Representing and Searching Sets of Strings

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Outline

- Radix search
- Ternary search tries

Sets of Strings

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- Several very important applications

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- Several very important applications

  E.g.,

  - dictionary (of words)
  - symbol table in a compiler
  - all kinds of key-based index
  - …

Symbol Table

- Operations

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- Operations
  - insert(Key)
  - delete(Key)
  - search(Key)
  - min()
  - max()

Dictionary

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Dictionary

- Operations
  - insert(Key)
  - search(Key)
- No *delete* operation
- Built once and searched many times

Binary Search

```
BINARYSEARCH(A, K)
first = 1
last = length(A)
while first ≤ last
    x = ⌈(first + last)/2⌉
    if A[x] == K
        return TRUE
    elseif first == last
        return FALSE
    elseif A[x] > K
        last = x − 1
    else fisrt = x + 1
return FALSE
```

```
TREE-SEARCH(T, K)
x = T.root
while x ≠ NIL and K ≠ x.key
    if K < x.key
        x = x.left
    else x = x.right
if x ≠ NIL
    return TRUE
else return FALSE
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- Complexity?

Binary Search

K ***is a string!***

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  - we must account for the ***complexity of string comparisons***

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- So, the complexity of **BinarySearch**(A, K) is $O(m \log n)$

What About a Hash Table

```
CHAINED-HASH-SEARCH(T, K)
    L = T[h(K)]
    return LIST-SEARCH(L, K)
```

```
HASH-SEARCH(T, K)
    for i = 1 to length(T)
        j = h(K, i)
        if T[j] == K
            return TRUE
        if T[j] == NIL
            return FALSE
    return FALSE
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  - and for the hash functions

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- Later in **BinarySearch**(A, K)
  - $A[x]$ gets closer and closer to K
  - so, **StringEquals**$(A[x], K)$ is likely to iterate for nearly m steps
  - problem is, **StringEquals**$(A[x], K)$ is likely to go through the same prefix of K many times
- So, since $m = \Theta(\log N)$, and **BinarySearch**(A, K) uses $\Theta(\log N)$ comparisons each one running in $O(m)$:

$$T(N, m) = O(\log^2 N)$$

A New Data Structure

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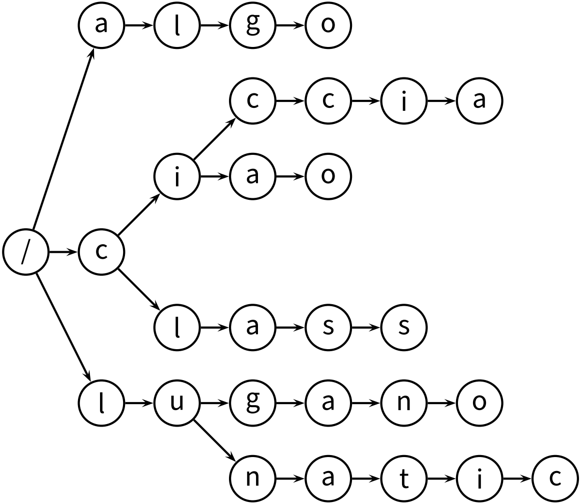

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- Keys with the same prefix share a branch of the tree
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- *Question:* how do we represent nodes and links?
  - one way would be to hold $|\Sigma|$ links
  - one for each character of the given alphabet Σ

Radix Trie

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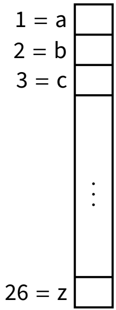

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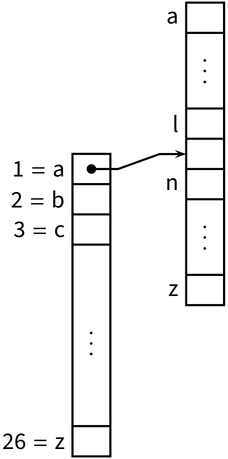

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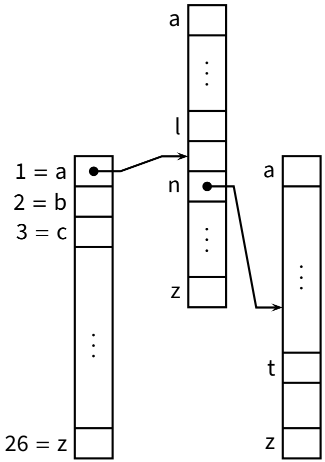

