String Matching Algorithms

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Outline

- Problem definition
- Naïve algorithm
- Knuth-Morris-Pratt algorithm
- Boyer-Moore algorithm

Problem

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  mi ritrovai per una selva oscura
  che la dritta via era smarrita…

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- A more challenging example: How many times does the string “110011” appear in the following text

  001111010101101001100011010111101101 0111
  011011100100101010101111011110110000101
  1011000010111111011110011000011111000100
  100101001011101110101101111010100110010 1
  001011100100001111111001001101110101101 0
  0110011011101001010010101000010100111110

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  Nel mezzo del cammin di nostra vita
  mi ritrovai per una selva oscura
  che la dritta via era smarrita…

  Find the string "trova"

- A more challenging example: How many times does the string "110011" appear in the following text

  0011110101011010011000110101111011010111
  0110111001001010101011111011110110000101
  10110000010111111011110011000011111000100
  1001010010111011101011011110101001100101
  0010111001000011111110010011011101011010
  011001101110100101001010100001010011110

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- Both T and P can be modeled as arrays
  - $T[1 \ldots n]$ and $P[1 \ldots m]$

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- Both T and P can be modeled as arrays
  - $T[1 \dots n]$ and $P[1 \dots m]$

- Pattern P occurs with ***shift*** s in T iff
  - $0 \leq s \leq n - m$
  - $T[s + i] = P[i]$ for all positions $1 \leq i \leq m$

Example

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$n = 14$

T	a	b	c	a	a	b	a	a	b	a	b	a	c	a

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$n = 14$

T														
	a	b	c	a	a	b	a	a	b	a	b	a	c	a

$m = 3$

P			
	a	b	a

- Result

- Problem: find all s such that
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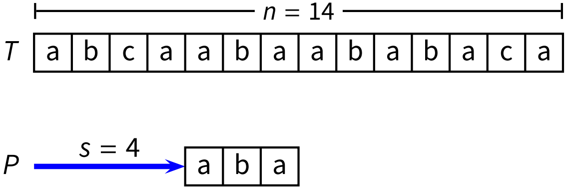

- Result
  $s = 4$

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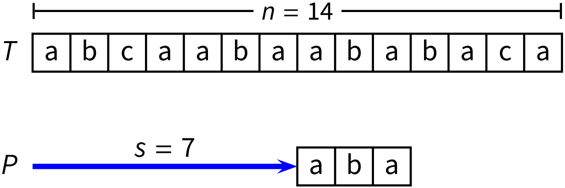

- Result

  $s = 4$

  $s = 7$

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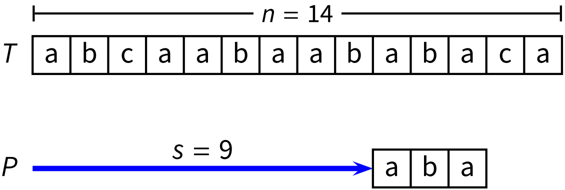

- Result

$s = 4$
$s = 7$
$s = 9$

Naïve Algorithm

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- For each position s in $0 \ldots n - m$, see if $T[s+i] = P[i]$ for all $1 \leq i \leq m$

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```
NAIVE-STRING-MATCHING(T, P)
n = length(T)
m = length(P)
for s = 0 to n − m
    if SUBSTRING-AT(T, P, s)
        OUTPUT(s)
```

Naïve Algorithm

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  for s = 0 to n − m
      if SUBSTRING-AT(T, P, s)
          OUTPUT(s)
```

```
SUBSTRING-AT(T, P, s)
  for i = 1 to length(P)
      if T[s + i] ≠ P[i]
          return FALSE
  return TRUE
```

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- Complexity of **Naive-String-Match** is $O((n-m+1)m)$

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$$T = \mathrm{a}^n, \quad P = \mathrm{a}^m$$

i.e.,

$$T = \overbrace{\mathrm{aa}\cdots\mathrm{a}}^{n}, \quad P = \overbrace{\mathrm{aa}\cdots\mathrm{a}}^{m}$$

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i.e.,

$$T = \overbrace{\mathrm{aa}\cdots\mathrm{a}}^{n}, \quad P = \overbrace{\mathrm{aa}\cdots\mathrm{a}}^{m}$$

So, $(n - m + 1)m$ is a tight bound, so the (worst-case) complexity of **Naive-String-Match** is

$$\boxed{\Theta((n - m + 1)m)}$$

Improvement Strategy

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- Observation

T	a	b	c	a	a	b	a	a	b	a	b	a	c	a

P	a	b	a

- Observation

T	a	b	c	a	a	b	a	a	b	a	b	a	c	a

$=$

P	a	b	a

- Observation

T	a	b	c	a	a	b	a	a	b	a	b	a	c	a
	=	=												
P	a	b	a											

- Observation

T	a	b	c	a	a	b	a	a	b	a	b	a	c	a
	$=$	$=$	$\neq$											
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- What now?

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- What now?
  - the naïve algorithm ***goes back to the second position in*** T and ***starts from the beginning of*** P

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	$=$	$=$	$\neq$											
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- What now?
  - the naïve algorithm ***goes back to the second position in*** T and ***starts from the beginning of*** P
  - can’t we simply move along through T?
  - why?

