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## **Invited conferences**

# **Hermite subdivision schemes: a dual de Rham-type approach**

**Costanza Conti**

University of Firenze, Italy

## **Abstract**

Hermite subdivision algorithms are mainly designed to interpolate functional values and associated derivatives. These schemes process non-scalar data (functional values and derivatives), and can be rewritten as non-stationary vector algorithms, although their non-stationarity is of a very specific kind.

In this talk we present new families of approximating subdivision schemes derived from interpolatory Hermite schemes via the so-called DeRham technique. This allows us to define vector subdivision schemes capable of generating smoother limit functions than the corresponding interpolatory Hermite subdivision schemes starting from the same initial (vector) data. These approximating schemes are dual in the sense that, at each iteration, two new points are inserted in between two old ones.

In the analysis of these dual Hermite schemes, we will pay particular attention to their vector-valued nature and we will establish their convergence/smoothness properties via classical techniques of vector subdivision analysis.

This is a joint work with Jean-Louis Merrien (INSA Rennes, France) and Lucia Romani (University of Milano-Bicocca, Italy).

# **Non-linear subdivision schemes for geometric objects**

**Nira Dyn**

Tel-Aviv University, Israel

## **Abstract**

This talk reviews non-linear subdivision schemes refining three types of geometric objects; curves, compact sets, and positive definite matrices. We present several such schemes together with their analysis; two extensions of corner cutting schemes to curves, two extensions of the Lane–Riesenfeld algorithm to sets, and two extensions of corner cutting schemes to positive definite matrices. In the latter case the analysis is rather general, while for the other two types of objects, the method of analysis is specific to each scheme.

The talk is based on joint works with Uri Itai and Elza Farkhi, on a work of Uri Itai and on a joint work of Nir Sharon and Uri Itai.

# Subdivision schemes and cascade algorithms in weighted spaces

**Bin Han**

University of Alberta, Canada

## **Abstract**

Subdivision schemes and cascade algorithms are often studied in the space  $C^k$  in computer graphics, and in the space  $L_2$  (or more generally, Sobolev spaces) in wavelet analysis. In this talk, first we shall discuss several motivations for study them in other spaces such as weighted  $L_p$  spaces. We shall provide a sufficient condition for the convergence of subdivision scheme and cascade algorithm in a weighted  $L_p$  space. In particular, we establish a necessary and sufficient condition for the subdivision scheme converges in the weight  $L_2$  space. As an application, we show that if the mask of a standard refinable function  $\phi$  in  $L_2(\mathbb{R})$  has exponential decay, then  $\phi$  must have exponential decay.

Finally we shall discuss the connections of the convergence of subdivision schemes in Sobolev spaces and wavelets in Sobolev spaces.

# **On design-related algorithms in subdivision-based modeling**

**Weiyin Ma**

City University of Hong Kong

## **Abstract**

Designing curves and surfaces from discrete input data and other constraints is a classic area of research in computer aided geometric design. Many topics have been widely studied in spline-based modeling. Typical examples include curve and surface design with positions, tangents and various other constraints, surface design from given input curves, and fair curve and surface design. This talk presents some related algorithms in the context of subdivision-based modeling. Many similar topics have also been addressed in parallel with the development of various subdivision schemes. The emphasis of this talk will be on curve-based subdivision surface design. Some open problems will also be highlighted for further discussions.

# **Analysis of Nonlinear Multi-Scale Schemes**

**Peter Oswald**

Jacobs University Bremen, Germany

## **Abstract**

We survey some results and open problems on the analysis nonlinear multi-scale algorithms and subdivision schemes. Motivation comes from the need of representing nonlinearly constrained data, the adequate representation of geometry, and data-dependent adaptive algorithms. One example (normal multi-scale transforms for surfaces) will be discussed in more detail. The talk is based on results obtained in collaboration with S. Harizanov, J. Moody, T. Shingel within a DFG-funded project "Nonlinear Subdivision".