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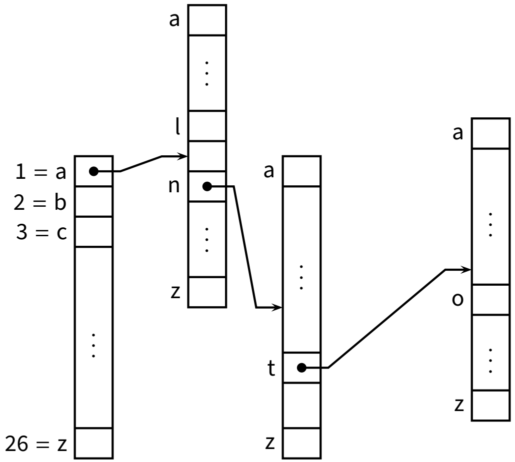

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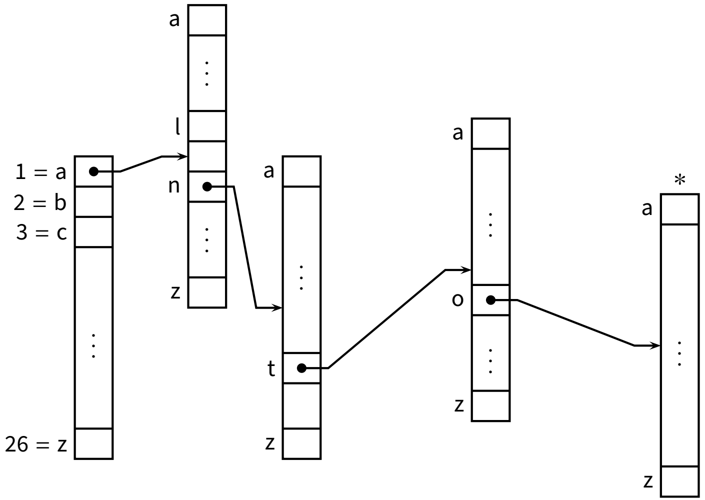

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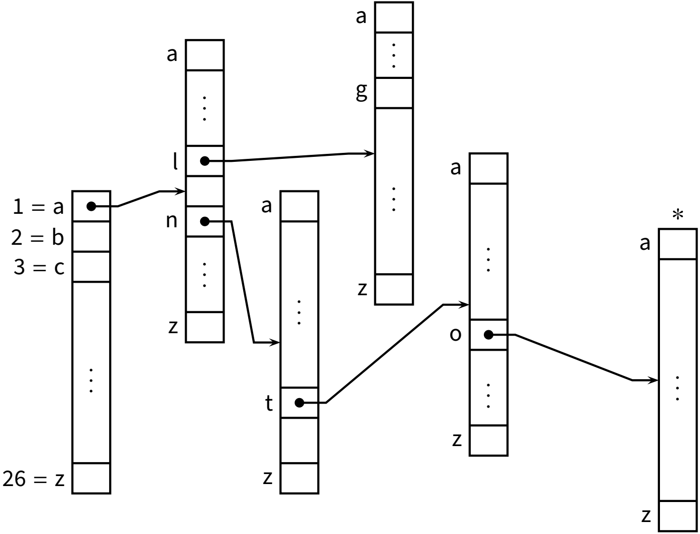

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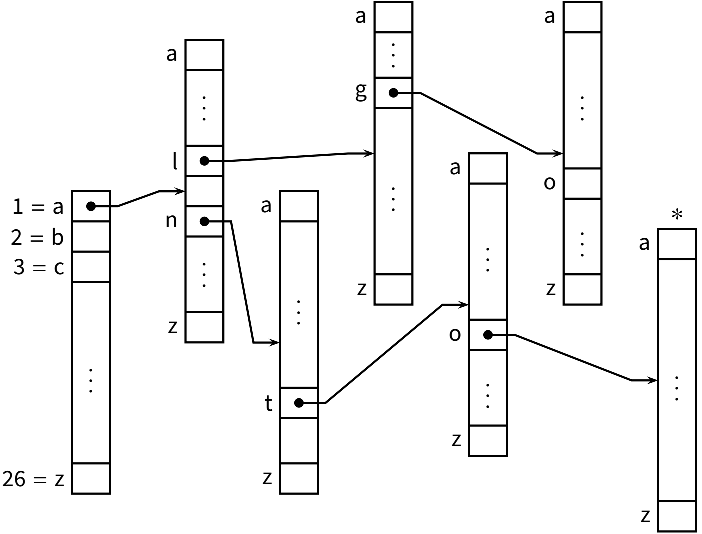

