More on Sorting:
Quick Sort and Heap Sort

Antonio Carzaniga

Faculty of Informatics
Università della Svizzera italiana

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Outline

- Another divide-and-conquer sorting algorithm
- The *heap*
- Heap sort

Sorting Algorithms Seen So Far

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Algorithm		**Complexity**		**In place?**
	worst	*average*	*best*	

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- *Basic step:* partition A in three parts based on a *chosen value* $v \in A$
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- *Can we partition A* ***in place****?*

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```
QuickSort(A, begin, end)
if begin < end
        q = Partition(A, begin, end)
        QuickSort(A, begin, q − 1)
        QuickSort(A, q + 1, end)
```

Partition

- Start with $q = 1$
  - i.e., *assume all elements are greater than the pivot*

- Scan the array left-to-right, starting at position 2

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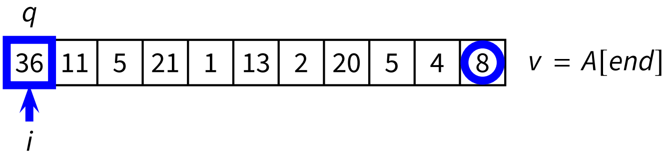

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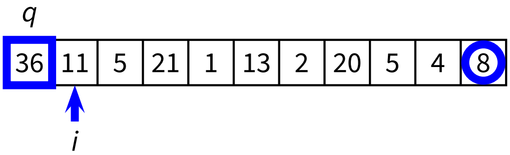

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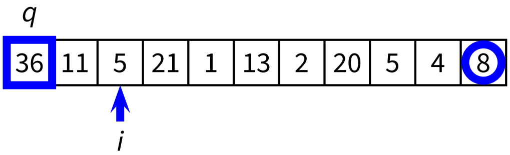

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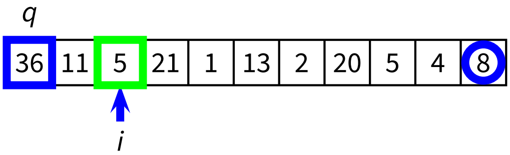

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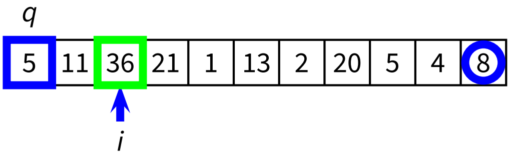

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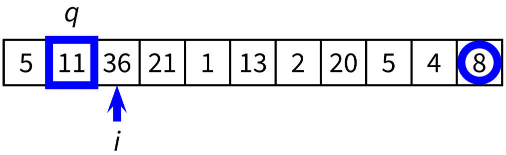

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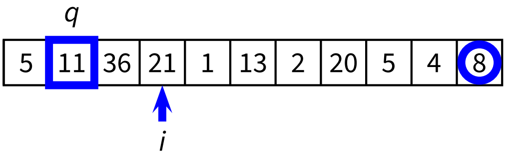

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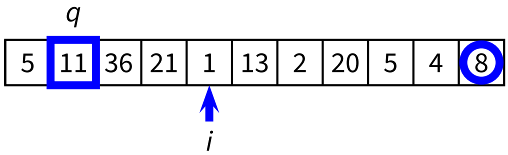

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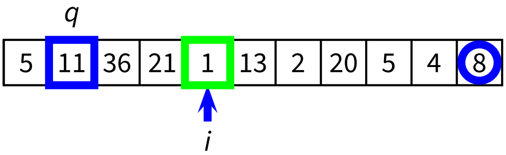

