A Few Basic Elements of Communication Security

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December 22, 2017

Some Advice

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- ***Make backups*** of your data
- ***Do NOT trust the network!***
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- ***Do NOT trust the network!***
- ***Use HTTPS*** instead of HTTP

- Understand the basics of public-key cryptography
- Communicate with ***end-to-end encryption*** (e.g., e-mail)
- use ***trusted certificates***
- ***Encrypt your confidential data*** (and make backups)
- use ***strong passwords***
- You might as well encrypt *all* your data
- Tools/technologies: ***ssh***, ***pgp*** (or ***gpg***)

Outline

- Communication security model
- Information-theoretic privacy
- Substitution ciphers
- Intro to modern cryptography
- One-time pad
- Block siphers
- Cryptographic hash functions
- Public-key cryptosystems

Communication Security

- *Communication model:* Alice sends a message m to Bob

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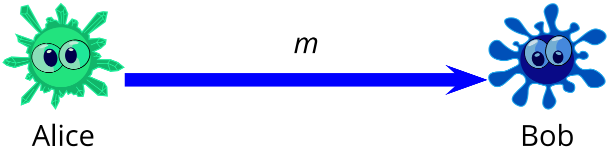

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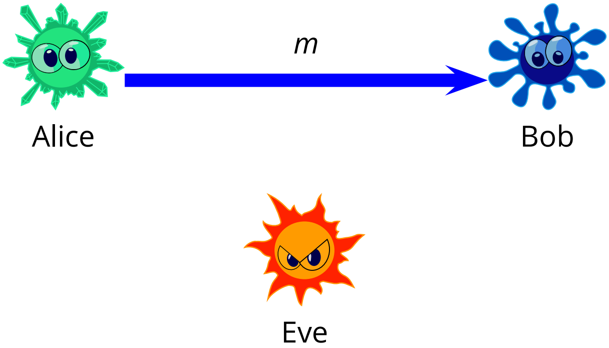

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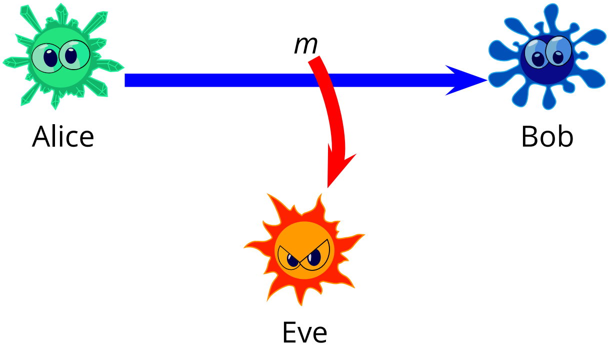

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  - can read the message

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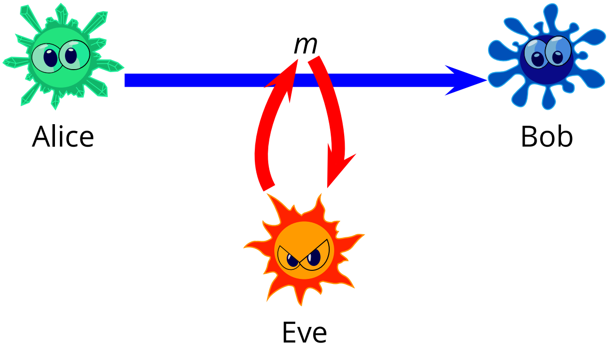

- *Passive adversary*
  - can read the message
- *Active adversary*
  - can modify the message

Goals

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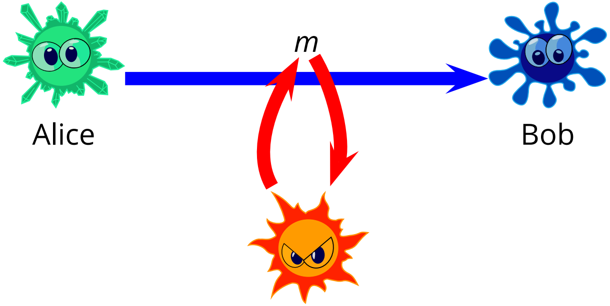

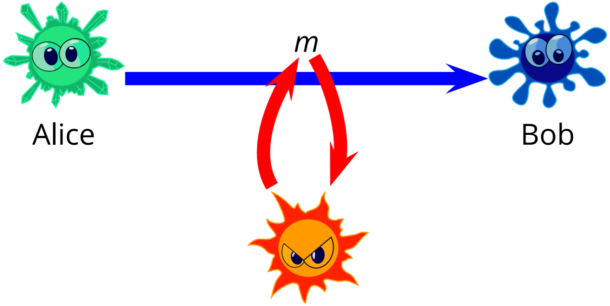

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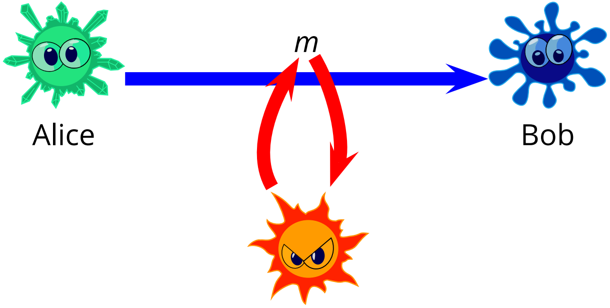