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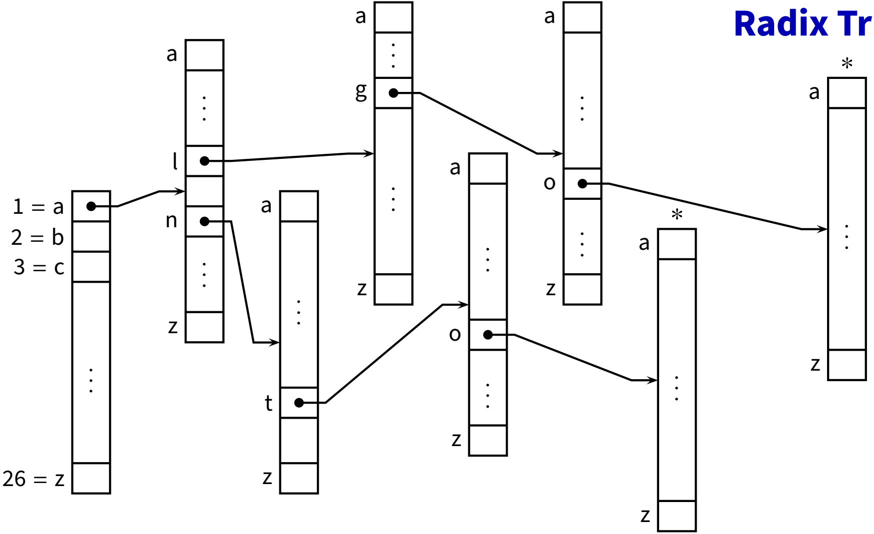

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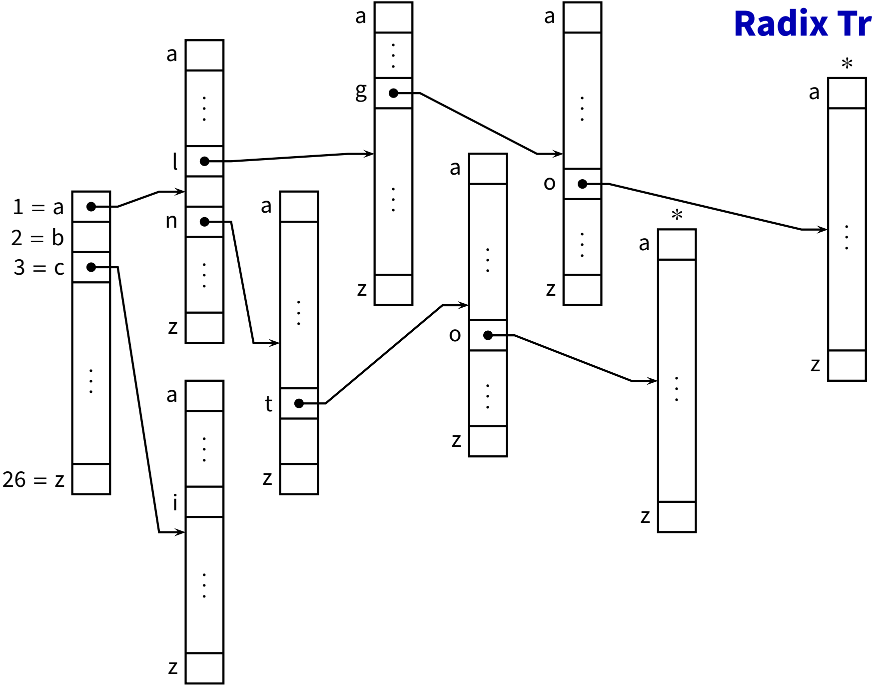

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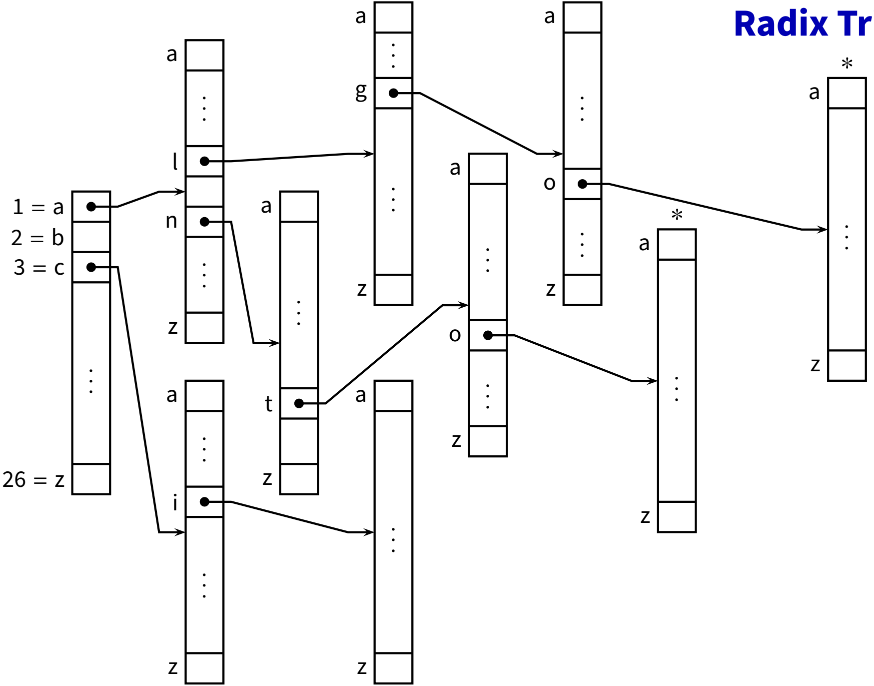

