A Few Basic Elements of Communication Security

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December 22, 2016

Some Advice

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- ***Make backups*** of your data
- ***Do NOT trust the network!***
- ***Use HTTPS*** instead of HTTP

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- ***Do NOT trust the network!***
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- Understand the basics of public-key cryptography
- Communicate with ***end-to-end encryption*** (e.g., e-mail)
- use ***trusted certificates***
- ***Encrypt your confidential data*** (and make backups)
- use ***strong passwords***
- You might as well encrypt *all* your data
- Tools/technologies: ***ssh***, ***pgp*** (or ***gpg***)

Outline

- Communication security model
- Information-theoretic privacy
- Substitution ciphers
- Intro to modern cryptography
- One-time pad
- Block siphers
- Cryptographic hash functions
- Public-key cryptosystems

Communication Security

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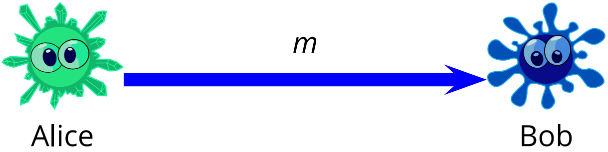

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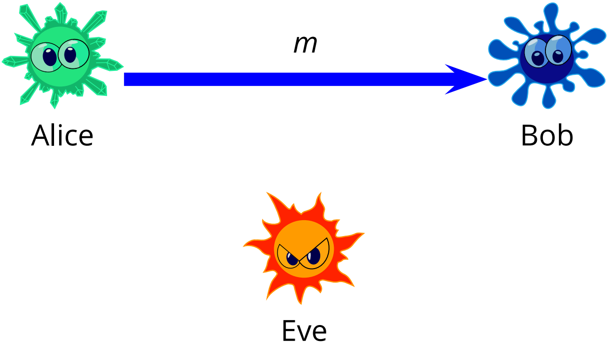

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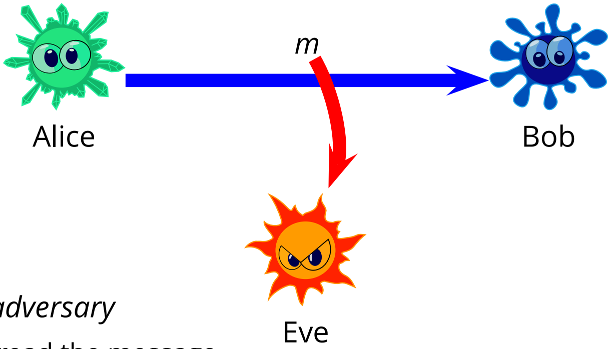

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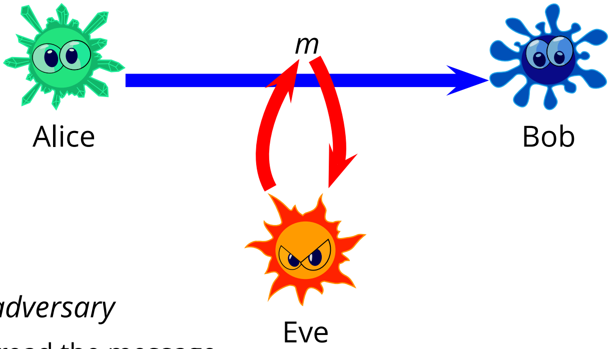

- *Passive adversary*
  - can read the message
- *Active adversary*
  - can modify the message

Goals

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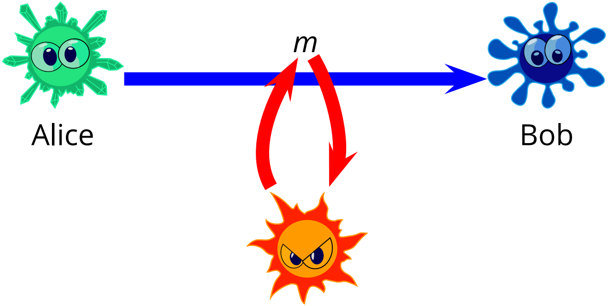

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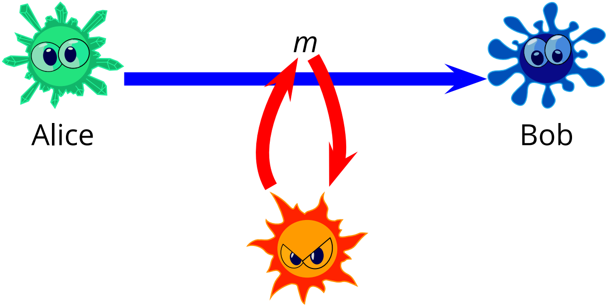