- ***Confidentiality (a.k.a., privacy):*** Alice wants to make sure that only Bob sees the message
- ***Message Integrity:*** Bob wants to make sure that the message he reads was exactly what Alice wrote

Goals (2)

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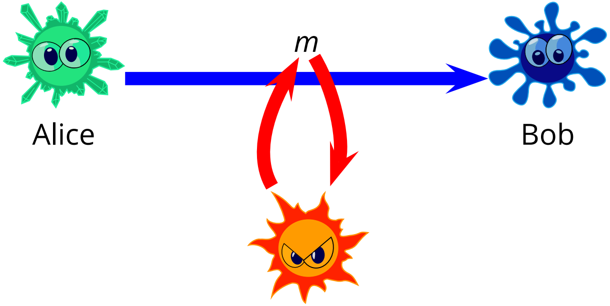
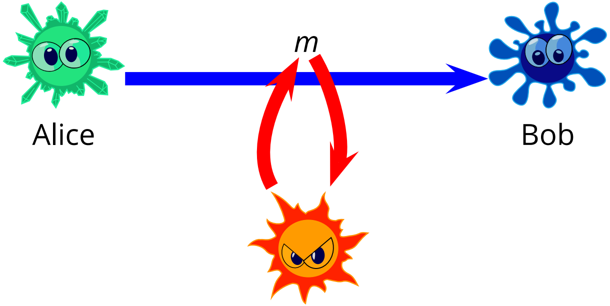
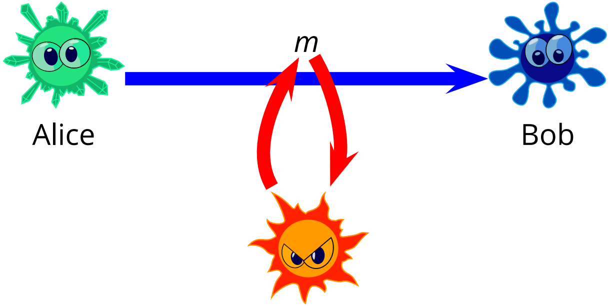




- ***End-point Authentication:*** Bob wants to make sure he is communicating with Alice



- ***End-point Authentication:*** Bob wants to make sure he is communicating with Alice
- ***Operational/system security:*** Alice and Bob want to maintain full control of their networks

What is Privacy, Exactly?

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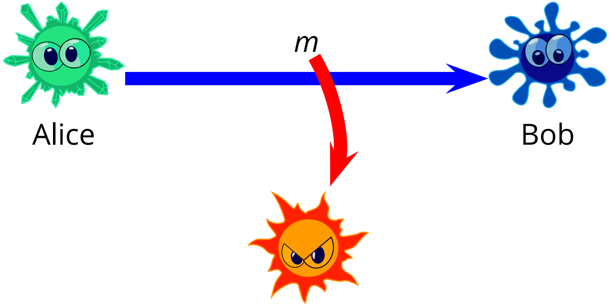

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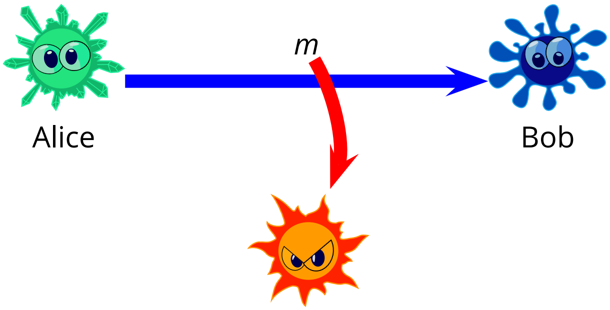

- Alice wants to make sure that only Bob “sees” the message
- What if Eve can *guess* the message?