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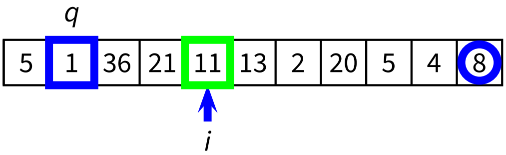

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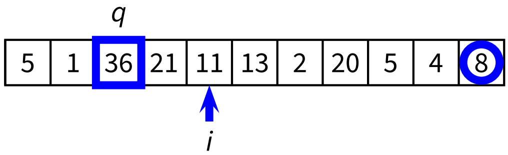

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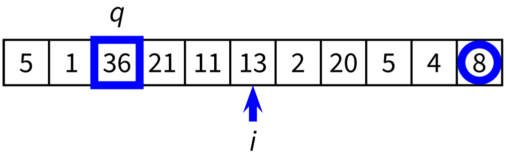

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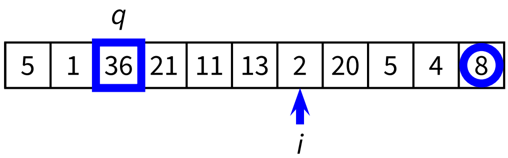

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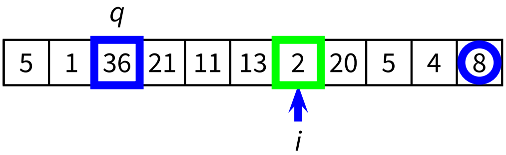

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		q								
5	1	$\boxed{2}$	21	11	13	$\boxed{36}$	20	5	4	8
						$\uparrow$ i				

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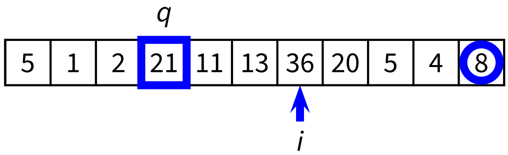

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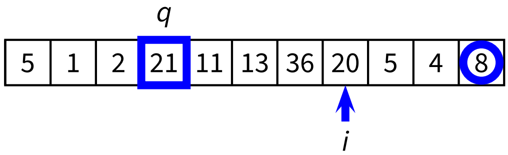

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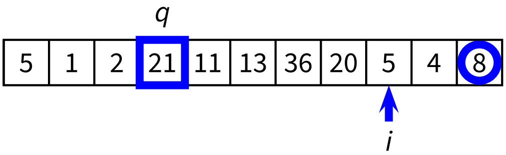

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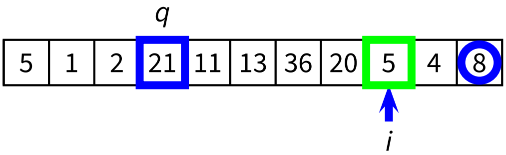

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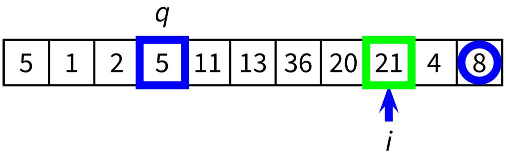

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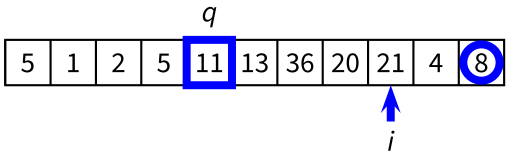

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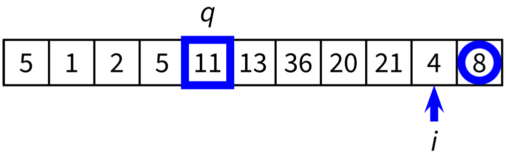

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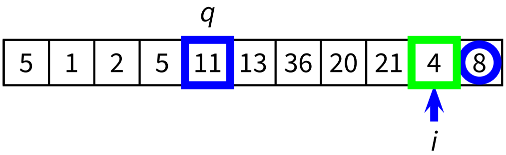

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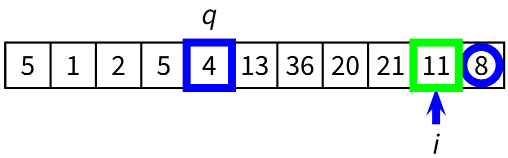

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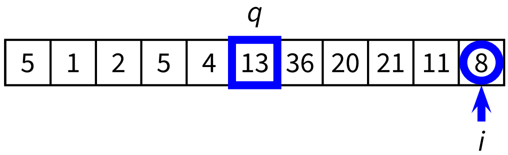

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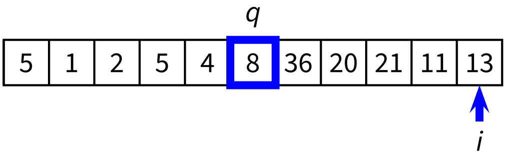

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5	1	2	5	4	$\boxed{8}$	36	20	21	11	13

Complete QuickSort Algorithm