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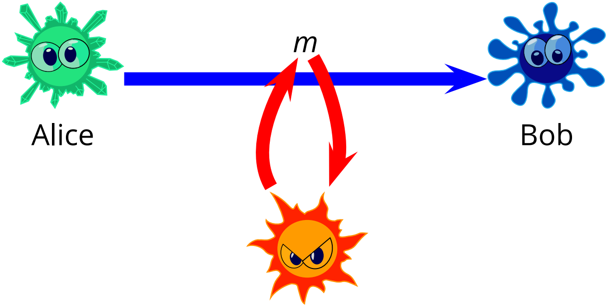

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- ***Message Integrity:*** Bob wants to make sure that the message he reads was exactly what Alice wrote

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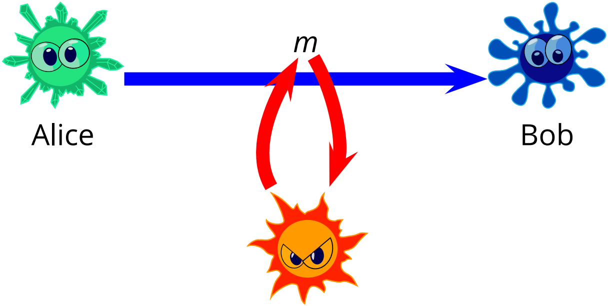

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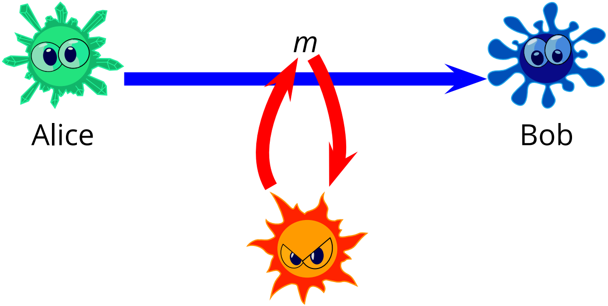

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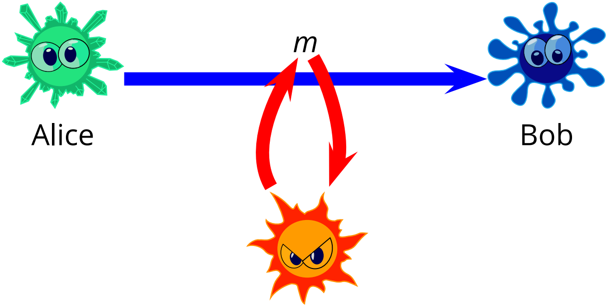

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- ***Operational/system security:*** Alice and Bob want to maintain full control of their networks

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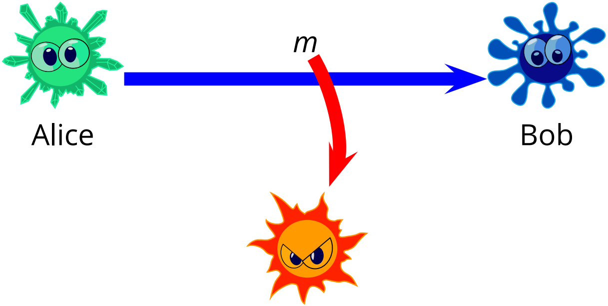

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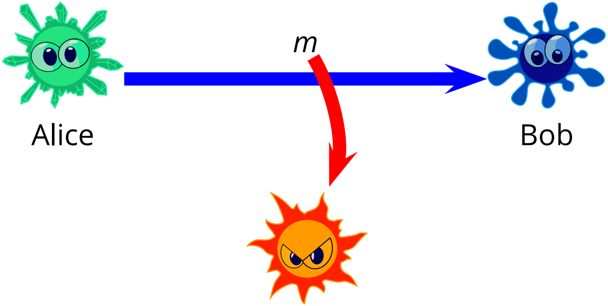

- Alice wants to make sure that only Bob “sees” the message
- What if Eve can *guess* the message?

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BUUBDL BU EBXO
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- How many possible ciphers?
  - How many key bits?