“Shift” Cipher

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- The ciphertext is

```
BUUBDL BU EBXO
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- How many possible ciphers?
  - How many key bits?

Substitution Cipher

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Example:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	_
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How many possible permutations?

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27!

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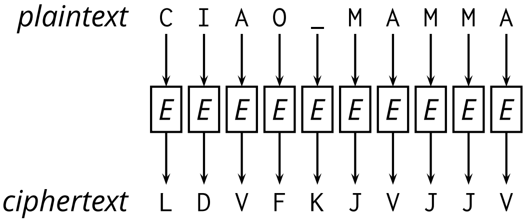
How many possible permutations?

$$27! = 10888869450418352160768000000 \approx 2^{93}$$

- Encrypting some text using a substitution cipher

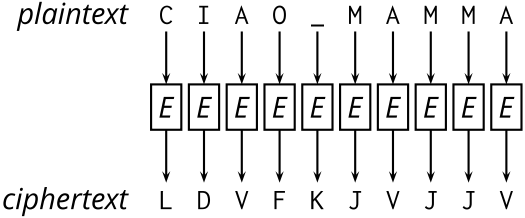
plaintext `C I A O _ M A M M A`

- Encrypting some text using a substitution cipher



- Problems?

- Encrypting some text using a substitution cipher



- Problems?
  - easy to break just by *guessing!*
  - ...

- Decrypt this ciphertext obtained by encrypting an English text with a substitution-cipher:

gbafoduayfbhbayvpyfhayoanbahbdl-brcubqyayfkyakddaibqakvbaxvbkybuabzpkd
yfkyayfbwakvbabquogbuanwayfbcvaxvbkyovagcyfaxbvykcqapqkdcbqkndbavctfyh
yfkyakioqtayfbhbakvbadclbadcnbvywakquayfbampvhpcyaolafkmmcqbhh
yfkyayoahbxpvbayfbhbavctfyhatorbvqibqyhakvbacqhycypybuakioqtaibq
ubvcrcqtayfbcvajphyamogbvhalvoiayfbaxoqhbqyaolayfbatorbvqbu
yfkyagfbqbrbvakqwaloviaolatorbvqibqyanbxoibhaubhyvpxycrbaolayfbhbabquh
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Modern cryptology

- Open and clear models
- Open algorithms (the only secret part is the key material)
- Well-defined *provable* security properties

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1. somebody (re-)designs a cryptosystem or protocol
2. somebody brakes it
3. go back to step 1

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   - prove this implication:

     primitive is secure $\Rightarrow$ ***protocol is secure***

Example: Factoring

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Problem: given N, find p and q

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...we don't know how to do better!

Not even Gauss could figure that out!

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  …in the sense that, if you can break SSH (efficiently) then you can also break RSA

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- Basic ingredients: cryptographic primitives