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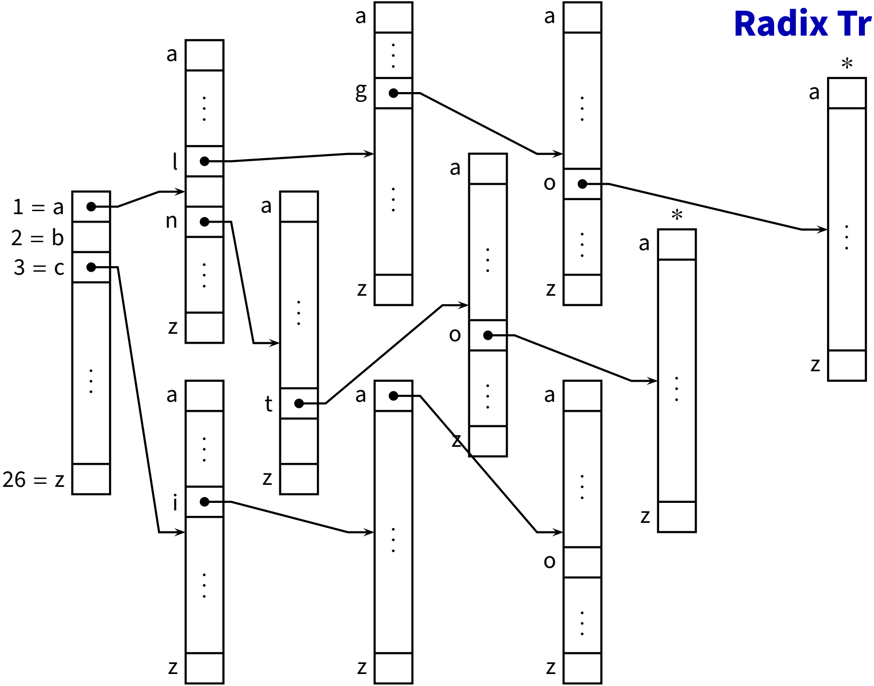

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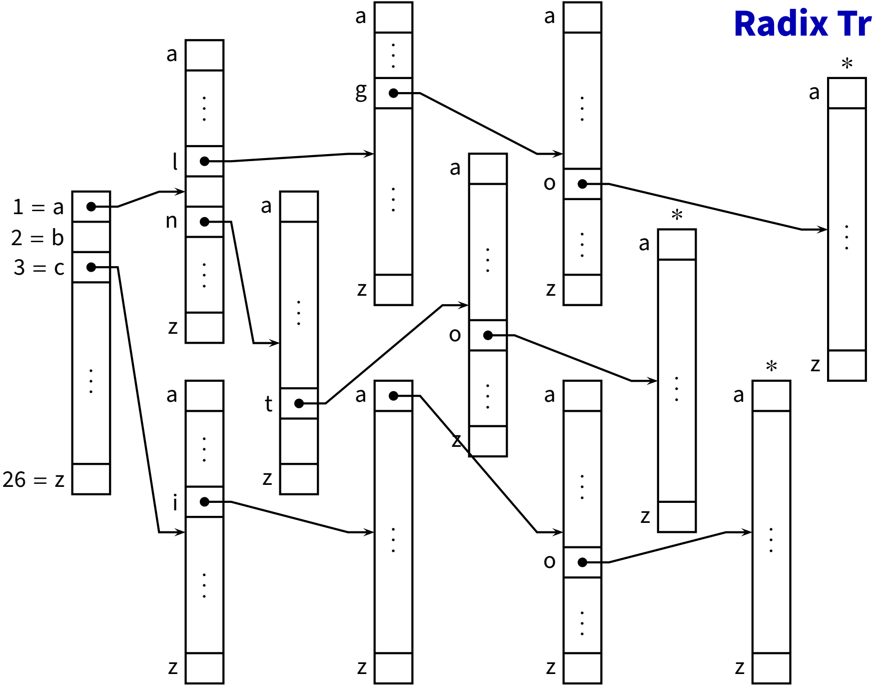