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How many possible permutations?

$$27! = 10888869450418352160768000000 \approx 2^{93}$$

- Encrypting some text using a substitution cipher

plaintext `C I A O _ M A M M A`

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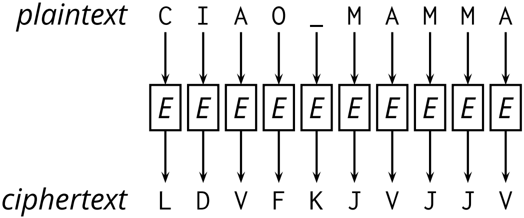

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Substitution Cipher

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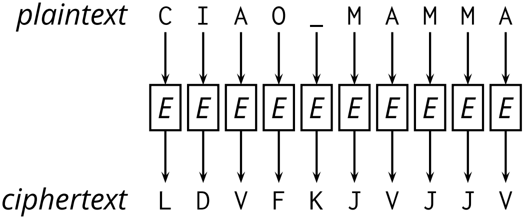

- Problems?
  - easy to break just by *guessing!*
  - . . .

Problem

- Decrypt this ciphertext obtained by encrypting an English text with a substitution-cipher:

gbafoduayfbhbayvpyfhayoanbahbdl-brcubqyayfkyakddaibqakvbaxvbkybuabzpkd
yfkyayfbwakvbabquogbuanwayfbcvaxvbkyovagcyfaxbvykcqapqkdcbqkndbavctfyh
yfkyakioqtayfbhbakvbadclbadcnbvywakquayfbampvhpcyaolafkmmcqbhh
yfkyayoahbxpvbayfbhbavctfyhatorbvqibqyhakvbacqhycypybuakioqtaibq
ubvcrcqtayfbcvajphyamogbvhalvoiayfbaxoqhbqyaolayfbatorbvqbu
yfkyagfbqbrbvakqwaloviaolatorbvqibqyanbxoibhaubhyvpxycrbaolayfbhbabquh
cyachayfbavctfyaolayfbambomdbayoakdybvaovayoaknodchfacy

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Modern cryptology

- Open and clear models
- Open algorithms (the only secret part is the key material)
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1. somebody (re-)designs a cryptosystem or protocol
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3. go back to step 1

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   - based on ***public*** and ***time-tested algorithms*** and/or well-studied ***hard mathematical problems***

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   - prove this implication:

     primitive is secure $\Rightarrow$ ***protocol is secure***

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. . . we don't know how to do better!

Not even Gauss could figure that out!

Example: RSA,… ⇒ SSH

Example: the RSA cryptosystem (primitive) and SSH (protocol)

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The Big Picture

- Basic ingredients: cryptographic primitives
  - secret-key (symmetric) cryptography (e.g., AES)
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  - . . .
- Applications
  - electronic commerce
  - secure shell
  - secure electronic mail
  - virtual private networks
  - . . .

Symmetric Encryption

S

R

S
M
E
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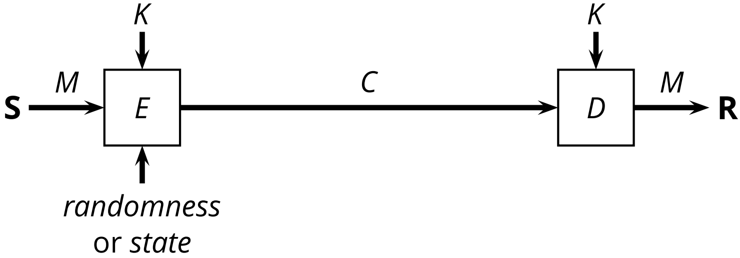

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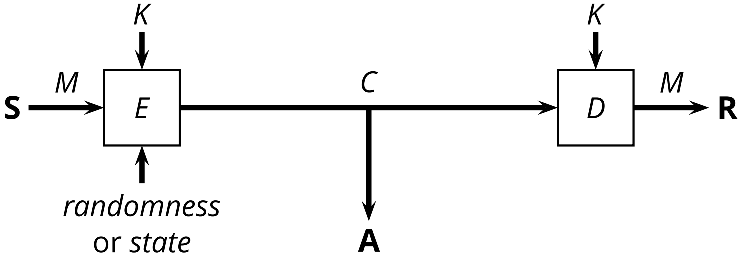

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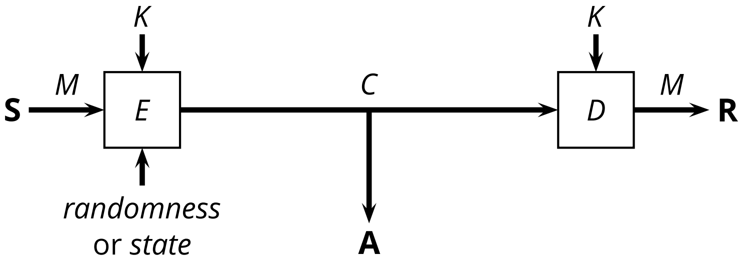