  - secret-key (symmetric) cryptography (e.g., AES)
  - public-key (asymmetric) cryptography (e.g., RSA)
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- Applications
  - electronic commerce
  - secure shell
  - secure electronic mail
  - virtual private networks
  - ...

Symmetric Encryption

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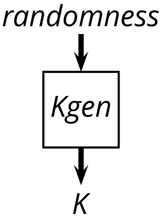

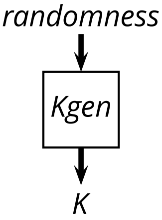

S

R


randomness
Kgen
K
S
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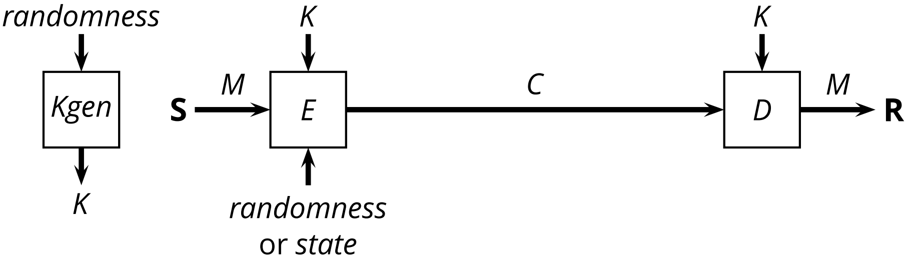
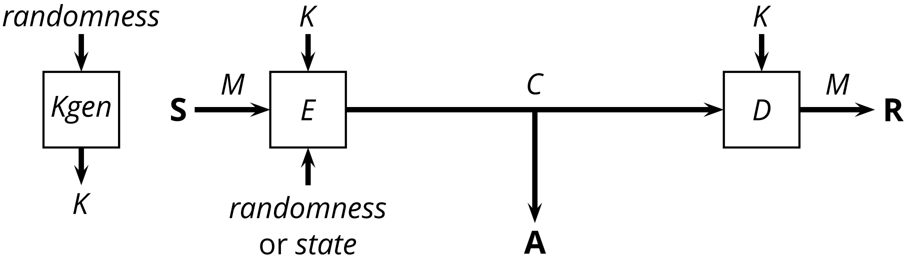

randomness
Kgen
K
S
M
K
E
randomness
or state
C
K
D
M
R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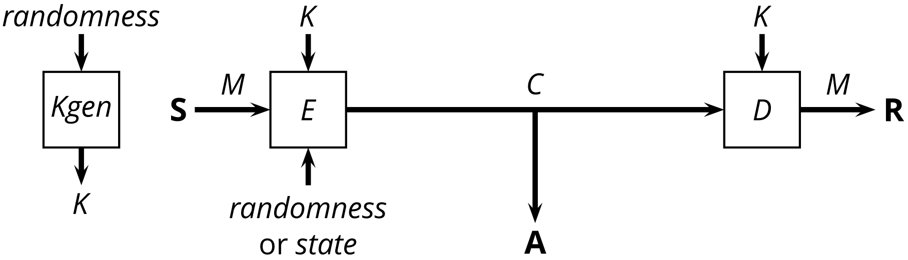

S	sender
R	receiver
A	adversary
E	encryption algorithm
D	dencryption algorithm
M	plaintext message
C	ciphertext message
K	key

One-Time Pad

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- *Scheme*
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  $$E(K,M) := M \oplus K$$
  the key K is then thrown away an never reused
  - decryption:
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- **Example:** M 0110010110111011

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- **Example:**

M	0110010110111011
K	1011000101000101

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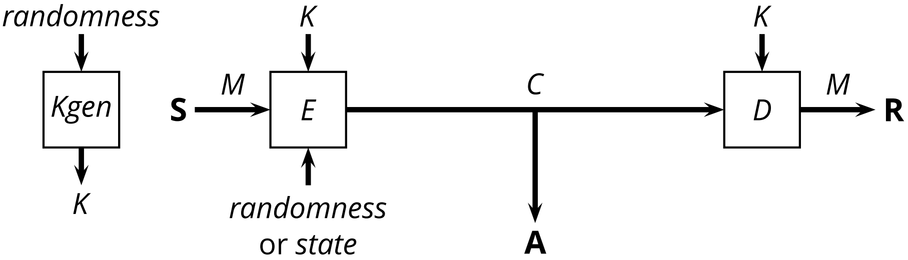
C	1101010011111110

Symmetric Encryption (2)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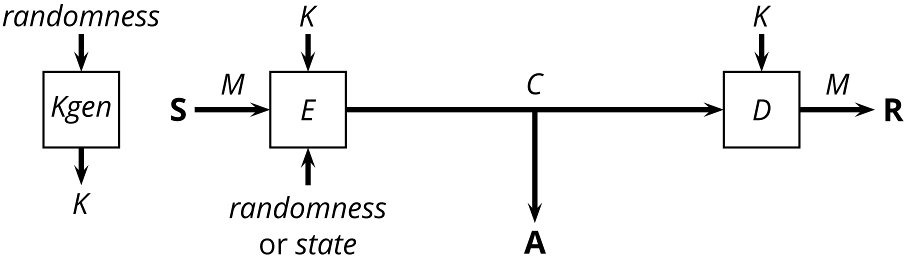

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Rules of the game:

- *Kgen*, *E* and *D* are *public* algorithms
- **A** can not “steal” the key *K*
- **A** can not “break into” **S** or **R**
- **A** might know something about *M*

A must guess *M* correctly to win the game

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- Is a substitution cipher perfectly secure?
- Is one-time-pad perfectly secure?

The Cost of Perfect Privacy

- *Perfect privacy* implies that

$$|\mathcal{K}| \geq |\mathcal{M}|$$

- *Perfect privacy* implies that

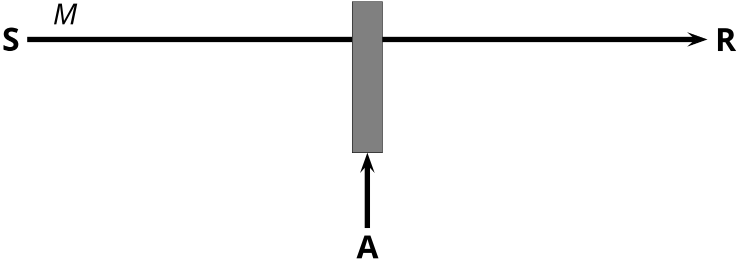
$$|\mathcal{K}| \geq |\mathcal{M}|$$

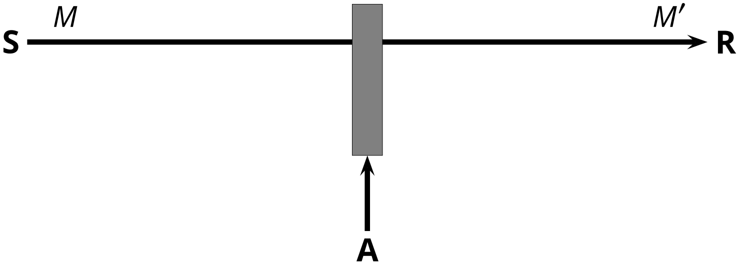
- *Proof:* assume not.
  Fix a possible ciphertext C, i.e., there is a message m and a key k such that $E_K(m) = C$, and $\Pr_{K \in \mathcal{K}}[E_K(m) = C] > 0$
  Let $P_C = \{m \in \mathcal{M}$ such that $E_k(m) = C$ for some $k\}$
