Algorithms and Data Structures

Course Introduction

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Università della Svizzera italiana

February 22, 2022

General Information

- On-line course information
  - on iCorsi: ***INF.B.SP22.05***
  - and on my web page: ***http://www.inf.usi.ch/carzaniga/edu/algo/***
  - previous edition also on-line: ***http://www.inf.usi.ch/carzaniga/edu/algo21s/***

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- Announcements
  - ***you are responsible for reading the announcements (posted through iCorsi)***

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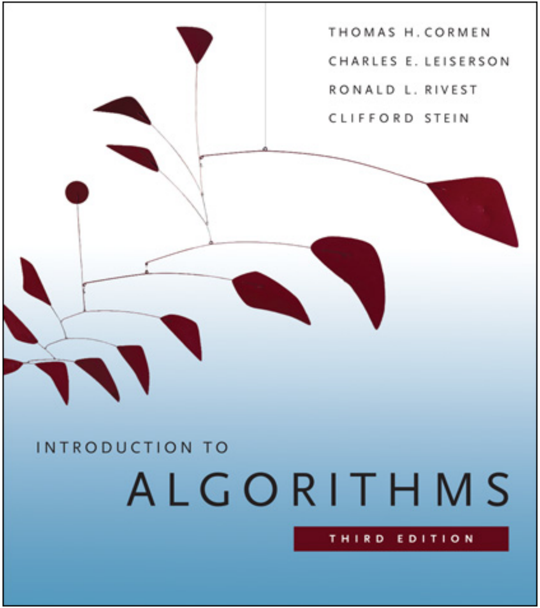
- Announcements
  - ***you are responsible for reading the announcements (posted through iCorsi)***
- Office hours
  - Antonio Carzaniga: *by appointment*
  - Dylan Robert Ashley: *by appointment*
  - Aditya Ramesh: *by appointment*
  - Morteza Rezaalipour: *by appointment*

Textbook

Introduction to Algorithms

Thomas H. Cormen
Charles E. Leiserson
Ronald L. Rivest
Clifford Stein

The MIT Press



Evaluation

- +20% homework
  - 3–5 assignments
  - grades added together, thus resulting in a weighted average
- +30% midterm exam
- +50% final exam
- ±10% instructor's discretionary evaluation
  - participation
  - extra credits
  - trajectory
  - ...

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- −100% plagiarism penalties

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- "material" means ideas, words, code, suggestions, corrections on one's work, etc.
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  - e.g., software libraries
  - ***always clearly identify the external material, and acknowledge its source!***
    Failing to do so means committing plagiarism.
  - the work will be evaluated based on its *added value*

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  - the work will be evaluated based on its *added value*
- Plagiarism or cheating on an assignment or an exam may result in
  - failing that assignment or that exam
  - losing one or more points *in the final note!*
- Penalties may be escalated in accordance with the regulations

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  - The grade of an assignment turned in more than two days late is 0

an introductory example…

Fundamental Ideas

Johannes Gutenberg invents movable type and the printing press in Mainz, circa 1450
(already known in China and Korea, circa 1200 CE)

Maybe More Fundamental Ideas

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Muhammad ibn Musa al-Khwārizmī

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  - these procedures were ***precise***, ***unambiguous***, ***mechanical***, ***efficient***, and ***correct***

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  - these procedures were ***precise***, ***unambiguous***, ***mechanical***, ***efficient***, and ***correct***
  - *they were **algorithms!***

Muhammad ibn Musa al-Khwārizmī

Algorithms are

the essence

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Example

- A sequence of numbers

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$$

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- The well-known Fibonacci sequence

Leonardo da Pisa (ca. 1170–ca. 1250)
son of Guglielmo “Bonaccio”
a.k.a. *Leonardo Fibonacci*

The Fibonacci Sequence

Mathematical definition:

$$F_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F_{n-1} + F_{n-2} & \text{if } n > 1 \end{cases}$$