S	sender
R	receiver
A	adversary
E	encryption algorithm
D	dencryption algorithm
M	plaintext message
C	ciphertext message
K	key

One-Time Pad

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  K 1011000101000101

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K	1011000101000101
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- *Proof:* assume not.
  Fix a possible ciphertext C, i.e., there is a message m and a key k such that $E_K(m) = C$, and $\Pr_{K \in \mathcal{K}}[E_K(m) = C] > 0$
  Let $P_C = \{m \in \mathcal{M}$ such that $E_k(m) = C$ for some $k\}$
  Since every k maps exactly one message m to C, and since we have fewer keys than messages, then there is an $m' \notin P_C$ such that no key k maps m' to C; therefore $\Pr_{K \in \mathcal{K}}[E_K(m') = C] = 0$, which violates the perfect-secrecy condition that for all m and m', $\Pr_{K \in \mathcal{K}}[E_K(m) = C] = \Pr_{K \in \mathcal{K}}[E_K(m') = C]$

Message Authenticity

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S

R

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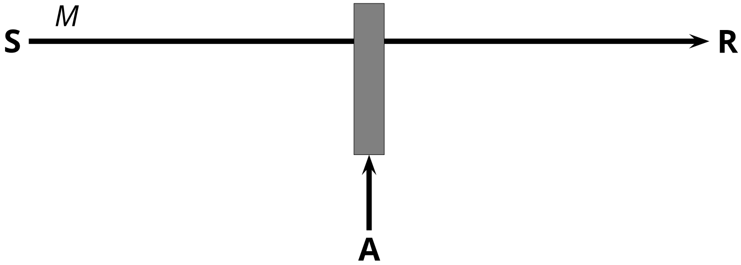
$$\mathbf{S} \xrightarrow{\quad M \quad} \mathbf{R}$$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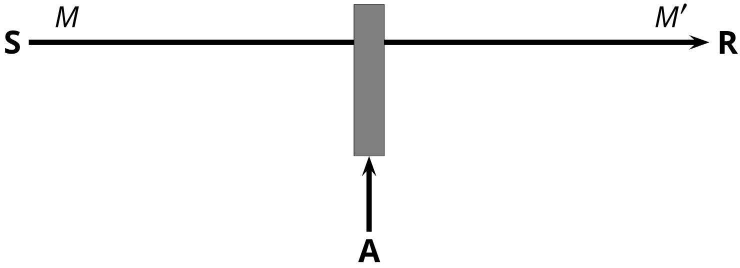
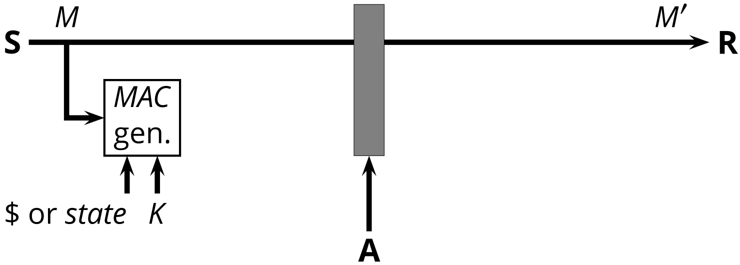
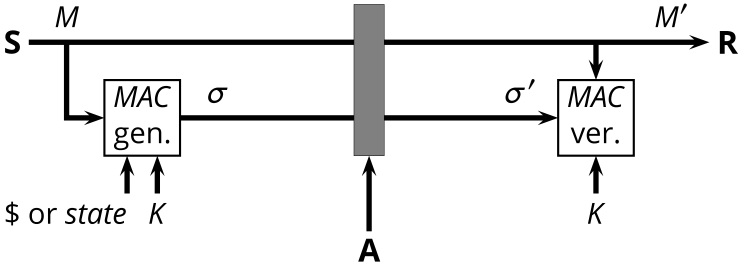
Message Authenticity




S
M
M′
R
MAC
gen.
$ or state
K
A


S
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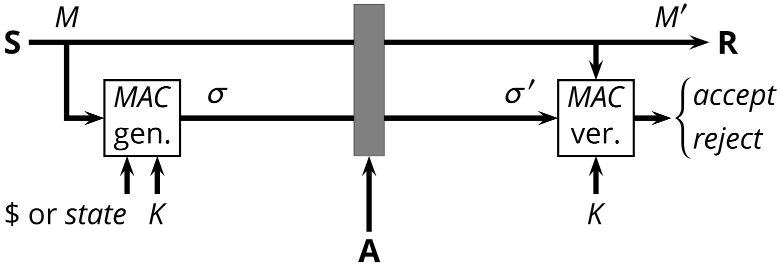