  Since every k maps exactly one message m to C, and since we have fewer keys than messages, then there is an $m' \notin P_C$ such that no key k maps m' to C; therefore $\Pr_{K \in \mathcal{K}}[E_K(m') = C] = 0$, which violates the perfect-secrecy condition that for all m and m', $\Pr_{K \in \mathcal{K}}[E_K(m) = C] = \Pr_{K \in \mathcal{K}}[E_K(m') = C]$

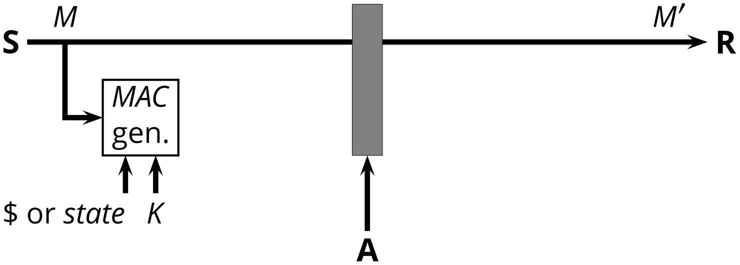
Message Authenticity

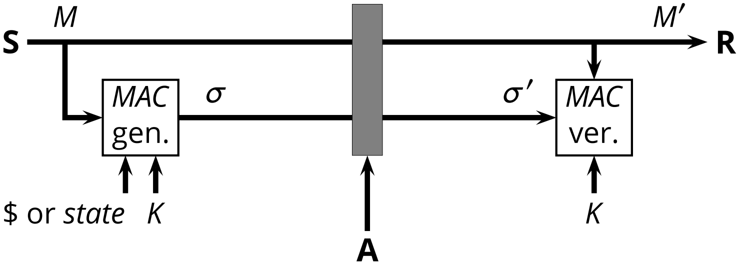
S R

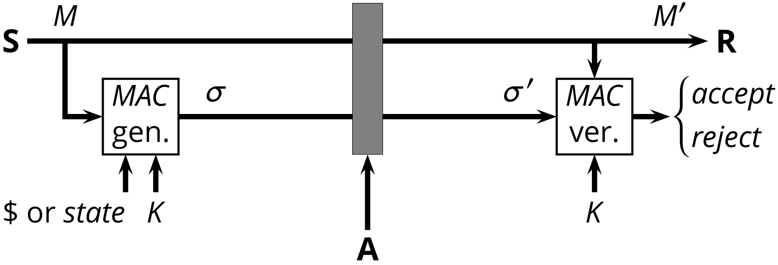
Message Authenticity

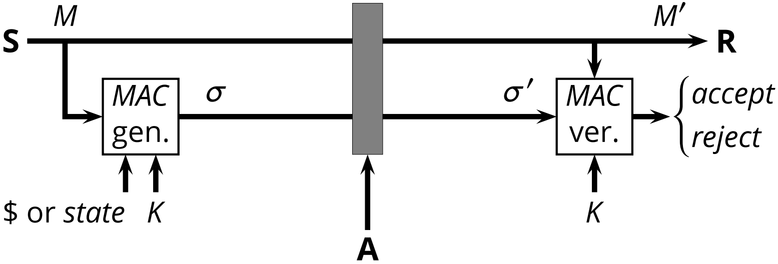

M
S
R
A


M
M′
S
R
A


M
S
M′
R
MAC
gen.
$ or state
K
A


S
M
M′
R
MAC
gen.
σ
σ′
MAC
ver.
$ or state
K
A
K


S
M
M′
R
MAC gen.
σ
σ′
MAC ver.
accept
reject
$ or state
K
K
A



σ	message authentication code (MAC)
K	key
\$	randomness
MAC gen.	MAC *generation* algorithm
MAC ver.	MAC *verification* algorithm

Asymmetric Encryption

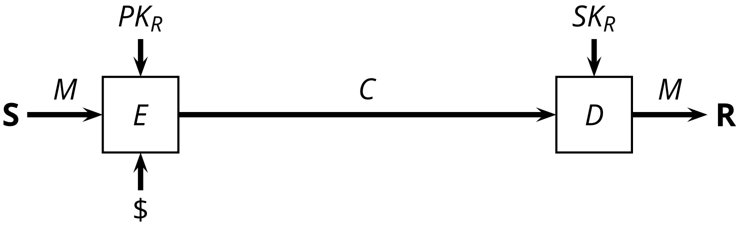
Asymmetric Encryption

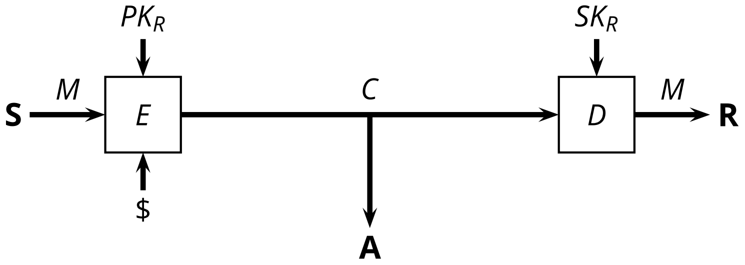
S **R**

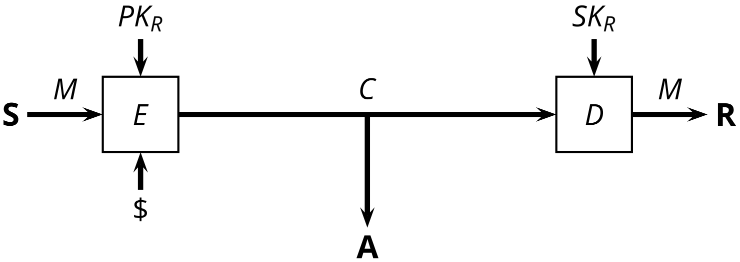

S
M
E
R

Asymmetric Encryption


PK_R
S
M
E
$
R

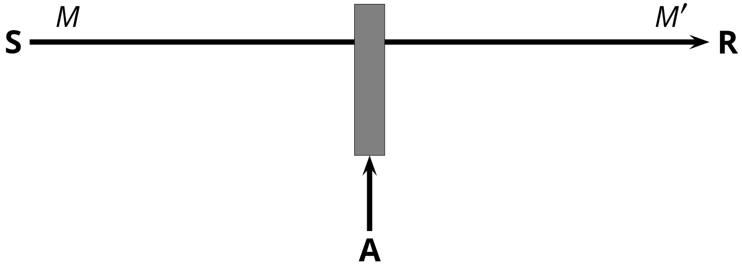

PK_R
SK_R
S
M
E
C
D
M
R
$

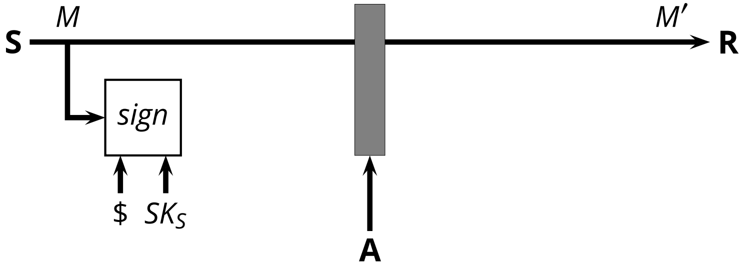

PK_R
SK_R
S
M
E
C
D
M
R
$
A

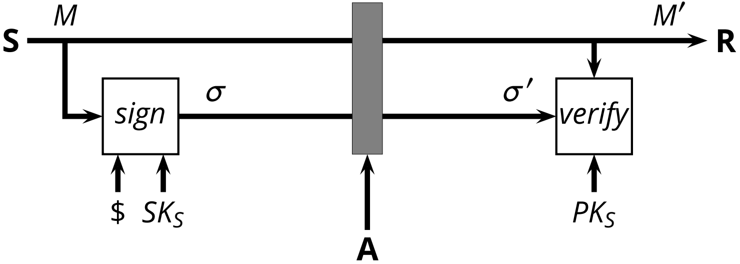


PK_R	receiver's *public key*
SK_R	receiver's *secret key*
M	plaintext message
C	ciphertext message

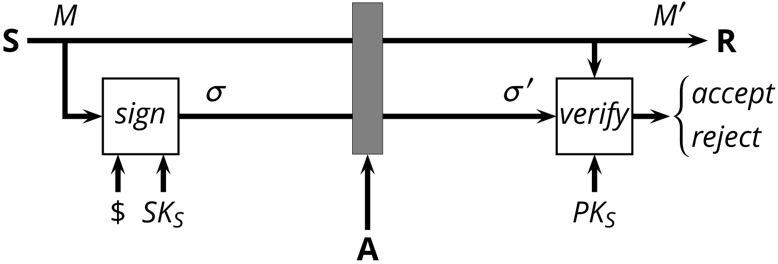
Digital Signatures

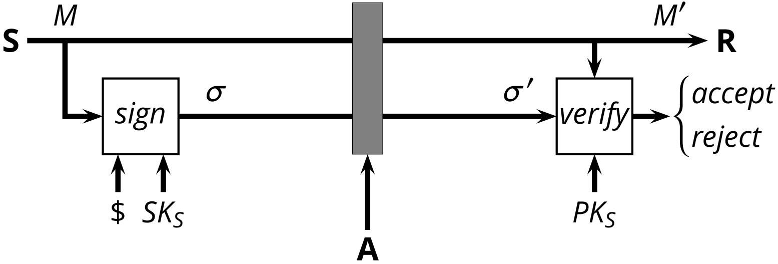

S
M
M′
R
A


S
M
M′
R
sign
$
SK_S
A



S
M
sign
σ
$
SK_S
A
M′
R
σ′
verify
PK_S


S
M
M′
R
sign
σ
σ′
verify
accept
reject
$ SK_S
A
PK_S



σ	digital signature
SK_S	sender's *secret key*
PK_S	sender's *public key*
\$	randomness
sign	*signing* algorithm
verify	*verification* algorithm

Primitives vs. Protocols

- ***Protocol***
  - an *algorithm*
  - solves a specific security problem (e.g., signing a message)

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- ***Protocol***
  - an *algorithm*
  - solves a specific security problem (e.g., signing a message)
- ***Primitive***
  - also an *algorithm*
  - the elementary subroutines of protocols
  - implement (try to approximate) well-defined mathematical object
  - embody “hard problems”

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  - given an initialization key K
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  - generates an infinite pseudo-random sequence of bits
- E.g., RC4

Padding with a Stream Cipher