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```
WRONG-STRING-MATCHING(T, P)
n = length(T)
m = length(P)
q = 0            // number of characters matched in P
s = 1
while s ≤ n
    s = s + 1
    if T[s] == P[q + 1]
        q = q + 1
        if q == m
            OUTPUT(s − m)
            q = 0
    else q = 0
```

Improvement Strategy (3)

- Example run of **Wrong-String-Matching**

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P	a	g	o

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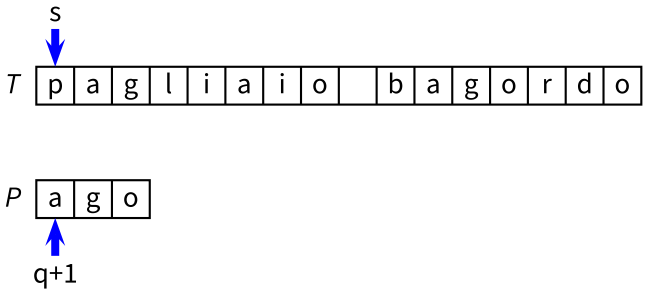

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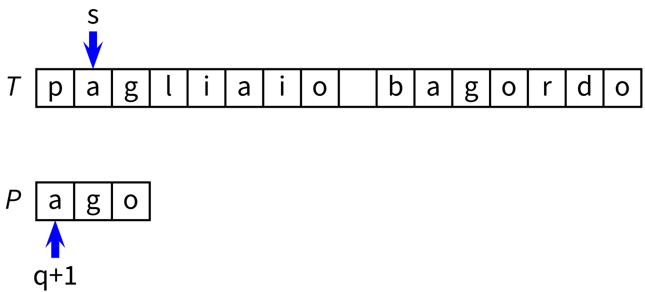

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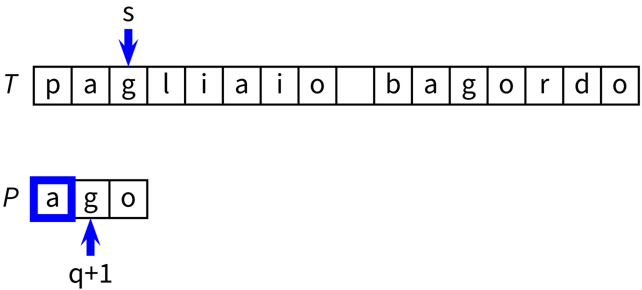

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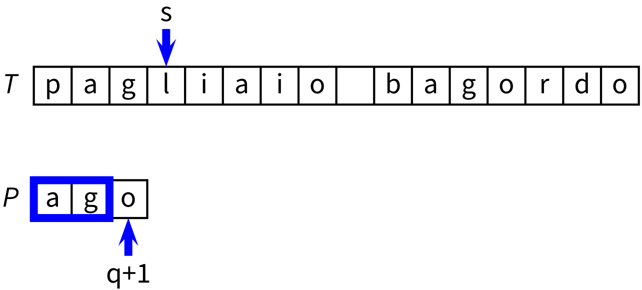

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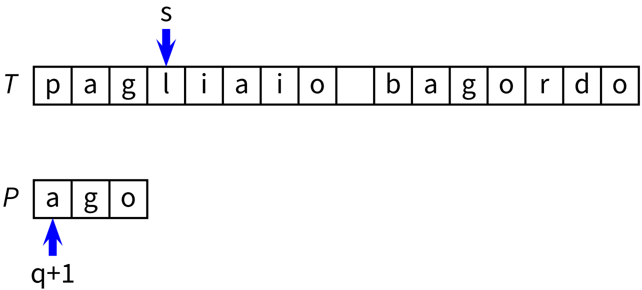

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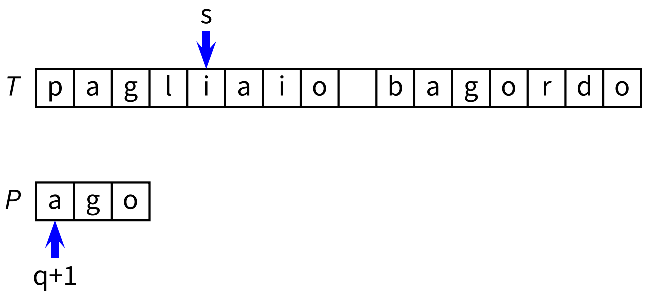

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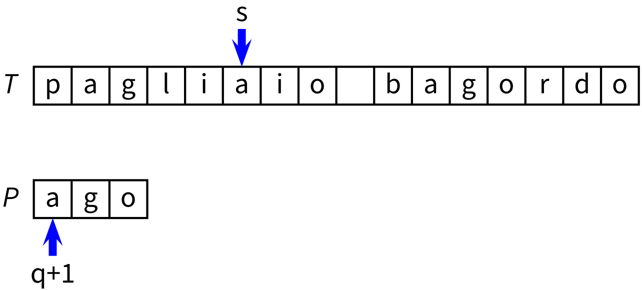

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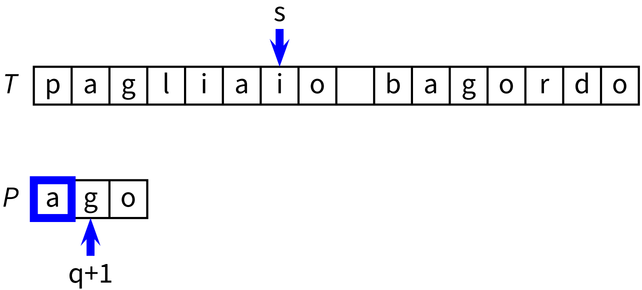

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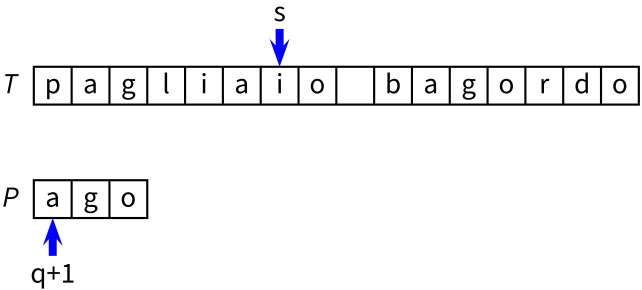

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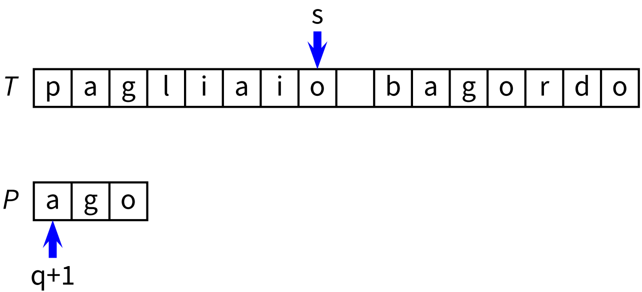