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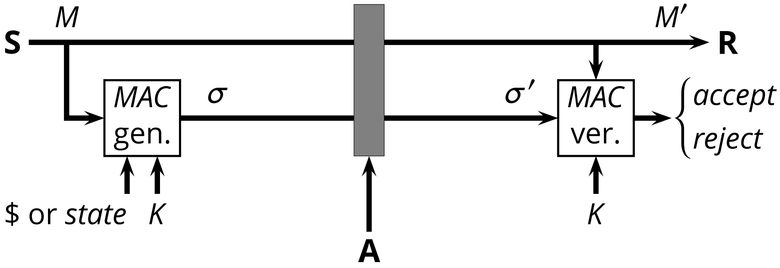


σ	message authentication code (MAC)
K	key
\$	randomness
MAC gen.	MAC *generation* algorithm
MAC ver.	MAC *verification* algorithm

Asymmetric Encryption

Asymmetric Encryption

S

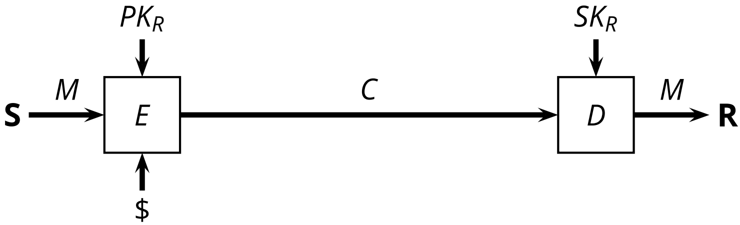
R

Asymmetric Encryption

Asymmetric Encryption

Asymmetric Encryption

Asymmetric Encryption



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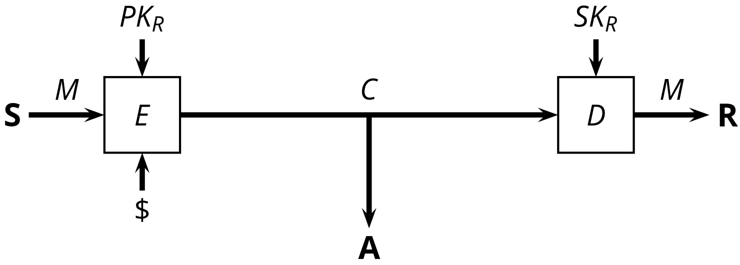
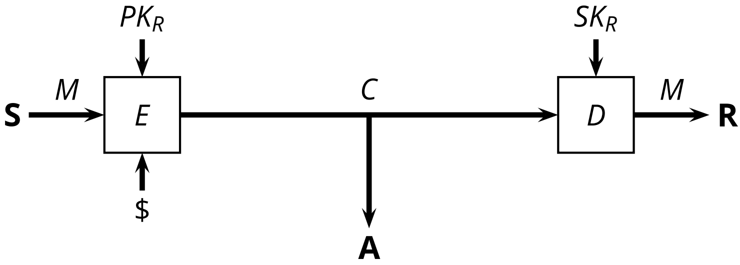
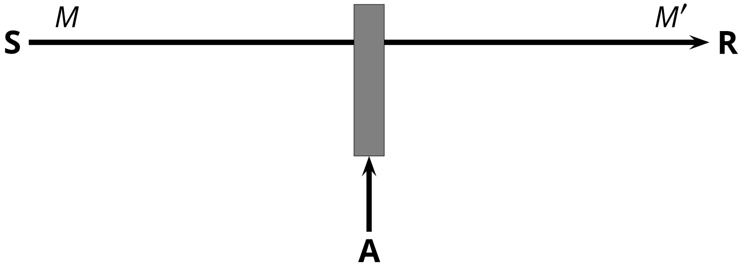
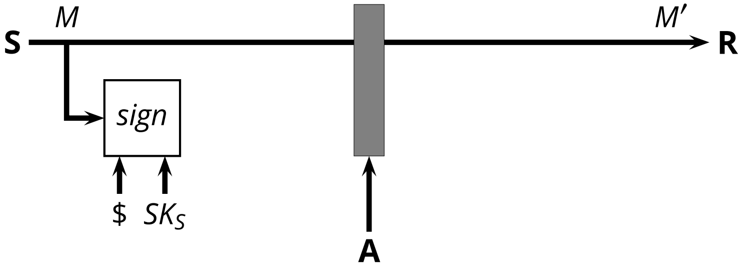