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- *Assumptions:* S and R share a secret key K and agree to use a stream cipher S_K
  - S and R maintain some *state:* position s initialized to $s = 0$

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- *Dencryption protocol*

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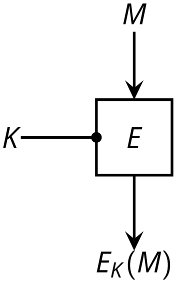

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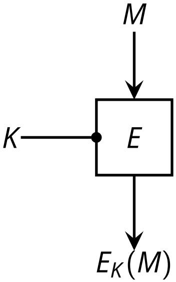

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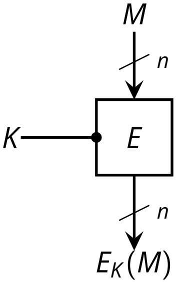

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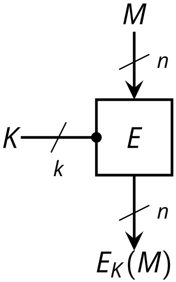

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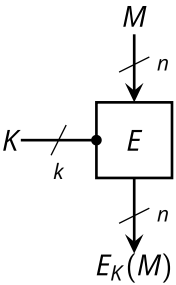

  - $E_K(\cdot)$ is a *permutation*, so $E_K^{-1}(\cdot)$ is always defined
  - fixed-length input and output (n)
  - fixed-length key (k)
  - e.g., DES, AES

An Encryption Protocol

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  - *Input:* k-bit key K, N-bit message M
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CBC(K, M)

$x \leftarrow 0^n$

for $i \leftarrow 0$ **to** $\lfloor |M|/n \rfloor$

  do $C[ni \ldots ni+n-1] \leftarrow E_K(x \oplus M[ni \ldots ni+n-1])$

    $x \leftarrow C[ni \ldots ni+n-1]$

return C

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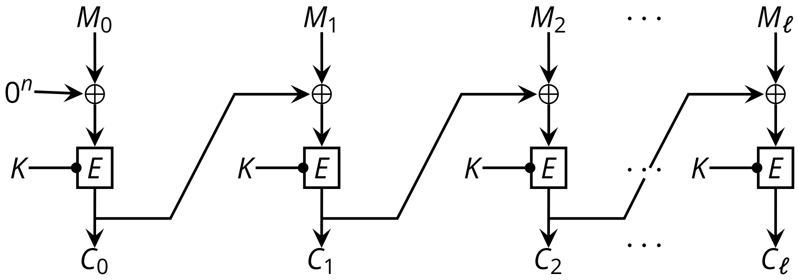

Exercise

- Write the decryption algorithm for CBC

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CBC-DECRYPT(K, C)

$x \leftarrow 0^n$

for $i \leftarrow 0$ **to** $\lfloor |C|/n \rfloor$

  do $M[ni \ldots ni + n - 1] \leftarrow x \oplus E_K^{-1}(C[ni \ldots ni + n - 1])$

    $x \leftarrow C[ni \ldots ni + n - 1]$

return M

An Encryption Protocol (2)

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  - ***any deterministic stateless protocol is insecure***
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- Is this CBC protocol secure?
  - ***any deterministic stateless protocol is insecure***
  - we need *state* and/or *randomness*
- What if $|M| \neq 0 \mod n$?
- Is CBC parallelizable?

CBC With Random IV

- ***CBC$:*** cipher block chaining with random IV

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```
CBC$-ENCRYPT(K, M)
if |M| = 0 ∨ |M| ≠ 0  mod n
    then return ⊥
M[1] · M[2] · · · M[ℓ] ← M
IV ←$ {0, 1}^n
C[0] ← IV
for i ← 1 to ℓ
        do C[i] ← E_K(C[i − 1] ⊕ M[i])
C ← C[1] · C[2] · · · C[ℓ]
return ⟨IV, C⟩
```