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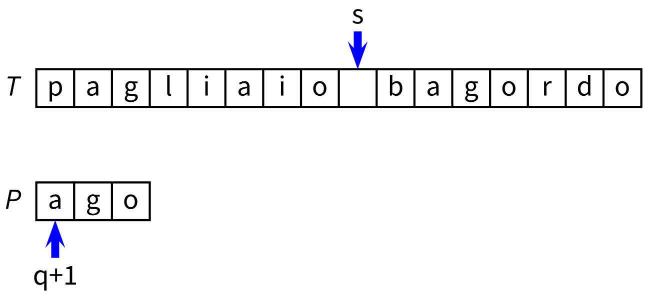

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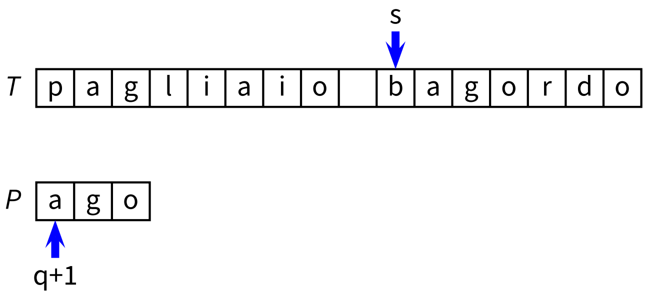

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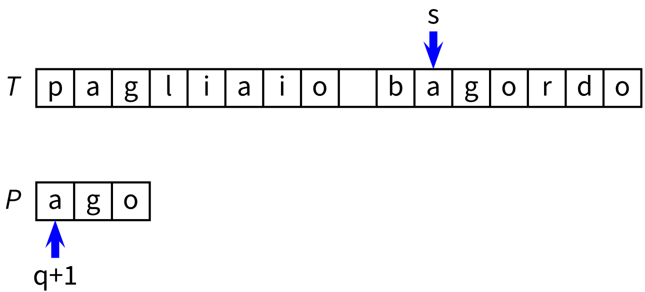

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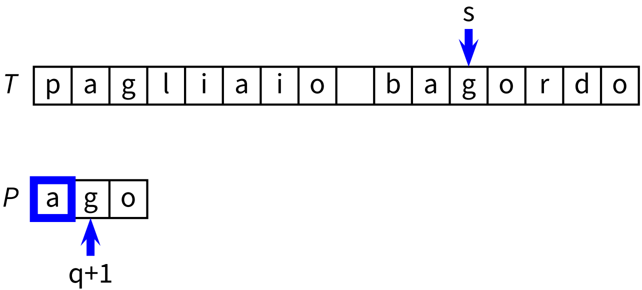

Improvement Strategy (3)

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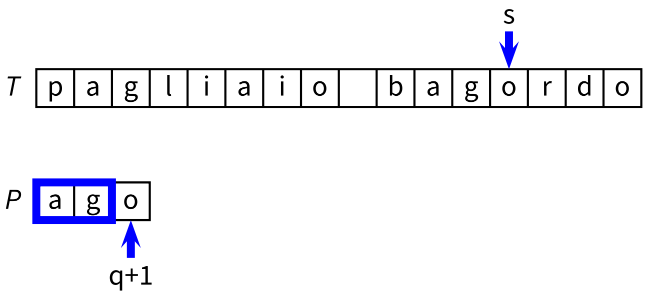

Improvement Strategy (3)

- Example run of **WRONG-STRING-MATCHING**

s

T	p	a	g	l	i	a	i	o		b	a	g	o	r	d	o

P	a	g	o

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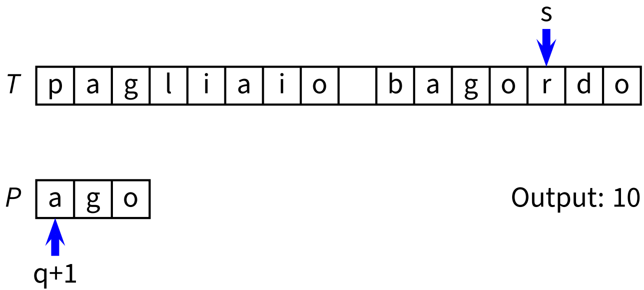

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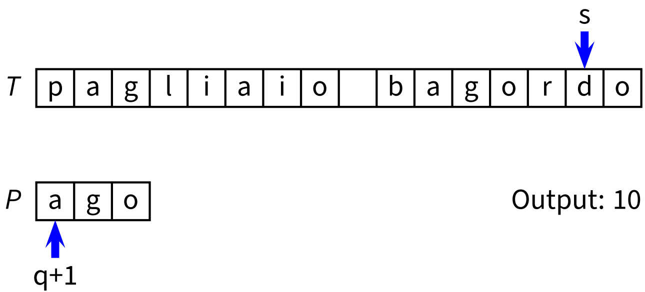

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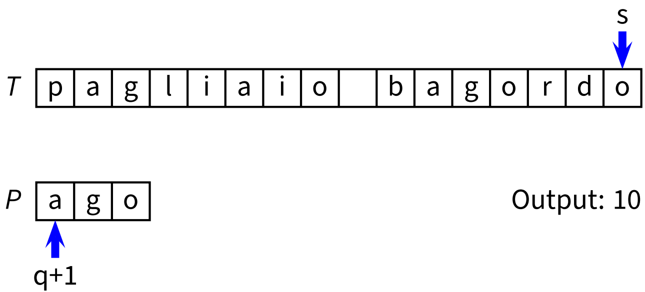

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T																
	p	a	g	l	i	a	i	o		b	a	g	o	r	d	o

P			
	a	g	o

Output: 10

- Done. Perfect!

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P	a	g	o

Output: 10

- Done. Perfect!
- Complexity: $\Theta(n)$

Improvement Strategy (4)

- What is wrong with **Wrong-String-Matching**?

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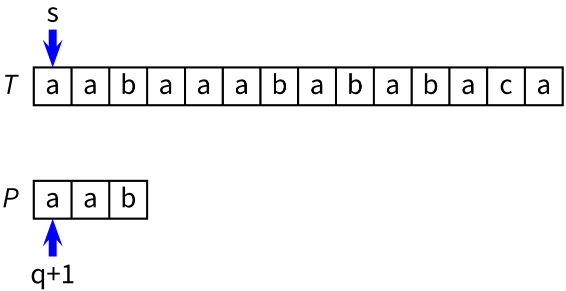