Radix Search

- Every element x has an array of links $x.links$
  - e.g., in “radix-256,” an element represents a *byte* in a string (of bytes)
- Every element x has a $x.value$ that is TRUE if that prefix corresponds to a string in the dictionary
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```
RADIXSEARCH(Root, K)
n = Root
for i = 1 to length(K)
    if n.links[K[i]] == NIL
        return FALSE
    else n = n.links[K[i]]
return n.value
```

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  - a better characterization (*Exercise:* figure this out!):

$$S(N, m) = \Theta\left(|\Sigma| \left[\frac{N-1}{|\Sigma|-1} + N\left(m - \frac{\log N}{\log |\Sigma|}\right)\right]\right)$$

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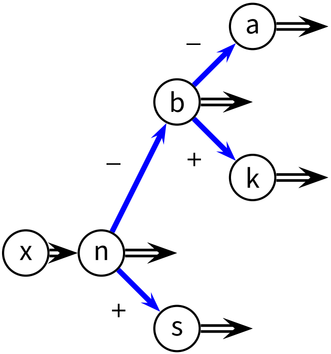

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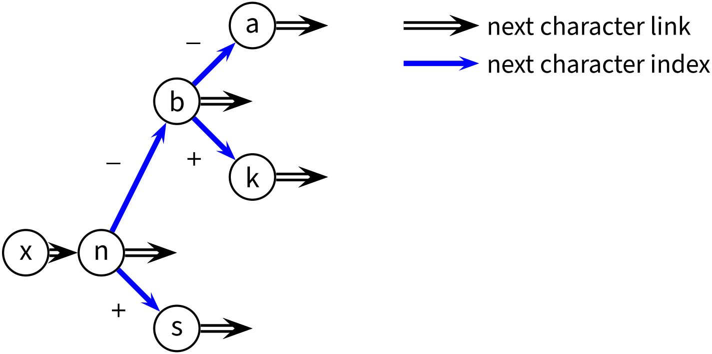