CBC With Random IV (2)

- ***CBC$:*** cipher block chaining with random IV (decryption)

CBC With Random IV (2)

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CBC$-DECRYPT$(K, IV, C)$

if $|C| = 0 \lor |C| \neq 0 \mod n$
  then return $\bot$
$C[1] \cdot C[2] \cdots C[\ell] \leftarrow C$
$C[0] \leftarrow IV$
for $i \leftarrow 1$ **to** ℓ
  do $M[i] \leftarrow C[i-1] \oplus E_K(C[i])$
$M \leftarrow M[1] \cdot M[2] \cdots M[\ell]$
return M

CBC With Stateful Counter

- ***CBCC:*** cipher block chaining with stateful counter

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CBCC-ENCRYPT(K, M)

static $ctr \leftarrow 0$
if $ctr \geq 2^n \vee |M| = 0 \vee |M| \neq 0 \mod n$
  then return $\perp$
$M[1] \cdot M[2] \cdots M[\ell] \leftarrow M$
$IV \leftarrow [ctr]_n$
$C[0] \leftarrow [ctr]_n$
for $i \leftarrow 1$ **to** ℓ
    do $C[i] \leftarrow E_K(C[i-1] \oplus M[i])$
$C \leftarrow C[1] \cdot C[2] \cdots C[\ell]$
$ctr \leftarrow ctr + 1$
return $\langle IV, C \rangle$

CBC With Stateful Counter (2)

- ***CBCC:*** cipher block chaining with stateful counter

CBC With Stateful Counter (2)

- ***CBCC:*** cipher block chaining with stateful counter

```
CBCC-Decrypt(K, IV, C)
if IV + |C| ≥ 2^n ∨ |C| = 0 ∨ |C| ≠ 0  mod n
    then return ⊥
C[1] · C[2] · · · C[ℓ] ← C
IV ← [ctr]_n
C[0] ← IV
for i ← 1 to ℓ
        do M[i] ← C[i − 1] ⊕ E_K^{−1}(C[i])
M ← M[1] · M[2] · · · M[ℓ]
return M
```

Counter Mode

- ***CTR$:*** counter mode with random initial counter

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  - *family of functions:* $F : \{0,1\}^k \times \{0,1\}^n \to \{0,1\}^n$

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```
CTR$-ENCRYPT(K, M)

R ←$ {0,1}^n
Pad ← F_K([R]_n)
for i ← 1 to ⌈|M|/n⌉ − 1
        do Pad ← Pad ·F_K([R + i]_n)
Pad ← first |M| bits of Pad
C ← M ⊕ Pad
return ⟨R, C⟩
```

- ***CTR$:*** counter mode with random initial counter (decryption)
  - *family of functions:* $F : \{0,1\}^k \times \{0,1\}^n \to \{0,1\}^n$

Counter Mode (2)

- ***CTR$:*** counter mode with random initial counter (decryption)
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CTR$-DECRYPT(K, R, C)
Pad ← F_K([R]_n)
for i ← 1 to ⌈|C|/n⌉ − 1
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Pad ← first |C| bits of Pad
M ← C ⊕ Pad
return M
```

- ***CTRC:*** counter mode with stateful counter
  - *family of functions:* $F : \{0,1\}^k \times \{0,1\}^n \longrightarrow \{0,1\}^n$

Counter Mode (3)

- ***CTRC:*** counter mode with stateful counter
  - *family of functions:* $F : \{0,1\}^k \times \{0,1\}^n \rightarrow \{0,1\}^n$

CTRC(K, M)

static $R \leftarrow 0$
$\ell \leftarrow \lceil |M|/n \rceil$
if $R + \ell - 1 \geq 2^n$
    then return $\perp$
$Pad \leftarrow F_K([R]_n)$
for $i \leftarrow 1$ **to** $\ell - 1$
        do $Pad \leftarrow Pad \cdot F_K([R+i]_n)$
$Pad \leftarrow$ first $|M|$ bits of Pad
$C \leftarrow M \oplus Pad$
$R \leftarrow R + \ell$
return $\langle R - \ell, C \rangle$