Asymmetric Encryption

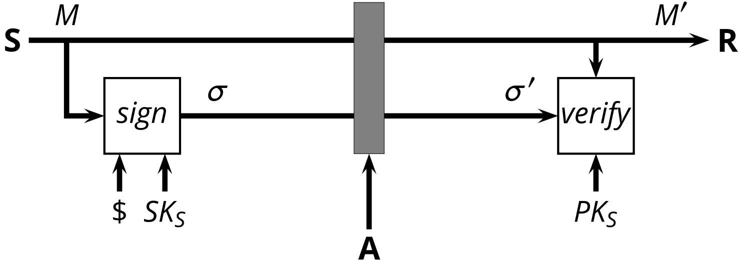


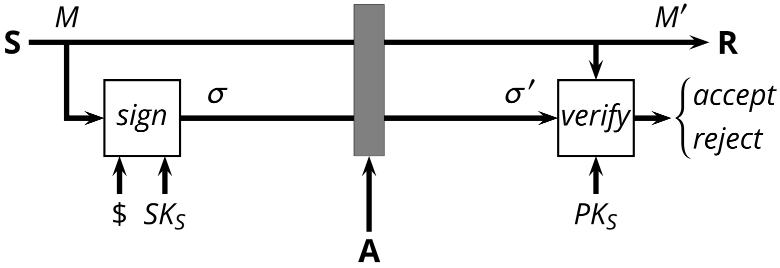
PK_R	receiver's *public key*
SK_R	receiver's *secret key*
M	plaintext message
C	ciphertext message

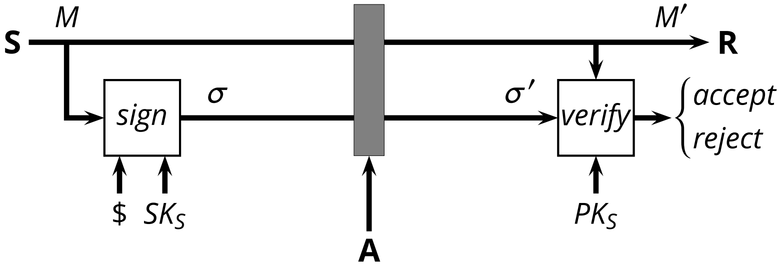

S
M
M′
R
A



M
M′
S
R
sign
$
SK_S
A


S
M
M′
R
sign
σ
σ′
verify
$
SK_S
PK_S
A


S
M
M′
R
sign
σ
σ′
verify
accept
reject
$
SK_S
PK_S
A



σ	digital signature
SK_S	sender's *secret key*
PK_S	sender's *public key*
$	randomness
sign	*signing* algorithm
verify	*verification* algorithm

Primitives vs. Protocols

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- ***Protocol***
  - an *algorithm*
  - solves a specific security problem (e.g., signing a message)

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- ***Primitive***
  - also an *algorithm*
  - the elementary subroutines of protocols
  - implement (try to approximate) well-defined mathematical object
  - embody “hard problems”

Stream Ciphers

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- A *stream cipher* is a generator of a *pseudo-random streams*

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  - given an initialization key K
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- E.g., RC4

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  - S and R maintain some *state:* position s initialized to $s = 0$

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- *Encryption protocol*

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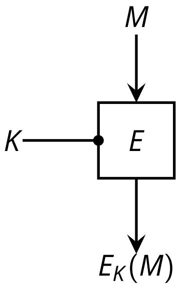

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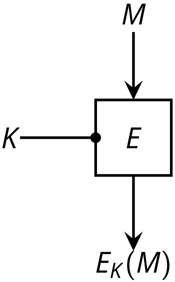

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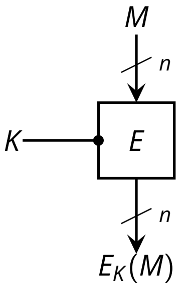

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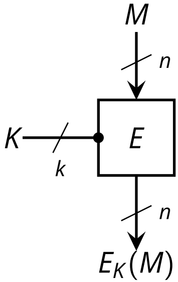

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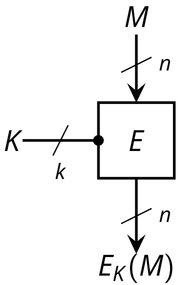

- $E_K(\cdot)$ is a *permutation*, so $E_K^{-1}(\cdot)$ is always defined
- fixed-length input and output (n)
- fixed-length key (k)
- e.g., DES, AES