# General midpoint schemes

**Hartmut Prautzsch**

Karlsruhe Institute of Technology, Germany

## Abstract

Subdivision means to generate sequences of polygons by computing affine combinations of vertices. Often these affine combinations are averages of vertices and if these are just midpoints, we call the subdivision scheme a midpoint scheme. We can distinguish between face and edge midpoints and there are face and edge midpoint schemes. For example the Doo-Sabin and the Catmull-Clark scheme are face midpoint schemes of degree 2 and 3. The simplest subdivision scheme is an edge midpoint scheme. While these schemes generate quadrilateral meshes, there are also midpoint schemes as Loops scheme for triangular meshes. Combinations of such simple midpoint schemes and certain variations with more general averaging operators lead to general midpoint schemes and infinite classes of such schemes. Recently, we have analyzed the smoothness of general midpoint subdivision surfaces for quadrilateral and triangular meshes and were able to prove  $C^1$  smoothness in particular at the extraordinary points. In this talk, I will give an overview of the results obtained and also show how these results help to construct  $C^k$  subdivision schemes for arbitrary meshes.

# **Analysis of geometric subdivision schemes**

**Ulrich Reif**

Darmstadt University of Technology, Germany

## **Abstract**

We present a general framework for analyzing geometric subdivision schemes, such as circle preserving subdivision. Rigorous global convergence criteria can be established numerically by the concept of  $r$ -contractivity, while local convergence is governed by the properties of the linear scheme given by the derivative of the geometric scheme at linear data.

# Why is finding a good polyhedron so hard?

**Malcolm Sabin**

Numerical Geometry Ltd., UK

## **Abstract**

For the last couple of years the Rainbow group in Cambridge has been looking for good fast algorithms for finding a sparse subdivision surface control polyhedron which gives a limit surface closely fitting given dense data. We deliberately didn't follow well trodden paths (incremental decimation, cross-field parametrisation) because there is no point is starting so far behind the front-runners, and because there is no apparent concensus that these methods really solve the problem.

We have tried two further main approaches, ridges, and Morse theory, and are finding detailed problems with both of these.

It's time to step back and think about the problem itself;

- what is the real objective ?
- what are the underlying issues which make it so hard to find a good approach ?
- might it be a question of 'horses for courses', different methods being appropriate for different kinds of dense data ?

# Multiple subdivision

**Tomas Sauer**

University of Passau, Germany

## **Abstract**

Multiple subdivision is a subdivision process where in each step a scheme can be chosen from a collection of stationary subdivision schemes in an arbitrary number of variable. These schemes can have different masks and/or different dilation matrices. Clearly, the limit function depends on this sequence of choices.

The talk will deal with the basic aspects of such schemes like (joint) refinability, continuity with respect to the sequence parameters and characterization of convergence. As an application and motivation, *shearlet subdivision* will show how the degrees of freedom in choosing the dilations can be used for capturing geometric information.

# Approximate Subdivision Surfaces and their Parameterization

**Scott Schaefer**

Texas A&M University, USA

## **Abstract**

This talk will discuss our approaches to approximating subdivision surfaces for GPU tessellation using a minimal set of patches. While the majority of the patches on a subdivision surface are simple polynomials, extraordinary patches have a more complex structure. We replace these extraordinary patches with a single, easy to evaluate patch that approximates the underlying subdivision surface. Moreover, we also discuss the parameterization of these surfaces and provide a simple, geometry independent method that the parameterization conforms better to the original Catmull-Clark surface. Our method handles patches that contain multiple extraordinary vertices and is efficient in both computation and memory bandwidth on the GPU.

# Gaussian vs. sparse stochastic processes: a unifying spline-based perspective

Michael Unser

EPFL Lausanne, Switzerland

## Abstract

The main point that is made in this talk is that splines and stochastic processes can be linked by some common operator formalism.