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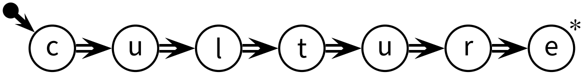
- A node *n* has three links
  - *n.lower* links to a node representing a "lower" character at the same position
  - *n.higher* links to a node representing a "higher" character at the same position
  - *n.equal* links to a node representing a character in the next position

Example

“culture”

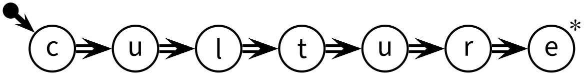
Example

"culture"



Example

“lugano”



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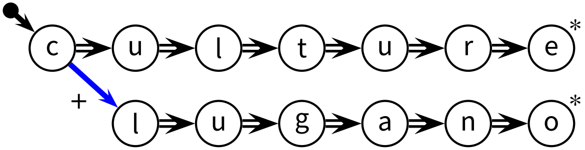

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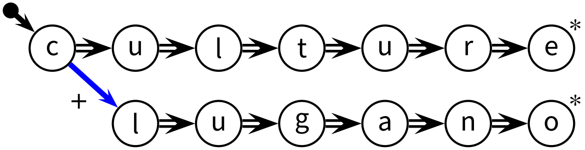

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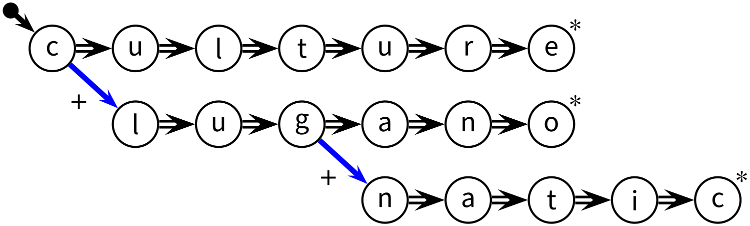

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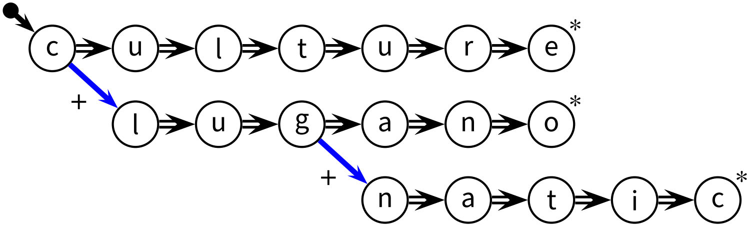

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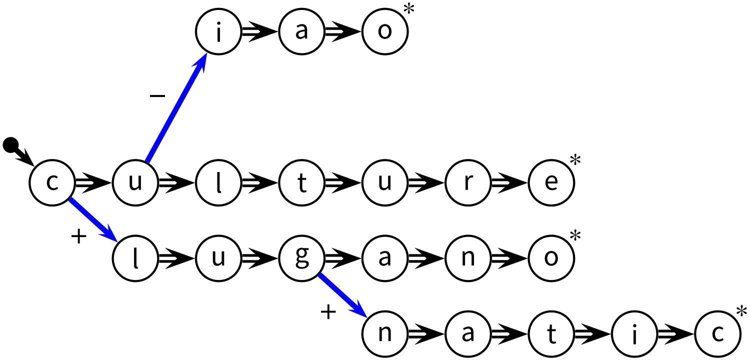