```
PARTITION(A, begin, end)
q = begin
v = A[end]
for i = begin to end
    if A[i] ≤ v
        swap A[i] and A[q]
        q = q + 1
return q − 1
```

```
QUICKSORT(A, begin, end)
if begin < end
    q = PARTITION(A, begin, end)
    QUICKSORT(A, begin, q − 1)
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```

Complexity of PARTITION

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$$T(n) = \Theta(n)$$

```
QuickSort(A, begin, end)
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```

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- Worst case

```
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- Worst case
  - $q = begin$ or $q = end$

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  - $q = \mathit{begin}$ or $q = \mathit{end}$
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Complexity of QUICKSORT (2)

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QUICKSORT(A, begin, end)
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- Best case
  - $q = \lceil n/2 \rceil$

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$$T(n) = \Theta(n \log n)$$

Sorting Algorithms Seen So Far

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Algorithm	Complexity			In place?
	worst	*average*	*best*	
Insertion-Sort	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n)$	yes
Selection-Sort	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	yes
Bubble-Sort	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	yes
Merge-Sort	$\Theta(n \log n)$	$\Theta(n \log n)$	$\Theta(n \log n)$	no

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QuickSort	$\Theta(n^2)$	$\Theta(n \log n)$	$\Theta(n \log n)$	yes

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QuickSort	$\Theta(n^2)$	$\Theta(n \log n)$	$\Theta(n \log n)$	yes
??	$\Theta(n \log n)$			***yes***

Binary Heap

- Our first real ***data structure***

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- Interface

Binary Heap

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  - **BUILD-MAX-HEAP**(A) rearranges A into a max-heap
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Binary Heap: Structure

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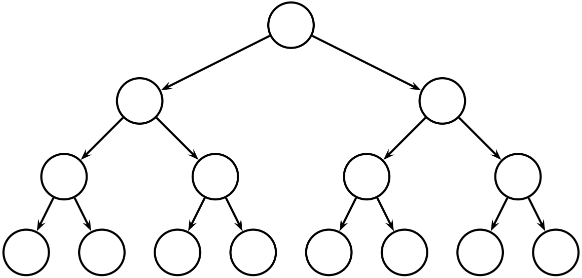

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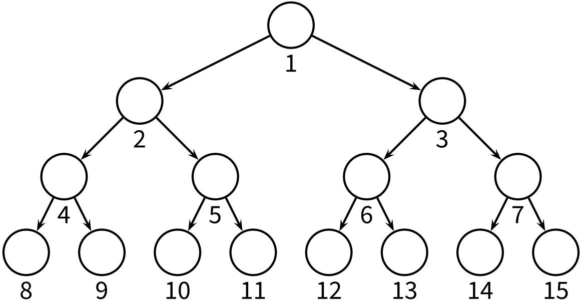

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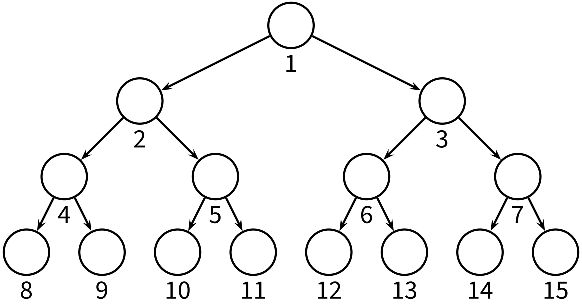