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```
CBC(K, M)
  x ← 0^n
  for i ← 0 to ⌊|M|/n⌋
      do C[ni ... ni + n − 1] ← E_K(x ⊕ M[ni ... ni + n − 1])
         x ← C[ni ... ni + n − 1]
  return C
```

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  - *Output:* N-bit ciphertext C

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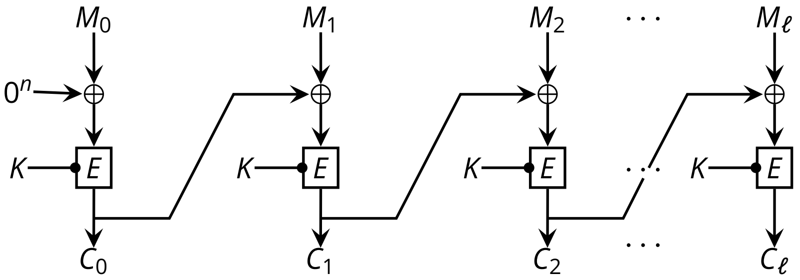

Exercise

- Write the decryption algorithm for CBC

- Write the decryption algorithm for CBC

CBC-Decrypt(K, C)

$x \leftarrow 0^n$

for $i \leftarrow 0$ **to** $\lfloor |C|/n \rfloor$

  do $M[ni \ldots ni + n - 1] \leftarrow x \oplus E_K^{-1}(C[ni \ldots ni + n - 1])$

    $x \leftarrow C[ni \ldots ni + n - 1]$

return M

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  - we need *state* and/or *randomness*
- What if $|M| \neq 0 \mod n$?
- Is CBC parallelizable?

CBC With Random IV

- ***CBC$:*** cipher block chaining with random IV

CBC With Random IV

- ***CBC\$:*** cipher block chaining with random IV

CBC\$-ENCRYPT(K, M)

if $|M| = 0 \vee |M| \neq 0 \mod n$

  then return $\perp$

$M[1] \cdot M[2] \cdots M[\ell] \leftarrow M$

$IV \xleftarrow{\$} \{0,1\}^n$

$C[0] \leftarrow IV$

for $i \leftarrow 1$ **to** ℓ

  do $C[i] \leftarrow E_K(C[i-1] \oplus M[i])$

$C \leftarrow C[1] \cdot C[2] \cdots C[\ell]$

return $\langle IV, C \rangle$

CBC With Random IV (2)

- ***CBC$:*** cipher block chaining with random IV (decryption)

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CBC$-$\textsc{Decrypt}(K, IV, C)$

if $|C| = 0 \lor |C| \neq 0 \mod n$
    then return $\perp$
$C[1] \cdot C[2] \cdots C[\ell] \leftarrow C$
$C[0] \leftarrow IV$
for $i \leftarrow 1$ **to** ℓ
    do $M[i] \leftarrow C[i-1] \oplus E_K(C[i])$
$M \leftarrow M[1] \cdot M[2] \cdots M[\ell]$
return M

CBC With Stateful Counter

- ***CBCC:*** cipher block chaining with stateful counter

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CBCC-ENCRYPT(K, M)

static $ctr \leftarrow 0$
if $ctr \geq 2^n \vee |M| = 0 \vee |M| \neq 0 \mod n$
  then return $\perp$
$M[1] \cdot M[2] \cdots M[\ell] \leftarrow M$
$IV \leftarrow [ctr]_n$
$C[0] \leftarrow [ctr]_n$
for $i \leftarrow 1$ **to** ℓ
    do $C[i] \leftarrow E_K(C[i-1] \oplus M[i])$
$C \leftarrow C[1] \cdot C[2] \cdots C[\ell]$
$ctr \leftarrow ctr + 1$
return $\langle IV, C \rangle$

CBC With Stateful Counter (2)

- ***CBCC:*** cipher block chaining with stateful counter

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CBCC-Decrypt(K, IV, C)

if $IV + |C| \geq 2^n \vee |C| = 0 \vee |C| \neq 0 \mod n$
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$C[1] \cdot C[2] \cdots C[\ell] \leftarrow C$
$IV \leftarrow [ctr]_n$
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for $i \leftarrow 1$ **to** ℓ
        do $M[i] \leftarrow C[i-1] \oplus E_K^{-1}(C[i])$
$M \leftarrow M[1] \cdot M[2] \cdots M[\ell]$
return M

- ***CTR\$:*** counter mode with random initial counter

Counter Mode

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  - *family of functions:* $F : \{0,1\}^k \times \{0,1\}^n \rightarrow \{0,1\}^n$

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CTR$-ENCRYPT$(K, M)$

$R \xleftarrow{\$} \{0,1\}^n$
$Pad \leftarrow F_K([R]_n)$
for $i \leftarrow 1$ **to** $\lceil |M|/n \rceil - 1$
    do $Pad \leftarrow Pad \cdot F_K([R+i]_n)$
$Pad \leftarrow$ first $|M|$ bits of Pad
$C \leftarrow M \oplus Pad$
return $\langle R, C \rangle$

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Counter Mode (3)

- ***CTRC:*** counter mode with stateful counter
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CTRC(K, M)

static $R \leftarrow 0$
$\ell \leftarrow \lceil |M|/n \rceil$
if $R + \ell - 1 \geq 2^n$
    then return $\perp$
$Pad \leftarrow F_K([R]_n)$
for $i \leftarrow 1$ **to** $\ell - 1$
        do $Pad \leftarrow Pad \cdot F_K([R + i]_n)$
$Pad \leftarrow$ first $|M|$ bits of Pad
$C \leftarrow M \oplus Pad$
$R \leftarrow R + \ell$
return $\langle R - \ell, C \rangle$

Counter Mode (4)

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```
CTRC-DECRYPT(K, R, C)
Pad ← F_K([R]_n)
for i ← 1 to ⌈|C|/n⌉ − 1
        do Pad ← Pad ·F_K([R + i]_n)
Pad ← first |C| bits of Pad
M ← C ⊕ Pad
return M
```