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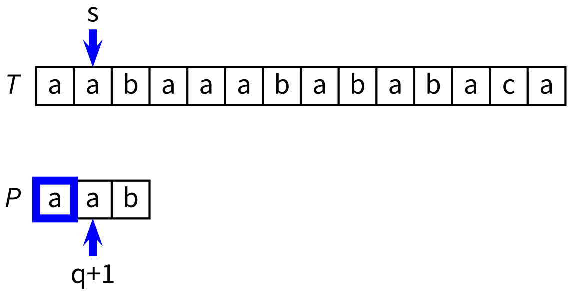

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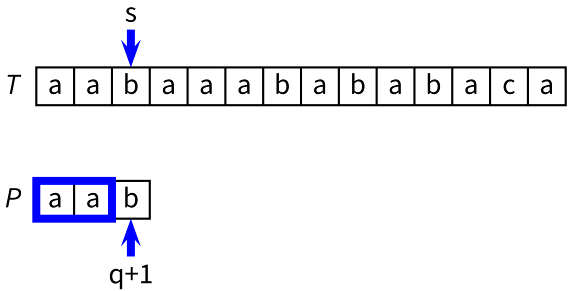

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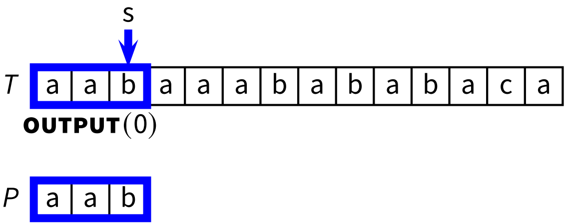

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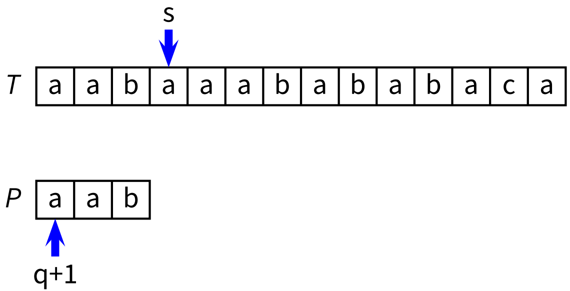

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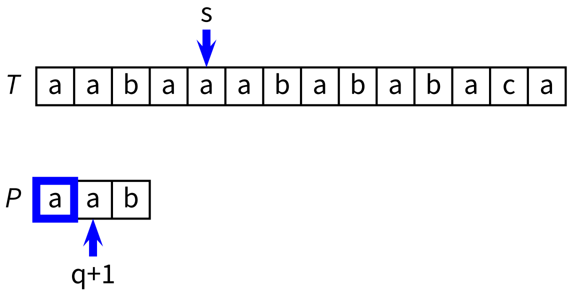

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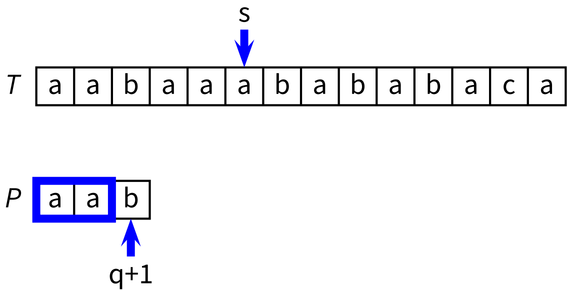

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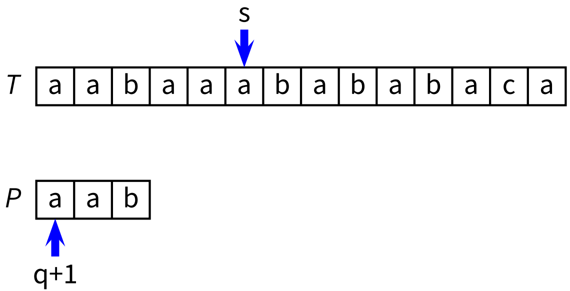

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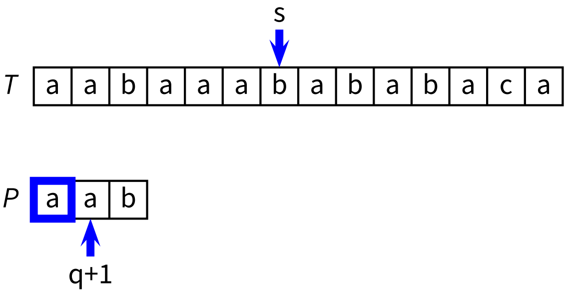

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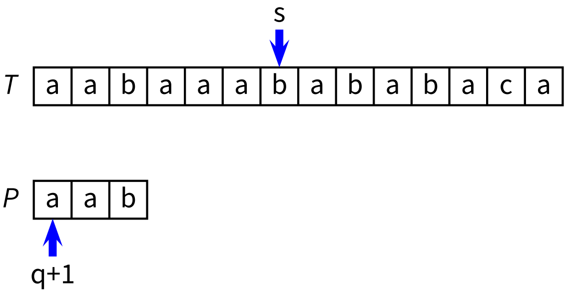

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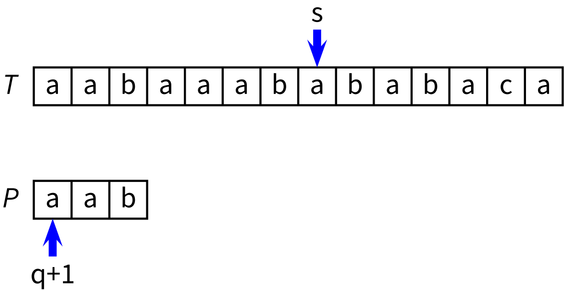

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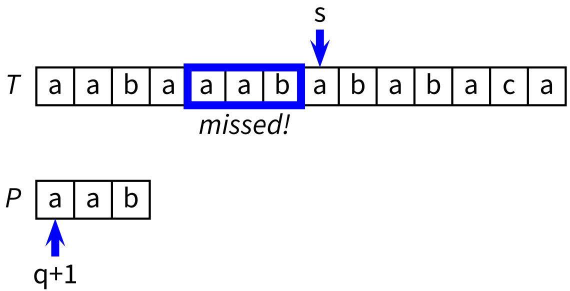

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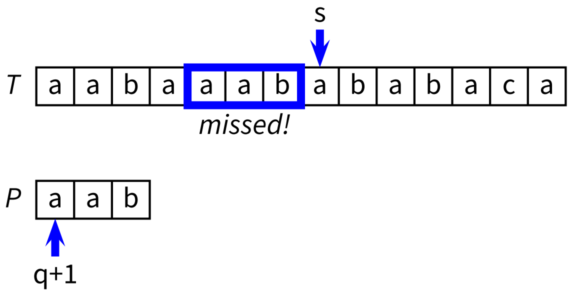

- So **WRONG-STRING-MATCHING** doesn't work, but it tells us something useful

Improvement Strategy (5)

- Where did **Wrong-String-Matching** go wrong?

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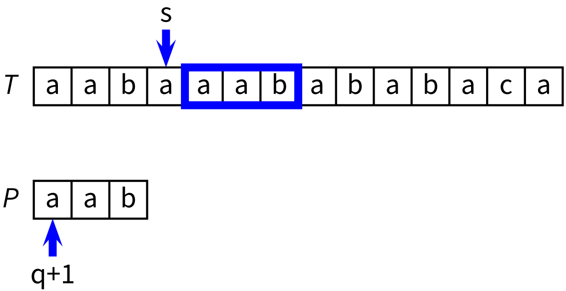