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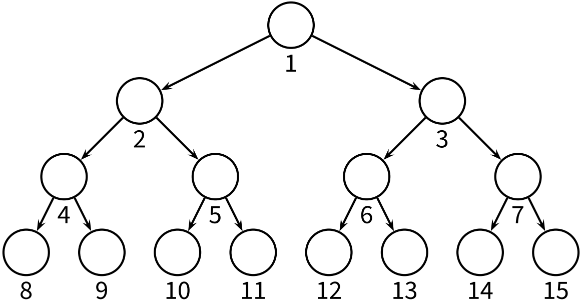

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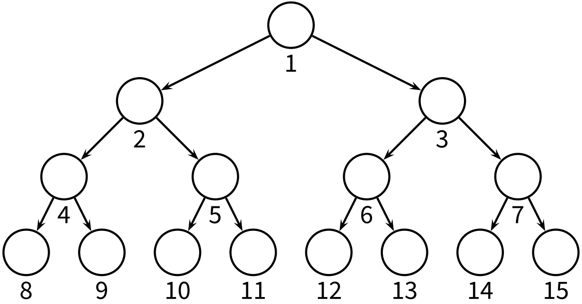

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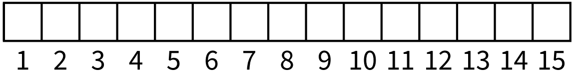

Binary Heap: Properties

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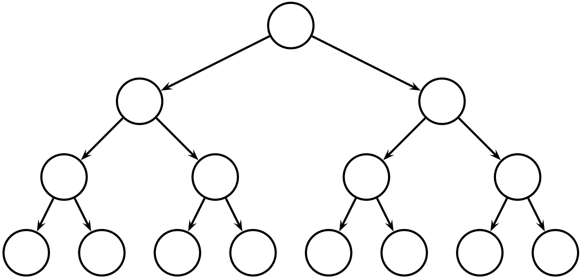

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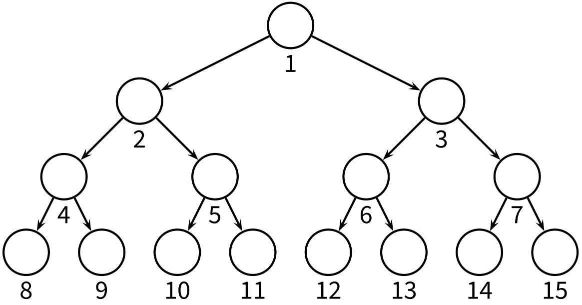

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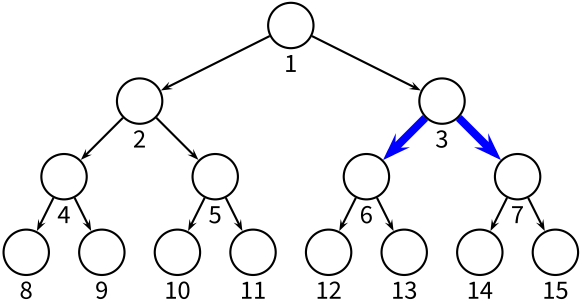

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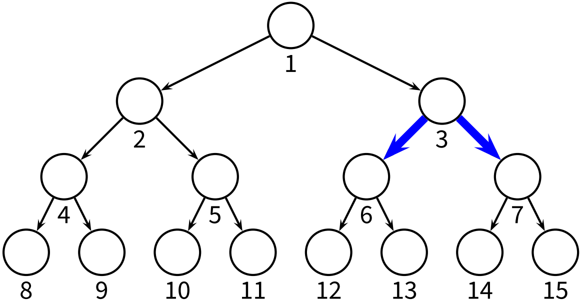

Parent(i)

  return $\lfloor i/2 \rfloor$

Left(i)

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Right(i)

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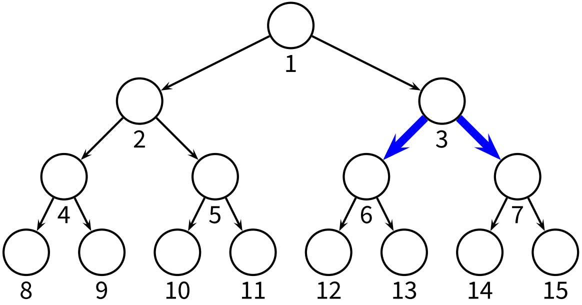

