Summary

Barycentric coordinates are often used in cage-based space deformation. **Shape-preserving** is an important property of space deformation. We review some barycentric coordinates with emphasis on their shape-preserving property and we try to improve them.

Review

Given a cage $P$ with vertices $V = \{v_i\}_{i \in I_v}$ and faces $T = \{t_i\}_{i \in I_t}$. Let $P'$ be the deformed cage with deformed vertices $V' = \{v'_i\}_{i \in I_v}$ and faces $T' = \{t'_i\}_{i \in I_t}$.

**Traditional barycentric coordinates**

E.g.: Mean Value Coordinates (MVC) and Harmonic Coordinates. Express a point $x$ inside $P$ as

$$x = F(x; P) = \sum_{i \in I_v} \varphi_i(x)v_i$$

with suitable coordinate functions $\varphi_i(\cdot)$.

The deformation is defined by

$$x \mapsto x' = F(x; P') = \sum_{i \in I_v} \varphi'_i(x)v'_i.$$

Properties: interpolatory, not shape-preserving

**Green Coordinates (GC)**

Express a point $x$ inside $P$ as

$$x = F(x; P) = \sum_{i \in I_v} \phi_i(x)v_i + \sum_{j \in I_t} \psi_j(x) n(l_j)$$

with suitable functions $\phi_i(\cdot)$ and $\psi_j(\cdot)$ and outward normal function $n(\cdot)$.

The deformation is defined by

$$x \mapsto x' = F(x; P') = \sum_{i \in I_v} \phi_i(x)v'_i + \sum_{j \in I_t} \psi_j(x) s_j n(l'_j).$$

Properties: shape-preserving, not interpolatory

A comparison

From [Lipman et al., SIGGRAPH 2008. Green Coordinates, Fig. 3].

**Improved barycentric coordinates**

1. Refine the cage $P$ and get $P$ with vertices $V = \{\bar{v}_i\}_{i \in I_v} \supseteq V$ and faces $T = \{\bar{t}_j\}_{j \in I_t}$.
2. Express a point $x$ inside $P$ as

$$x = F(x; P) = (1 - \alpha) \sum_{i \in I_v} \phi_i(x)\bar{v}_i + \alpha \sum_{j \in I_t} \psi_j(x)n(l_j)$$

with suitable functions $\phi_i(\cdot)$, $\psi_j(\cdot)$, outward normal function $n(\cdot)$ and parameter $\alpha \in [0, 1]$.
3. Compute the deformed cage $P'$ with vertices $V' = \{\bar{v}'_i\}_{i \in I_v}$ and faces $T' = \{\bar{t}'_j\}_{j \in I_t}$:
   - for **constrained vertices** $\bar{v}_j$, i.e., $\bar{v}_j \in V$ and $v'_j$ is to be interpolated: set $v'_j = v'_j$.
   - for all other **unconstrained vertices** $\bar{v}_j$, compute $v'_j$ by minimizing some energy, e.g., stretching energy + bending energy.
4. The deformation is defined by

$$x \mapsto x' = F(x; P') = (1 - \alpha) \sum_{i \in I_v} \phi_i(x)v'_i + \alpha \sum_{j \in I_t} \psi_j(x)s_j n(l'_j).$$

Advantage: shape-preserving, interpolatory (if $\alpha = 0$)

Disadvantage: In step 3 a nonlinear optimization may occur, which may not converge and may increase computing time.