T	a	a	b	a	a	a	b	a	b	a	b	a	c	a

P	a	a	b

- Where did **WRONG-STRING-MATCHING** go wrong?



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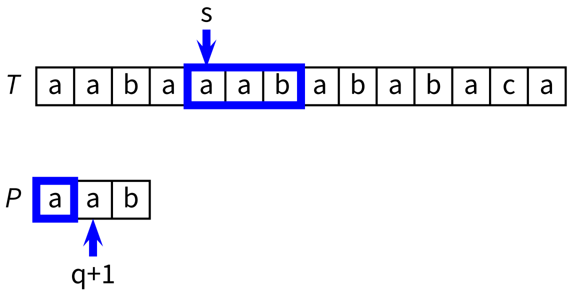

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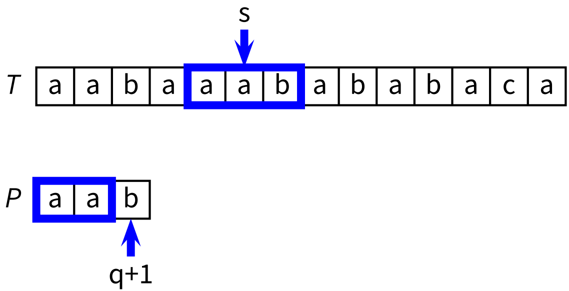

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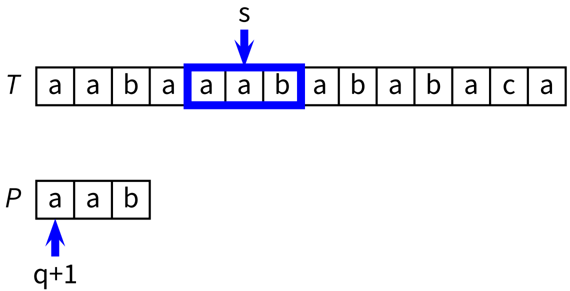

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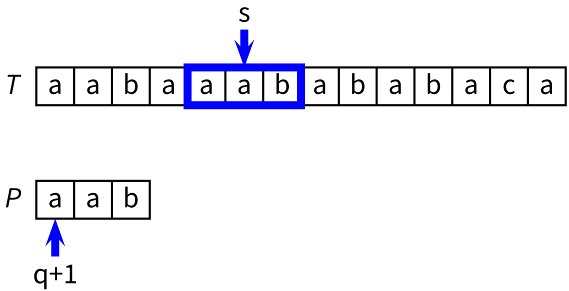

- Wrong: by going all the way back to $q = 0$ we throw away a good prefix of P that we already matched

Improvement Strategy (6)

- Another example

Improvement Strategy (6)

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T	a	b	a	b	a	b	a	c	b	a	c	b	c	a

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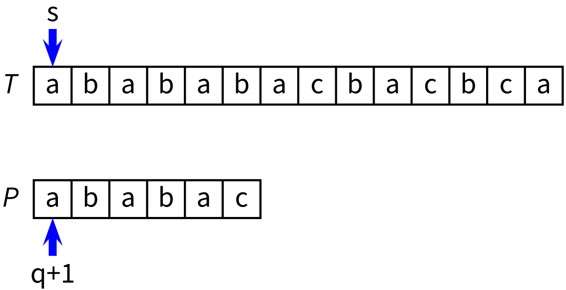

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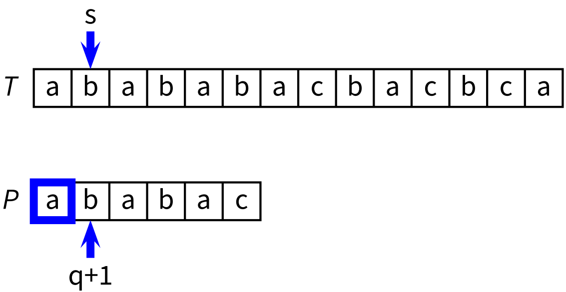

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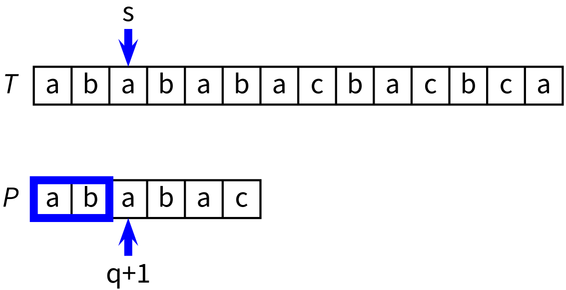

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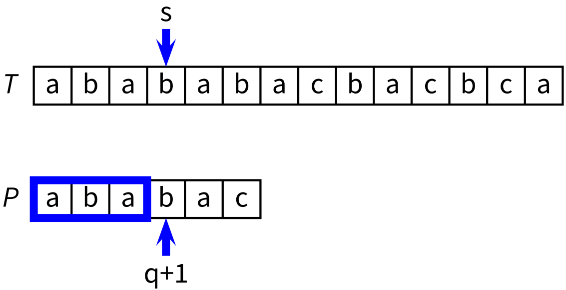

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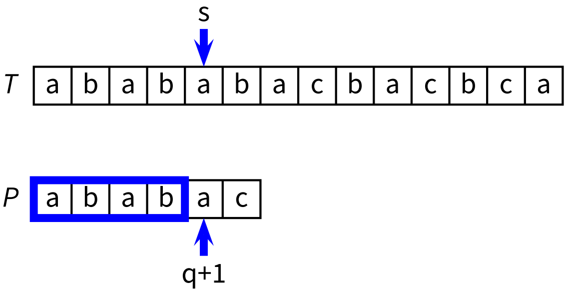

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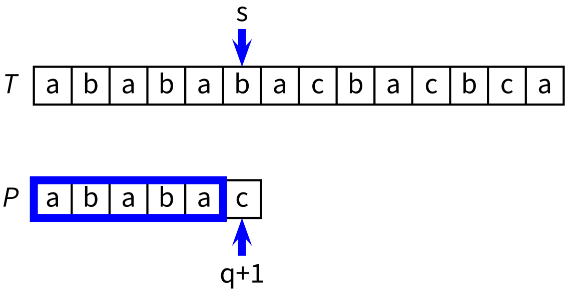

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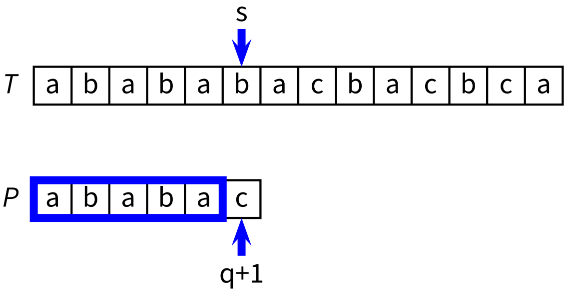