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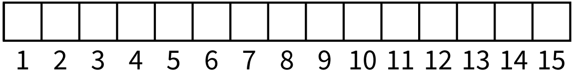
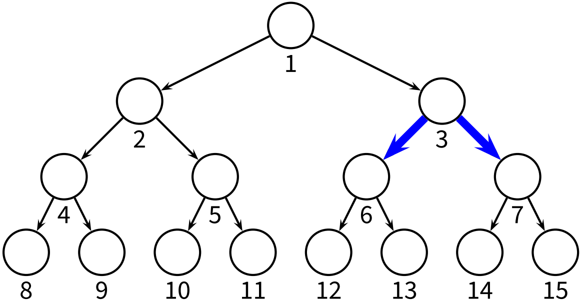
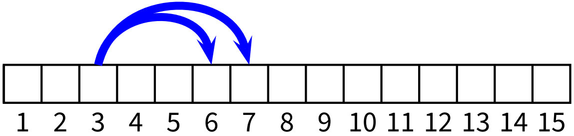


Binary Heap: Properties





PARENT(i)

  return $\lfloor i/2 \rfloor$

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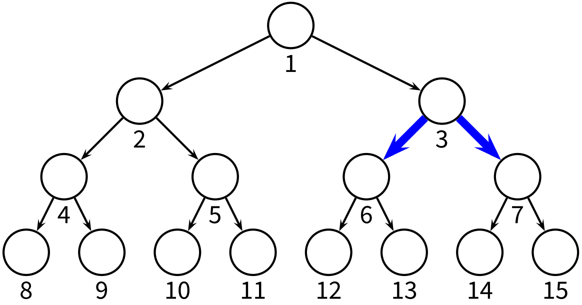

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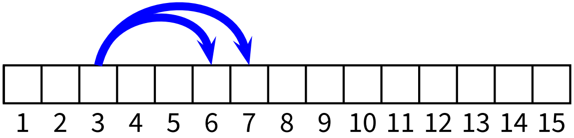

- ***Max-heap property:*** for all $i > 1$ $\boxed{A[\textbf{PARENT}(i)] \geq A[i]}$

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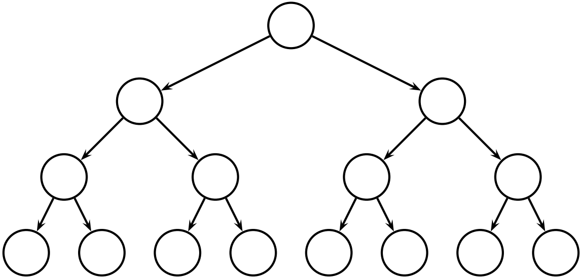

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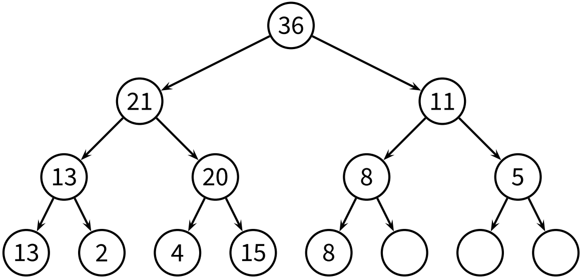

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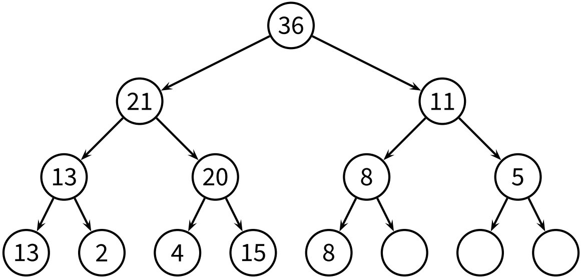

- Where is the max element?