The stochastic processes of interest are generalized functions that are solutions of (possibly unstable) stochastic differential equations (SDE). They constitute an extension of the classical Lévy processes on the one hand, and the Gaussian stationary processes on the other. Fractal processes such fractional Brownian motion are also part of the family. These generalized processes are described by a general innovation model that is specified by: 1) a whitening operator (e.g., fractional derivative), which shapes their second-order moments, and 2) a Lévy exponent  $f(\omega)$ , which controls the sparsity of the (non-Gaussian) innovations (white Lévy noise).

We show how the underlying SDEs can be solved by defining some appropriate inverse operator subject to some (adjoint)  $L_p$  boundedness constraint. The method results in a complete characterization of the processes via their characteristic form (the infinite-dimensional counterpart of the characteristic function). This allows us to prove that they admit a sparse representation in a wavelet bases. We provide evidence that wavelets allow for a better  $N$  term approximation than the classical Karhunen-Loève transform (KLT), except in the Gaussian case. We also highlight a fundamental connection between the construction of maximally localized basis functions (B-splines) and statistical decoupling (including a link with the Markov property).

# Approximation of scattered manifold-valued data

**Thomas Yu**

Drexel University, USA

## Abstract

Differential geometers have a trick called partition-of-unity which allows them to ‘think globally and act locally’ when they want to craft certain objects (tensor fields, embeddings etc.) on a manifold. Approximation theorists have a similar trick, except that they usually work on an Euclidean domain and the kind of partitions-of-unity they created are more sophisticated and specialized which give them approximation power.

Motivated by an application in dimensionality reduction of dynamical system (which I will briefly discuss), I address the problem of approximating a function with an Euclidean domain and the range being a fixed manifold. Now, none of the tricks mentioned above apply directly. But with a few twists based on Karcher’s Riemannian center of mass, the moving least square method, and a new version of proximity analysis adapted to scattered data, we found a natural solution to the problem with provable approximation and smoothness properties. When the manifold of interest is a symmetric space (e.g. sphere, Grassmannians, matrix Lie groups, etc.), the approximation method is also computationally efficient.

The project above is not directly related to subdivision, but if time allows, I wish to outline some of our new results in two other projects related to subdivision methods:

- On the necessity of proximity analysis of nonlinear subdivision schemes, based on joint work with Tom Duchamp and Gang Xie
- Willmore flow and Subdivision Surfaces, based on joint work with Jingmin Mia Chen and Andrew Zigerelli

# Conferences

# **Geometric Curve Subdivision**

**Kai Hormann**

University of Lugano, Switzerland

## **Abstract**

While linear subdivision schemes for curves and surfaces are well-understood by now, we still have but a few tools for analyzing the non-linear setting, although non-linear schemes often yield better results than their linear counterparts. In this talk we discuss a geometric condition which guarantees that a nonlinear interpolating subdivision scheme produces tangent continuous limit curves. As an example, we present two non-linear variants of the classical 4-point scheme which satisfy this condition. We further demonstrate some ideas for generating limit curves with higher orders of smoothness.

The talk is based on joint work with Thomas Cashman, Nira Dyn, and Ulrich Reif.

# **Subdivision algorithms for computing exact offsets of B-spline curves**

**Lucia Romani**

University of Milano-Bicocca, Italy

## **Abstract**

We propose a new class of stationary subdivision algorithms to solve the problem of computing the exact offset of B-spline curves in a very efficient way, providing at the same time the offset curve at a certain distance  $d$  and its generator, for any arbitrary order  $n \geq 3$ . The subdivision schemes for lower order B-splines offsetting are described in great detail, their convergence and smoothness analysis is presented and their performance is demonstrated by some examples.

This is a joint work with Victoria Hernández and Jorge Estrada (ICIMAF Havana, Cuba).