- ***CTRC:*** counter mode with stateful counter (decryption)
  - *family of functions:* $F : \{0,1\}^k \times \{0,1\}^n \rightarrow \{0,1\}^n$

Counter Mode (4)

- ***CTRC:*** counter mode with stateful counter (decryption)
  - *family of functions:* $F : \{0,1\}^k \times \{0,1\}^n \rightarrow \{0,1\}^n$

CTRC-DECRYPT(K, R, C)

$Pad \leftarrow F_K([R]_n)$
for $i \leftarrow 1$ **to** $\lceil |C|/n \rceil - 1$
    do $Pad \leftarrow Pad \cdot F_K([R+i]_n)$
$Pad \leftarrow$ first $|C|$ bits of Pad
$M \leftarrow C \oplus Pad$
return M

- *MAC generation*
  - *Input:* k-bit key K, N-bit message M
  - *Output:* n-bit message authentication code σ

Authentication Protocol

- *MAC generation*
  - *Input:* k-bit key K, N-bit message M
  - *Output:* n-bit message authentication code σ

- *CBC with random IV*
  - use a block cipher $E : \{0,1\}^k \times \{0,1\}^n \rightarrow \{0,1\}^n$
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MAC(K, M)

$IV \xleftarrow{\$} \{0,1\}^n$
$C \leftarrow IV$
for $i \leftarrow 0$ **to** $\lfloor |M|/n \rfloor$
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return $\langle IV, C \rangle$

- *MAC generation*
  - *Input:* k-bit key K, N-bit message M
  - *Output:* n-bit message authentication code σ

- *CBC with random IV*
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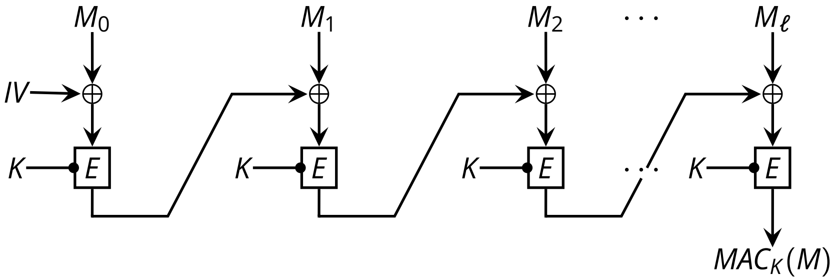

CBC MAC: Generation

- ***CBC MAC:*** cipher block chaining MAC with random IV

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CBC-MAC\$$(K, M)$

if $|M| = 0 \vee |M| \neq 0 \mod n$

  then return $\perp$

$M[1] \cdot M[2] \cdots M[\ell] \leftarrow M$

$IV \xleftarrow{\$} \{0,1\}^n$

$C \leftarrow IV$

for $i \leftarrow 1$ **to** ℓ

  do $C \leftarrow E_K(C \oplus M[i])$

return $\langle IV, C \rangle$

CBC MAC: Verification

- ***CBC MAC:*** cipher block chaining MAC with random IV

CBC MAC: Verification

- ***CBC MAC:*** cipher block chaining MAC with random IV

CBC-MAC\$-VERIFY(K, IV, σ, M)

if $|M| = 0 \vee |M| \neq 0 \mod n$
    then return $\perp$
$M[1] \cdot M[2] \cdots M[\ell] \leftarrow M$
$C \leftarrow IV$
for $i \leftarrow 1$ **to** ℓ
        do $C \leftarrow E_K(C \oplus M[i])$
if $C = \sigma$
    then return ACCEPT
    else return REJECT

Cryptographic Hash Functions

- *Cryptographic Hash:* $H : \{0,1\}^* \to \{0,1\}^n$

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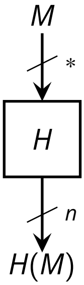

- *Cryptographic Hash:* $H : \{0,1\}^* \rightarrow \{0,1\}^n$
  - $H(\cdot)$ is a good *hash* function when (***informally***)

$$\forall m \in \{0,1\}^*, h \in \{0,1\}^n, \Pr[H(m) = h] = \frac{1}{2^n}$$

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  - it is "difficult" to find *collisions*

$$\text{find } m_1, m_2 \in \{0,1\}^* : m_1 \neq m_2, H(m_1) = H(m_2)$$

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  - it is "difficult" to find *collisions*

$$\text{find } m_1, m_2 \in \{0,1\}^* : m_1 \neq m_2, H(m_1) = H(m_2)$$

  - it is "difficult" to find a *preimage*

$$\text{given } m \in \{0,1\}^*, \text{ find } m' : H(m') = m$$

- *Cryptographic Hash:* $H : \{0,1\}^* \rightarrow \{0,1\}^n$
  - $H(\cdot)$ is a good *hash* function when (***informally***)

$$\forall m \in \{0,1\}^*, h \in \{0,1\}^n, \Pr[H(m) = h] = \frac{1}{2^n}$$

  - it is "difficult" to find *collisions*

$$\text{find } m_1, m_2 \in \{0,1\}^* : m_1 \neq m_2, H(m_1) = H(m_2)$$

  - it is "difficult" to find a *preimage*

$$\text{given } m \in \{0,1\}^*, \text{ find } m' : H(m') = m$$

  - e.g., SHA-1

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  - secure electronic mail
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