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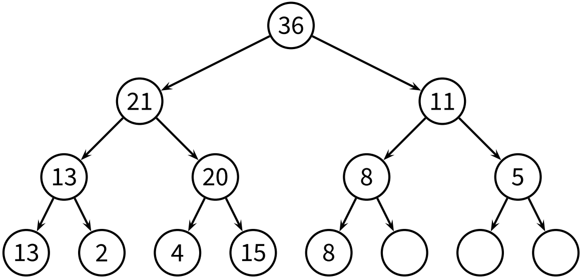

- Where is the max element?

- How can we implement **HEAP-EXTRACT-MAX**?

- **HEAP-EXTRACT-MAX** procedure
  - extract the max key
  - rearrange the heap to maintain the *max-heap property*

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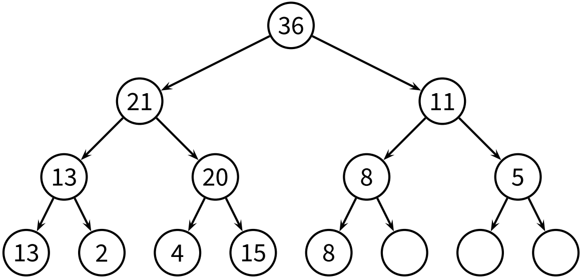

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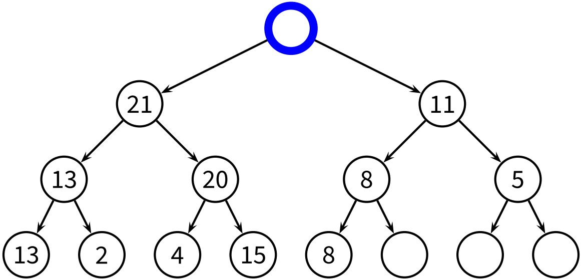

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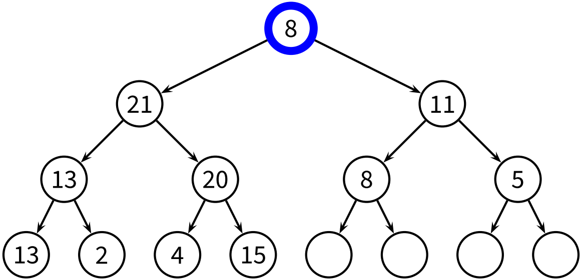

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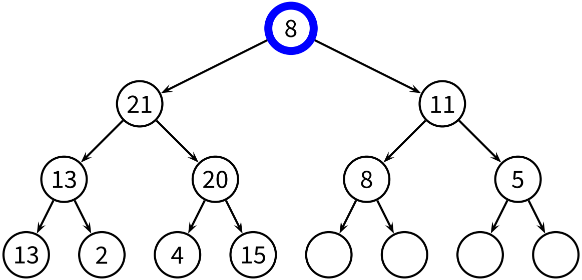

- Now we have two subtrees where the *max-heap property* holds

- **MAX-HEAPIFY**(A, i) procedure
  - *assume:* the *max-heap property* holds in the subtrees of node i
  - *goal:* rearrange the heap to maintain the *max-heap property*

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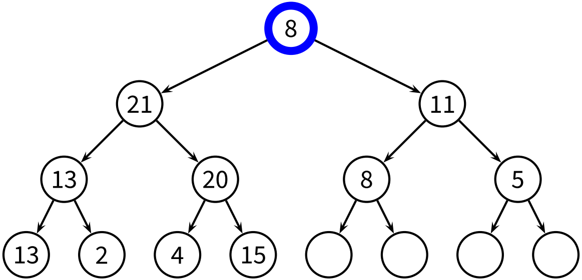

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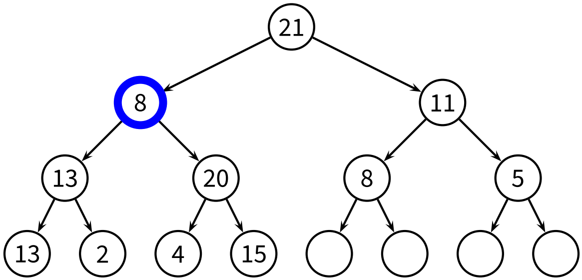