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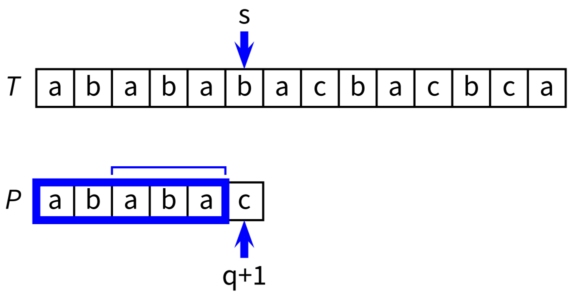

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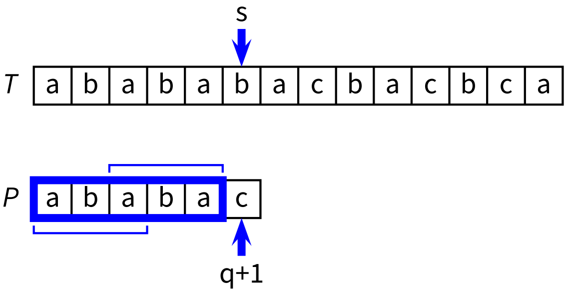

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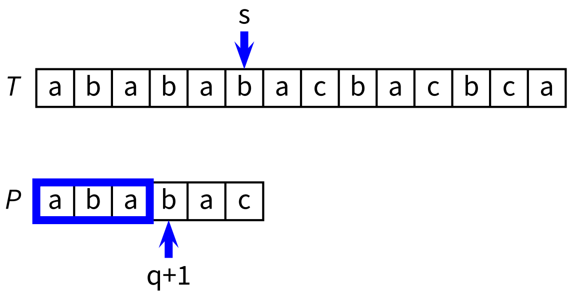

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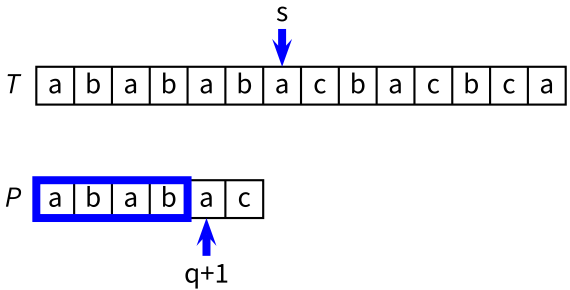

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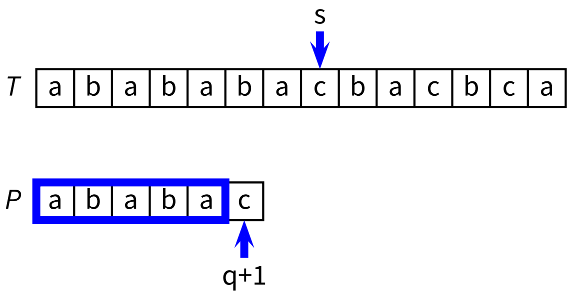

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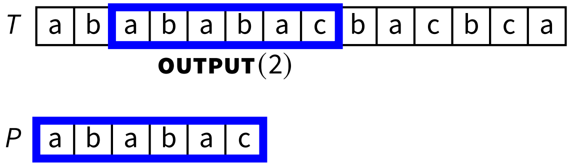

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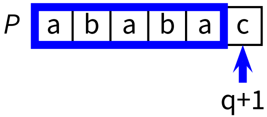

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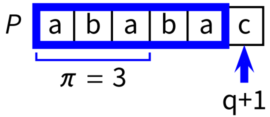

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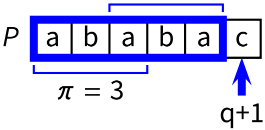

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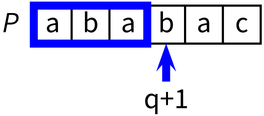

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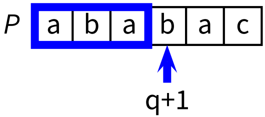

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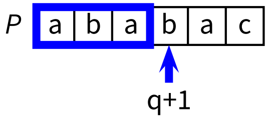

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- In essence, this is the Knuth-Morris-Pratt algorithm

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- Example

P	a	b	a	b	a	c
π	0	0	1	2	3	0

The Knuth-Morris-Pratt Algorithm

