30
20
10
0
20
25
30
35
40
45
50
55
n

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  - with different languages you get different performances
  - compiler optimizations can make a difference
- However, the differences are not substantial
  - *all* implementations sooner or later seem to hit a wall…
- Conclusion: ***the problem is with the algorithm***

Complexity of Our First Algorithm

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  We'll call it the algorithm's ***computational complexity***

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$$T(n) = T(n-1) + T(n-2) + 3$$

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2       return n
3   return PINGALA(n − 1) + PINGALA(n − 2)
```

$$T(1) = T(2) = 2$$

$$T(n) = T(n-1) + T(n-2) + 3 \quad \Rightarrow T(n) \geq P(n)$$

Complexity of Our First Algorithm (2)

- So, let's try to understand how $T(n) =$ grows with n

$$T(n) \geq T(n-1) + T(n-2)$$

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$$T(n) \geq (\sqrt{2})^n \approx (1.4)^n$$

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- $T(n)$ ***grows exponentially*** with n—and that is ***really bad!***
- Can we do better?

A Better Algorithm

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Idea: we can avoid repeating the same computations over and over again

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```
PINGALA-MEM(n, M)
if n ≤ 2
    return n
if M == ∅
    M = array of n NIL elements
if M[n] == NIL
    M[n] = PINGALA-MEM(n − 1, M) + PINGALA-MEM(n − 2, M)
return M[n]
```

An Even Better Algorithm

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Idea: we can build $P(n)$ from the ground up, with just a couple of extra variables!

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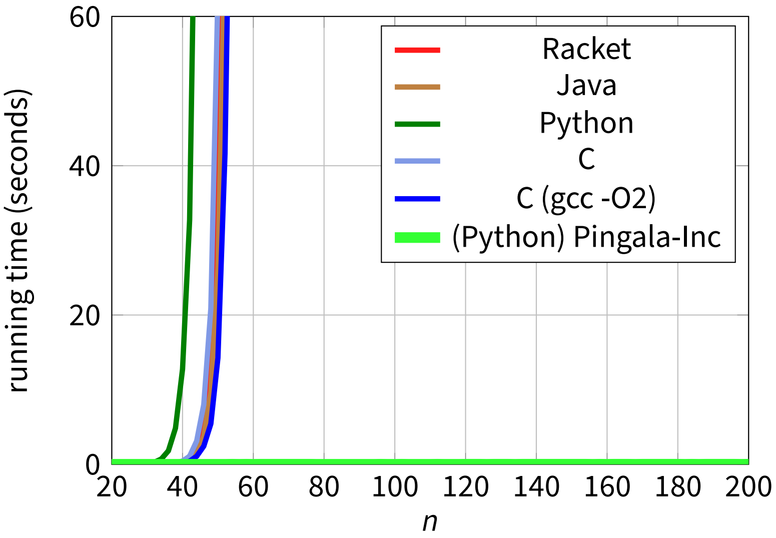
```
PINGALA-INC(n)
if n ≤ 2
        return n
pprev = 1
prev = 2
for i = 3 to n
        P = prev + pprev
        pprev = prev
        prev = P
return P
```

Results



running time (seconds)
60
40
20
0
20
40
60
80
100
120
140
160
180
200
n
Racket
Java
Python
C
C (gcc -O2)
(Python) Pingala-Inc

Complexity of PINGALA-INC

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$T(n) =$

Complexity of Pingala-Inc

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$T(n) = 4 + 5(n - 2)$

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```

$$T(n) = 4 + 5(n - 2) = 5n + \ldots$$

Complexity of PINGALA-INC

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$T(n) = 4 + 5(n-2) = 5n + \ldots = O(n)$

The *complexity* of **PINGALA-INC**(n) is ***linear*** in n