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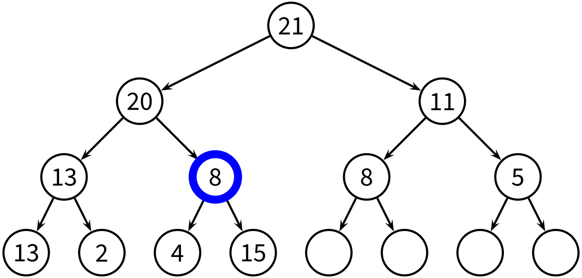

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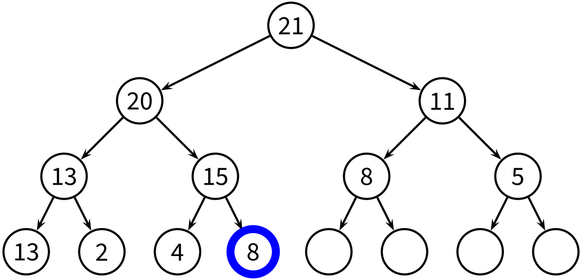

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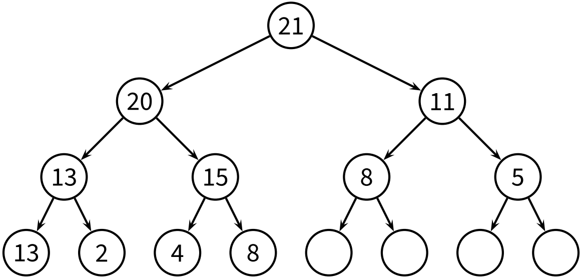