# Contributed talks

# Subdivision surfaces for CAD: integration through parameterization and local correction

M. Antonelli<sup>1</sup>, C.V. Beccari<sup>2</sup>, G. Casciola<sup>2</sup> and S. Morigi<sup>2</sup>

<sup>1</sup>University of Padova, Italy

<sup>2</sup> University of Bologna, Italy

## Abstract

The main roadblock that has limited the usage of subdivision surfaces in computer-aided design (CAD) systems is the lack of quality and precision that a model must achieve for being suitable in the engineering and manufacturing phases of design. The second roadblock concerns the integration into the modeling workflow, that, for engineering purposes, means providing a precise and controlled way of defining and editing models eventually composed of different geometric representations.

This talk documents the experience in the context of a European project whose goal was the integration of subdivision surfaces in a CAD system. To this aim, a new CAD system paradigm with an extensible geometric kernel is introduced, where any new shape description can be integrated through the two successive steps of parameterization and evaluation, and a heterogeneous boundary representation is used to easily model different kinds of shapes. In this way, the newly introduced geometric description automatically inherits any pre-existing CAD tools, and it can interact in a natural way with the other geometric representations supported by the CAD system.

To overcome the irregular behavior of subdivision surfaces in the neighborhood of extraordinary points, we locally modify the limit surface of the subdivision scheme so as to tune the analytic properties without affecting its geometric shape. Such correction is inspired by a polynomial blending approach, which generates multipatch surfaces evaluable at constant time at arbitrary parameter values.

Some modeling examples will demonstrate the benefits of the proposed integration, and some tests will confirm the effectiveness of the proposed local correction method.

# Interpolatory subdivision surfaces with augmented parameterization and their analysis

C.V. Beccari<sup>1</sup>, G. Casciola<sup>1</sup> and L. Romani<sup>2</sup>

<sup>1</sup>University of Bologna, Italy

<sup>2</sup>University of Milano-Bicocca, Italy

## Abstract

Subdivision curves and surfaces have so far become an effective alternative to the well known parametric splines. While approximating subdivision schemes represent a generalization of either uniform or non uniform splines, this is no longer true for interpolating subdivision methods, which require ad hoc definition and analysis. In this respect, interpolatory schemes parameterized over the uniform grid have been investigated in detail and by now fully understood. Oppositely, interpolatory methods with non-uniform parameterization have been addressed in some recent papers, but limited to the curve case. For curves, the major benefit of the non-uniform setting with respect to the uniform, is due to the empirical observation that, in analogy to what happens with splines, the quality of the interpolant improves significantly.

In this talk we focus on a class of non-uniform, linear, interpolating subdivision schemes derived by upsampling from fundamental bases and featured by a piecewise uniform parameterization. We investigate how these schemes can be used to define non tensor-product  $C^1$  surfaces interpolating grids of control points in such a way to maintain the good behavior provided by a proper parameterization. To this aim, the piecewise uniform parameterization is generalized to a so called augmented parameterization, with an updating relation between the parameters of the  $k$ -th level grid of points and the initial ones. Moreover we derive a method which exploits the particular parameters configuration to analyze the smoothness of the limit surface.

# Exponential B-splines and the design of active contours and surfaces for biomedical image analysis

R. Delgado-Gonzalo and Michael Unser  
EPFL Lausanne, Switzerland

## Abstract

Snakes are effective tools for image segmentation. Within a 2D image, a snake is a 1D curve that evolves from an initial position, which is usually specified by a user, toward the boundary of an object. Within a 3D image, a snake is represented by a 2D surface. In the literature, these methods are also known as active contours or active surfaces.

Research has been fruitful in this area, and many snake variants have emerged. Among them, we are interested in the spline-based kind, where the curve or the surface is described continuously by some coefficients (a.k.a. control points) using basis functions. These snakes have become popular because it is possible for the user to interact with them, not only when specifying their initial position, but also during the segmentation process. This is often achieved by allowing the user to specify anchor points the curve or surface should go through.

Our interest is to characterize the spline-like integer-shift-invariant bases involved in the design of this kind of snakes. We prove that any compact-support function that reproduces a subspace of the exponential polynomials can be expressed as the convolution of an exponential B-spline with a compact-support distribution. As a direct consequence of this factorization theorem, we show that the minimal-support basis functions of that subspace are linear combinations of derivatives of exponential B-splines. These minimal-support basis functions form a natural multiscale hierarchy, which we utilize to design fast multiresolution algorithms and subdivision schemes for the representation of closed geometric curves and surfaces. This makes them attractive from a computational point of view.