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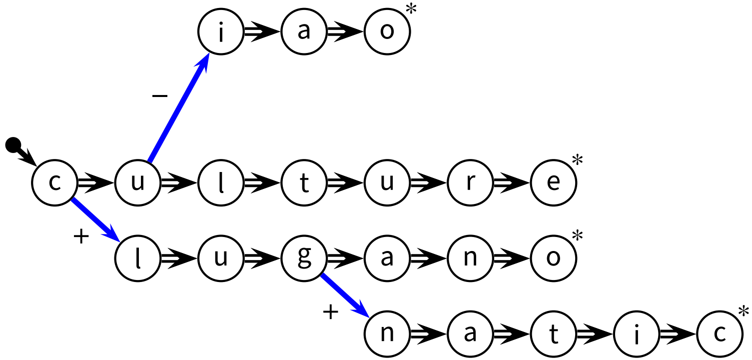

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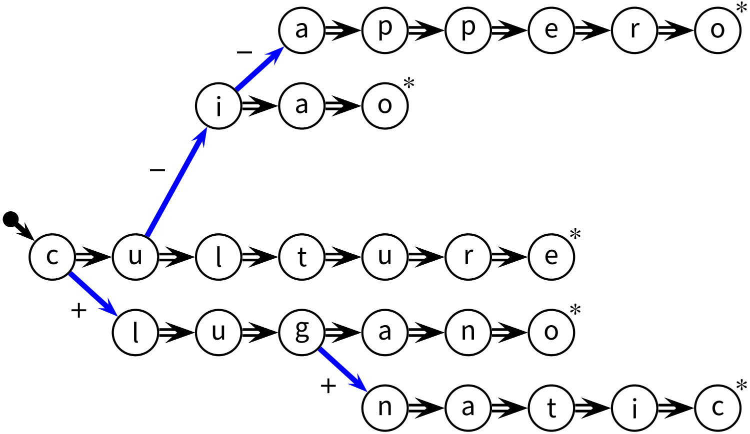

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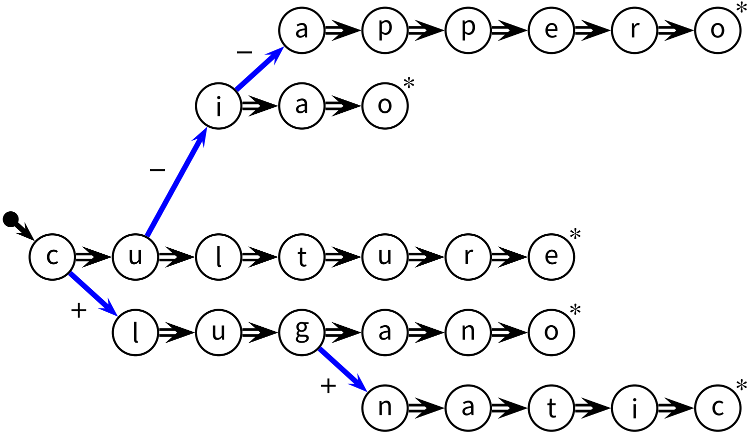

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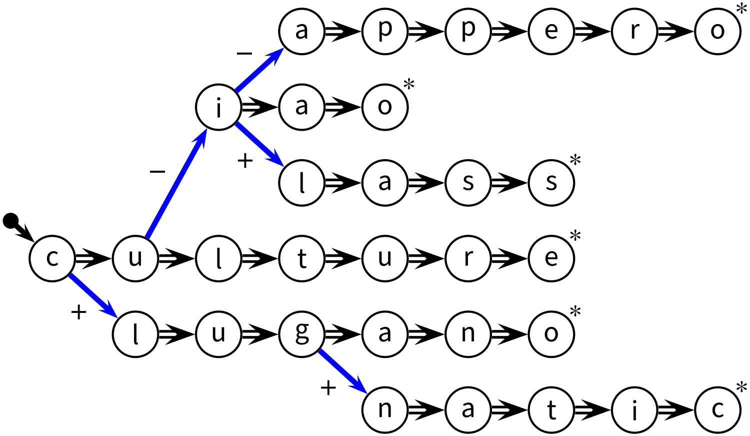

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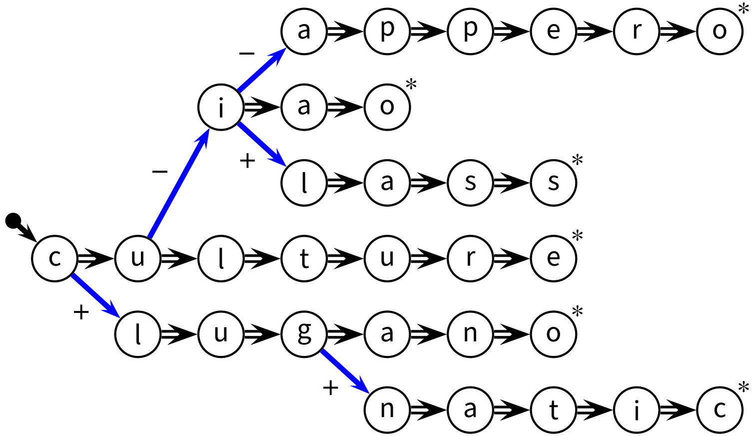

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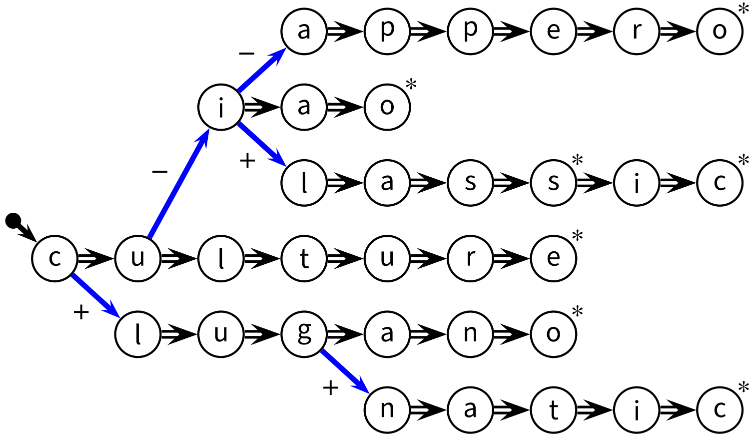

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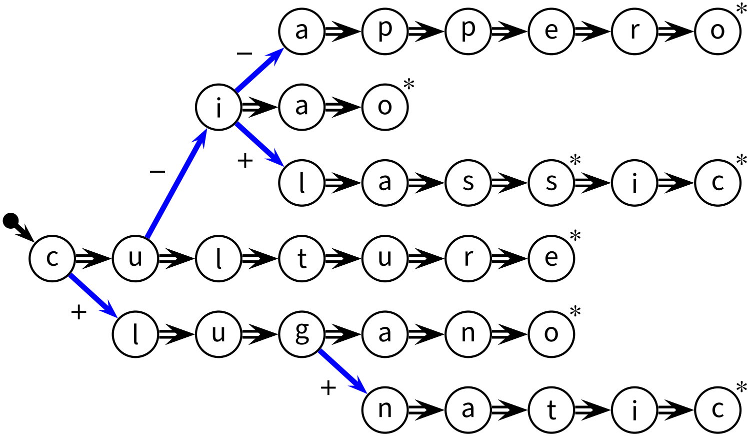

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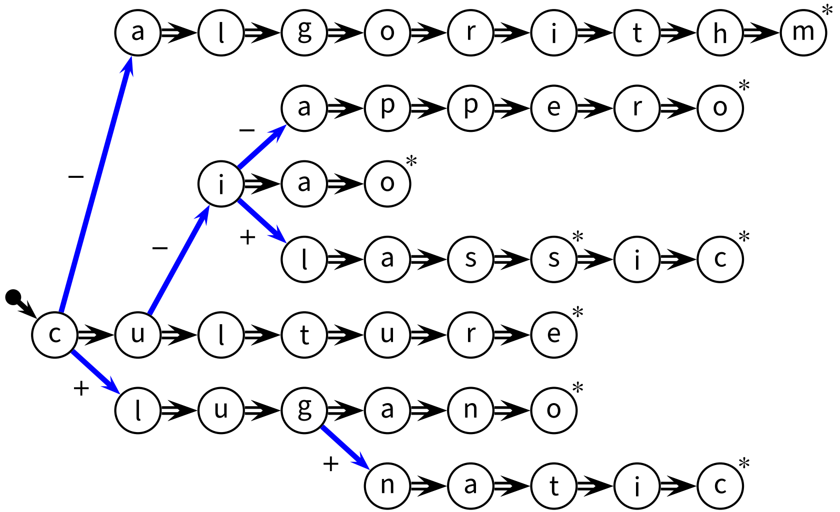

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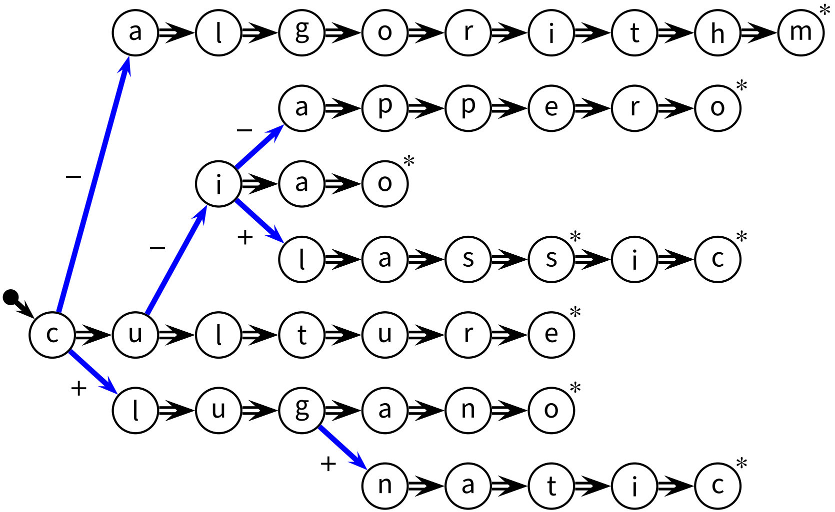
“algo”