```
KMP-STRING-MATCHING(T, P)
n = length(T)
m = length(P)
π = PREFIX-FUNCTION(P)
q = 0                          // number of character matched
for i = 1 to n                 // scan the text left-to-right
    while q > 0 and P[q + 1] ≠ T[i]
        q = π[q]               // no match: go back using π
    if P[q + 1] == T[i]
        q = q + 1
    if q == m
        OUTPUT(i − m)
        q = π[q]               // go back for the next match
```

Prefix Function Algorithm

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- In fact, **Prefix-Function** is remarkably similar to **KMP-String-Matching**

Prefix-Function(P)

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m = length(P)
π[1] = 0
k = 0
for q = 2 to m
    while k > 0 and P[k + 1] ≠ P[q]
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Prefix Function at Work

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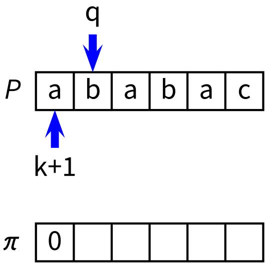

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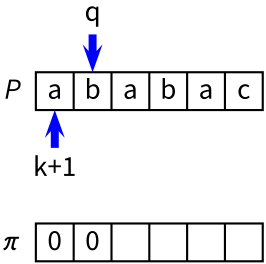

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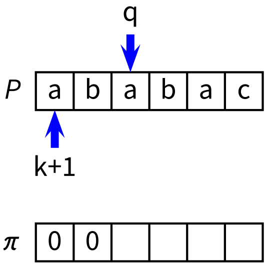

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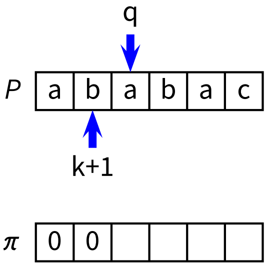

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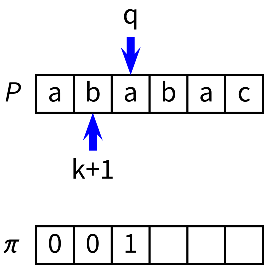

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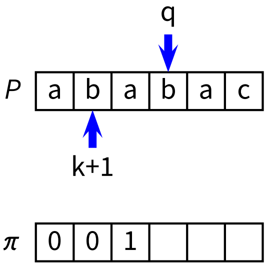

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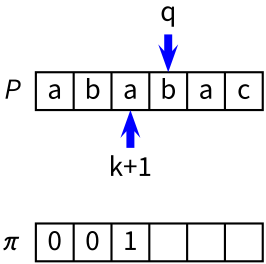

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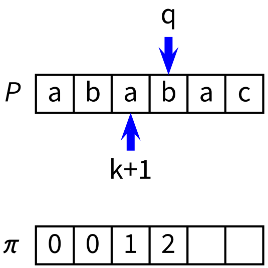

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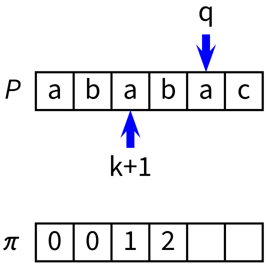

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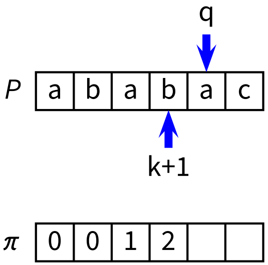

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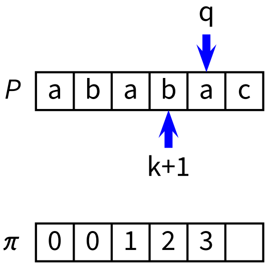

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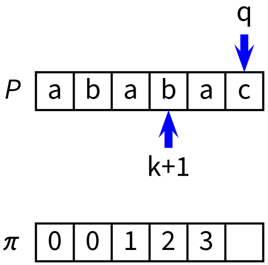

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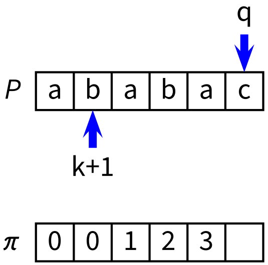

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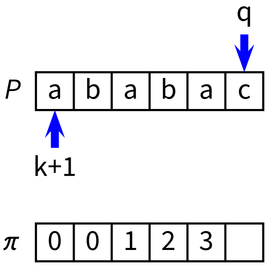

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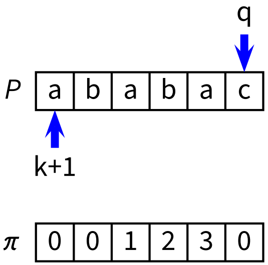

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- $O(m)$ for the pre-processing of the pattern
- The complexity analysis is non-trivial
- Can we do better?

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  - KMP will *always* go through *at least n* character comparisons