Finally, we illustrate our scheme by building efficient active contours and surfaces capable of exactly reproducing ellipses in 2D and ellipsoids in 3D irrespective of their position and orientation.

# **Non-uniform interpolatory curve subdivision via splines**

**K. Karciauskas<sup>1</sup> and J. Peters<sup>2</sup>**

<sup>1</sup> Vilnius University, Lithuania

<sup>2</sup> University of Florida, USA

## **Abstract**

New non-uniform interpolatory  $C^1$  4-point,  $C^2$  and  $C^3$  6-point curve subdivision schemes, based on sampling local low-degree spline fits, will be presented. These schemes are compared to recent constructions by Beccari, Casciola, Romani and to spline constructions. Higher than least-degree non-uniform schemes of the same family lead to a point of discussion: unlike the uniform case, higher degree reproduction results for very non-uniform configurations in poorer shape.

# **Subdivision surfaces with features**

**J. Kosinka**

University of Cambridge, UK

## **Abstract**

We will give a brief overview of techniques for modelling subdivision surfaces with features. Then, we will discuss a new approach based on arbitrary degree subdivision surfaces, the NURBS-compatible subdivision framework, which allows modelling with sharp features via multiple knots.

## Connecting the JSR to set-valued trees

C. Möller

Darmstadt University of Technology, Germany

### Abstract

The joint spectral radius (JSR) of a matrix family generalizes the concept of the spectral radius. It characterizes the maximal asymptotic growth of the norm of matrix products as the number of factors tends to infinity.

In our talk we establish a connection between the JSR and a special class of trees whose nodes code finite or infinite sets of matrix products. This leads to both theoretical results and an algorithm for the calculation of the JSR based on a depth first search on such a tree.

# **Quad meshing from a Morse-Smale complex and its simplification methods**

**D. Shin**, J. Kosinka, M. Sabin, N. Dodgson  
University of Cambridge, UK

## **Abstract**

This presentation describes a quad meshing method which can extract a quad from a Morse-Smale (MS) complex and present various quad simplification methods to produce quality control meshes for Catmull-Clark subdivision. Since the proposed method defines a quad from a MS complex, each node of a quad can be labelled as maximum, minimum or saddle. Additionally, each maximum (or minimum) node has a disjoint surface segment in a dual (or primal) graph. Motivated by this, we have developed a simple but practical quad simplification method which can avoid some artefact caused by conventional simplification methods. The proposed quad simplification is designed to collapse an edge between saddle and minimum (or maximum) followed by local quad remeshing, so that it is important to sort the deletion queue correctly to minimise the distortion from the simplification. To address this, we used the edge distance weighted by the total curvature of a minimum or maximum node connected to a collapsing saddle. This presentation will summarise some preliminary results of various simplification methods and demonstrate the corresponding subdivision results.

# Commutator Estimate for Nonlinear Subdivision

P. Oswald and T. Shingel  
Jacobs University Bremen, Germany

## Abstract

Nonlinear multiscale algorithms often involve nonlinear perturbations of linear coarse-to-fine prediction operators  $S$ . In order to control these perturbations, estimates of the “commutator”  $FS - SF$  of  $S$  with a sufficiently smooth map  $F$  are needed. Such estimates in terms of bounds on higher-order differences of the underlying mesh sequences have been explicitly or implicitly proved in various papers, in particular, in connection with manifold-valued multiscale schemes. Most of these papers deal with the univariate case, and are based on very technical arguments. We will give an outline of a compact (and in our opinion technically less tedious) proof of commutator estimates in terms of local best approximation by polynomials instead of bounds on differences, covering multiscale diadic  $S$ . An application to the analysis of normal multiscale algorithms for surface representation is outlined.

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