Authentication Protocol

- *MAC generation*
  - *Input:* k-bit key K, N-bit message M
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$\mathrm{MAC}(K, M)$

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return $\langle IV, C \rangle$

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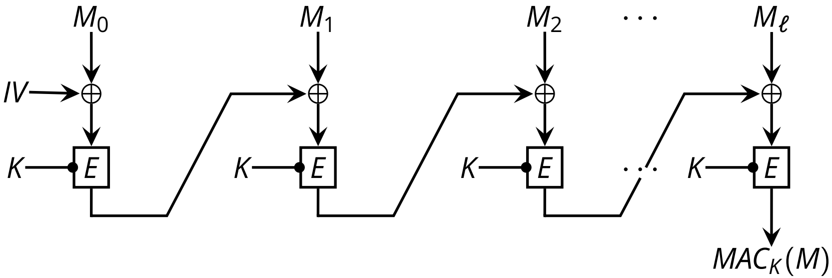

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CBC MAC: Generation

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CBC-MAC\$$(K, M)$

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  then return $\perp$

$M[1] \cdot M[2] \cdots M[\ell] \leftarrow M$

$IV \xleftarrow{\$} \{0,1\}^n$

$C \leftarrow IV$

for $i \leftarrow 1$ **to** ℓ

  do $C \leftarrow E_K(C \oplus M[i])$

return $\langle IV, C \rangle$

CBC MAC: Verification

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CBC-MAC\$-$\textsc{Verify}(K, IV, \sigma, M)$

if $|M| = 0 \lor |M| \neq 0 \mod n$
    then return $\perp$
$M[1] \cdot M[2] \cdots M[\ell] \leftarrow M$
$C \leftarrow IV$
for $i \leftarrow 1$ **to** ℓ
        do $C \leftarrow E_K(C \oplus M[i])$
if $C = \sigma$
    then return $\textsc{Accept}$
    else return $\textsc{Reject}$

Cryptographic Hash Functions

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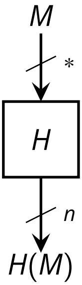

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  - e.g., SHA-1

Summary

- Basic ingredients: cryptographic primitives
  - secret-key (symmetric) cryptography (e.g., AES)
  - public-key (asymmetric) cryptography (e.g., RSA)
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- Recipes: cryptographic protocols
  - certificates (e.g., X.509)
  - secure transport (e.g., TLS, IPSec)
  - ...

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- Basic ingredients: cryptographic primitives
  - secret-key (symmetric) cryptography (e.g., AES)
  - public-key (asymmetric) cryptography (e.g., RSA)
  - cryptographic hash functions (e.g., SHA-1)
  - stream ciphers (e.g., RC4)

- Recipes: cryptographic protocols
  - certificates (e.g., X.509)
  - secure transport (e.g., TLS, IPSec)
  - ...

- Applications
  - electronic commerce
  - secure shell
  - secure electronic mail
  - virtual private networks
  - ...