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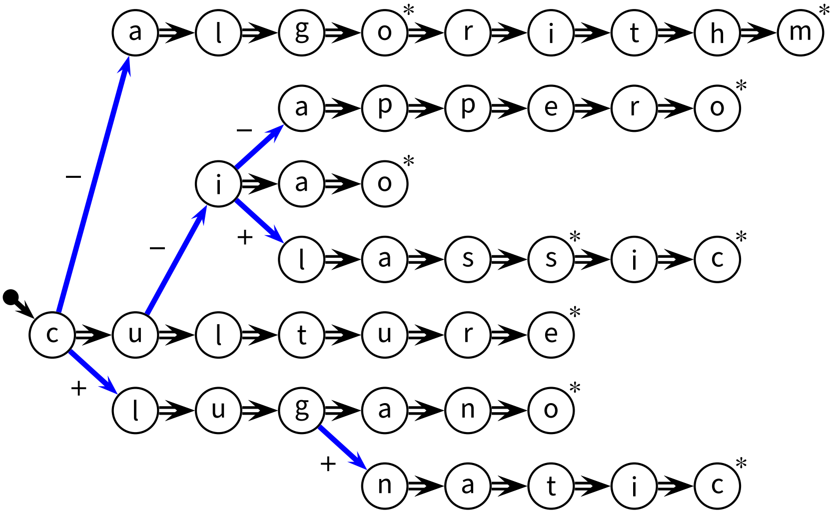

TST Search

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```
TSTSearch(T, K)
for i = 1 to |K|
    if i > 1
        T = T.equal
    while T ≠ NIL and K[i] ≠ T.character
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    if T == NIL
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- Complexity?

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- Complexity? Non-trivial…

TST Insertion

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- Recursion starts with $root = \textbf{TSTInsert}(root, K, 1)$

```
TSTInsert(T, K, i)
if T == NIL
        T = NewNode(K[i])
if K[i] < T.character
        T.lower = TSTInsert(T.lower, K, i)
elseif K[i] > T.character
        T.higher = TSTInsert(T.higher, K, i)
elseif K[i] == T.character
        if i < |K|
                T.equal = TSTInsert(T.equal, K, i + 1)
        else T.value = TRUE
return T
```