  - it fixes our “wrong” algorithm in the case of *periodic* patterns and texts

- Perhaps there’s another algorithm that works better on the average case
  - e.g., in the absence of periodic patterns

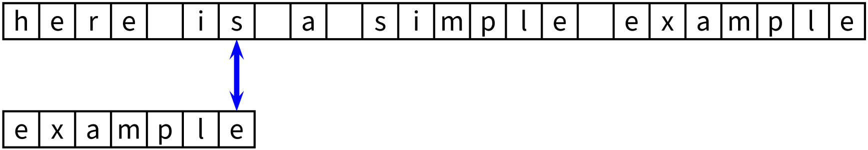
A New Strategy

h	e	r	e		i	s		a		s	i	m	p	l	e		e	x	a	m	p	l	e

A New Strategy

h	e	r	e		i	s		a		s	i	m	p	l	e		e	x	a	m	p	l	e

e	x	a	m	p	l	e



- We match the pattern right-to-left

A New Strategy

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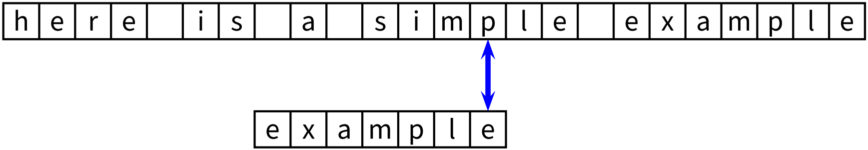

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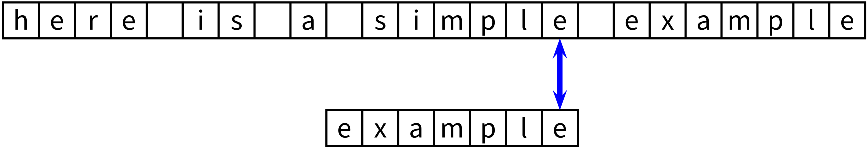

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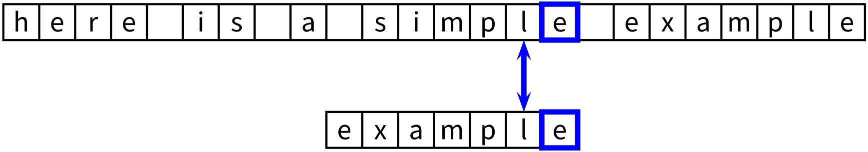

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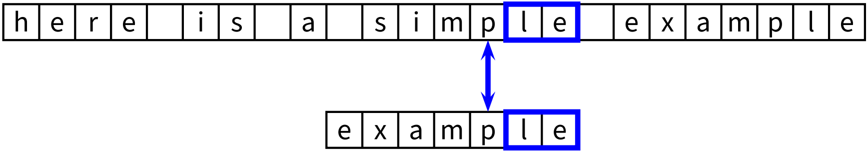

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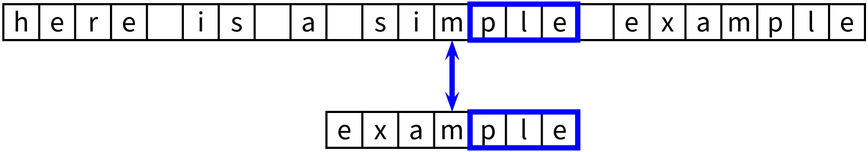

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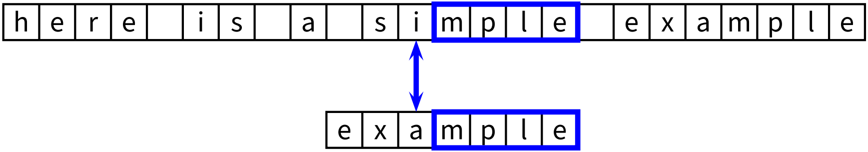

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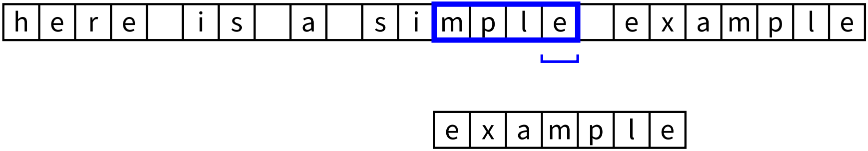

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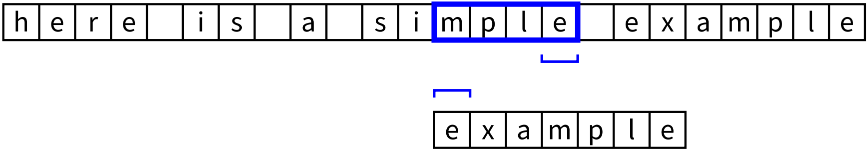

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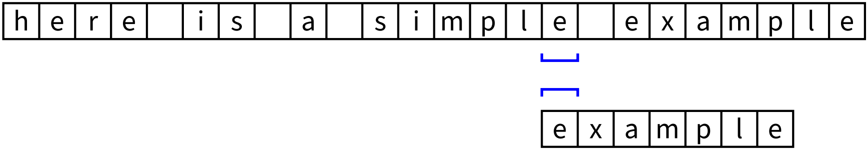

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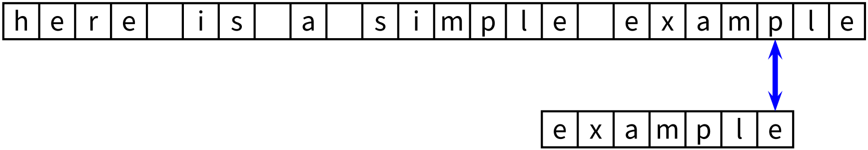

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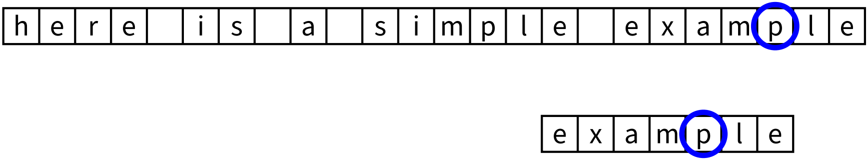

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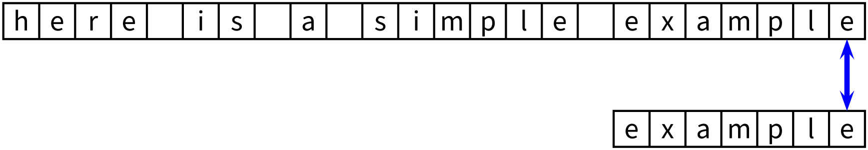

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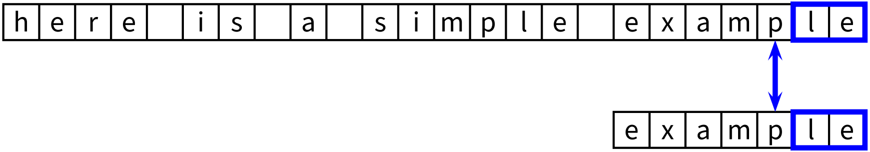

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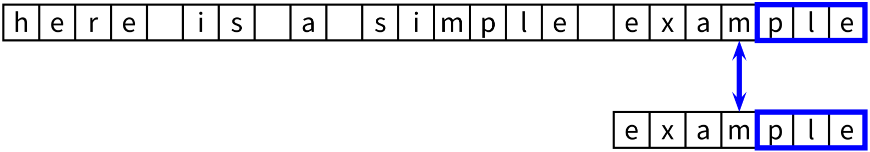

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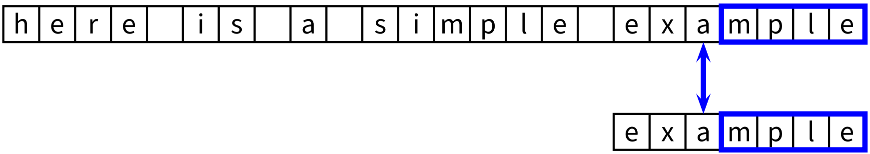

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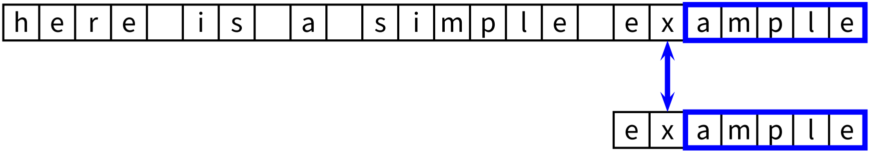

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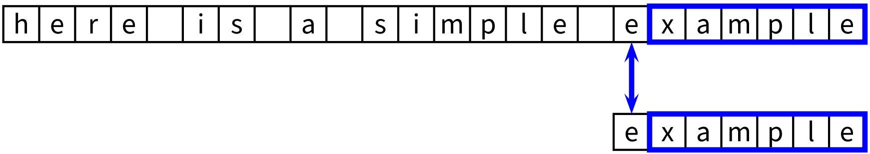

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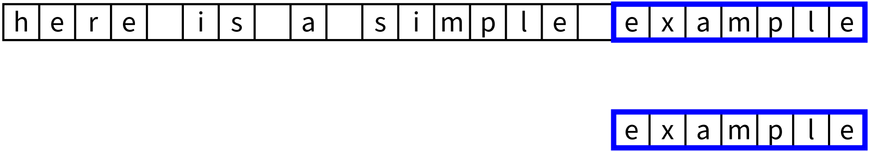

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- In essence, this is the Boyer-Moore algorithm

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- The search phase can be as low as $O(n/m)$ in common cases
- In practice, Boyer-Moore is the fastest string-matching algorithm for most applications