Max-Heapify

```
MAX-HEAPIFY(A, i)
l = LEFT(i)
r = RIGHT(i)
if l ≤ A.heap-size and A[l] > A[i]
        largest = l
else largest = i
if r ≤ A.heap-size and A[r] > A[largest]
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if largest ≠ i
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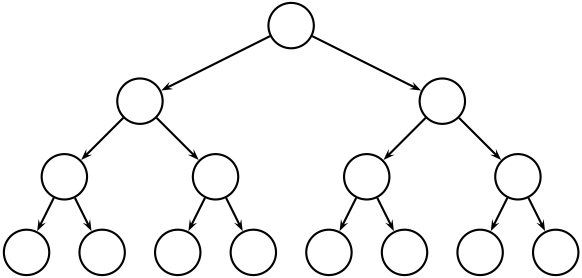

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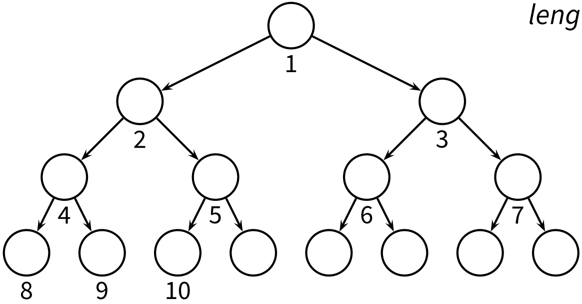

$length(A) = 10$

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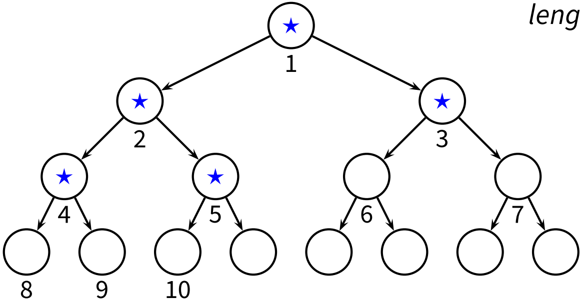

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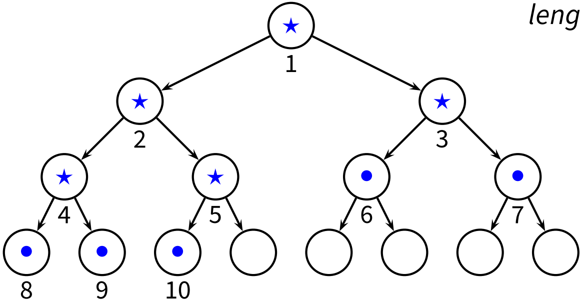

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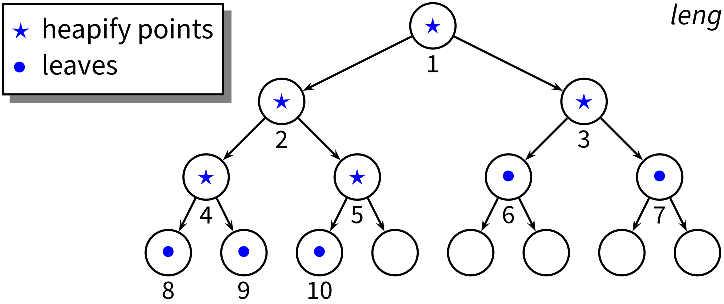

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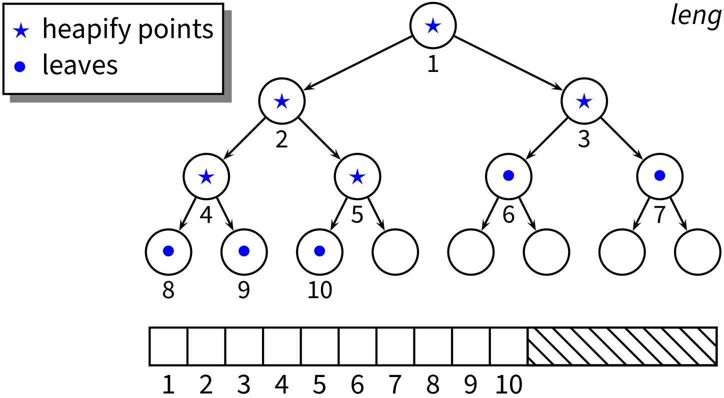

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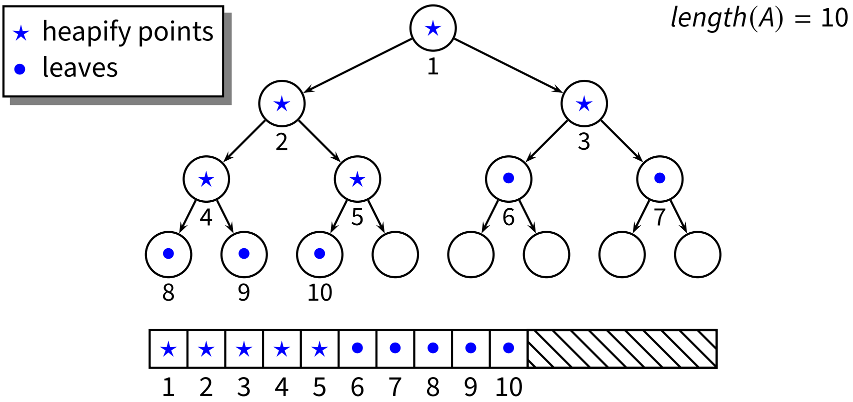

- Idea: we can use a heap to sort an array

Heap Sort

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HEAP-SORT(A)
BUILD-MAX-HEAP(A)
for i = length(A) downto 1
        swap A[i] and A[1]
        A.heap-size = A.heap-size − 1
        MAX-HEAPIFY(A, 1)
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- What is the complexity of **HEAP-SORT**?

$$\boxed{T(n) = \Theta(n \log n)}$$

- Benefits
  - in-place sorting; worst-case is $\Theta(n \log n)$

Summary of Sorting Algorithms

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Algorithm		Complexity		In place?
	worst	*average*	*best*	
Insertion-Sort				

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INSERTION-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n)$	yes
SELECTION-SORT				

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INSERTION-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n)$	yes
SELECTION-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	yes
BUBBLE-SORT				

Summary of Sorting Algorithms

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	worst	*average*	*best*	
Insertion-Sort	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n)$	yes
Selection-Sort	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	yes
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