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Implementation on a computer:

Racket

```
(define (F n)
  (cond
    ((= n 0) 0)
    ((= n 1) 1)
    (else (+ (F (- n 1)) (F (- n 2))))))
```

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Implementation on a computer:

Java

```
public class Fibonacci {
  public static int F(int n) {
    if (n == 0) {
      return 0;
    } else if (n == 1) {
      return 1;
    } else {
      return F(n-1) + F(n-2);
    }
  }
}
```

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Implementation on a computer:

C or C++

```
int F(int n) {
  if (n == 0) {
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Implementation on a computer:

Python

```
def F(n):
  if n == 0:
    return 0
  elif n == 1:
    return 1
  else:
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```

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Implementation on a computer:

very concise C/C++ (or Java)

```
int F(int n) {
  return (n<2)?n:F(n-1)+F(n-2);
}
```

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Implementation on a computer:

“pseudo-code”

```
F(n)
if n == 0
        return 0
elseif n == 1
        return 1
else return F(n − 1) + F(n − 2)
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Questions on Our First Algorithm

```
FIBONACCI(n)
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   - for every valid input, does it terminate?
   - if so, does it do the right thing?

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3. Can we do better?

Correctness

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- The algorithm is clearly correct
  - assuming $n \geq 0$

Performance

- How long does it take?

Performance

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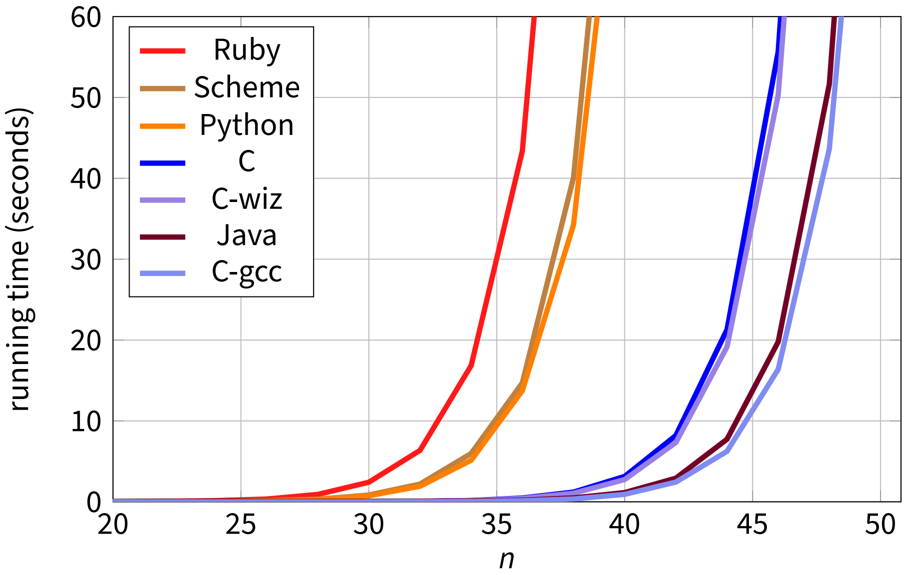
  Let's try it out…

Results



Ruby
Scheme
Python
C
C-wiz
Java
C-gcc
running time (seconds)
0
10
20
30
40
50
60
20
25
30
35
40
45
50
n

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  - it is better to let the compiler do the optimization
  - simple language tricks don't seem to pay off
- However, the differences are not substantial
  - *all* implementations sooner or later seem to hit a wall...
- Conclusion: ***the problem is with the algorithm***

Complexity of Our First Algorithm

- We need a mathematical characterization of the performance of the algorithm

  We'll call it the algorithm's ***computational complexity***

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$T(0) = 2;\ T(1) = 3$

$T(n) = T(n-1) + T(n-2) + 3 \quad \Rightarrow T(n) \geq F_n$

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- Can we do better?

A Better Algorithm

$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$

A Better Algorithm

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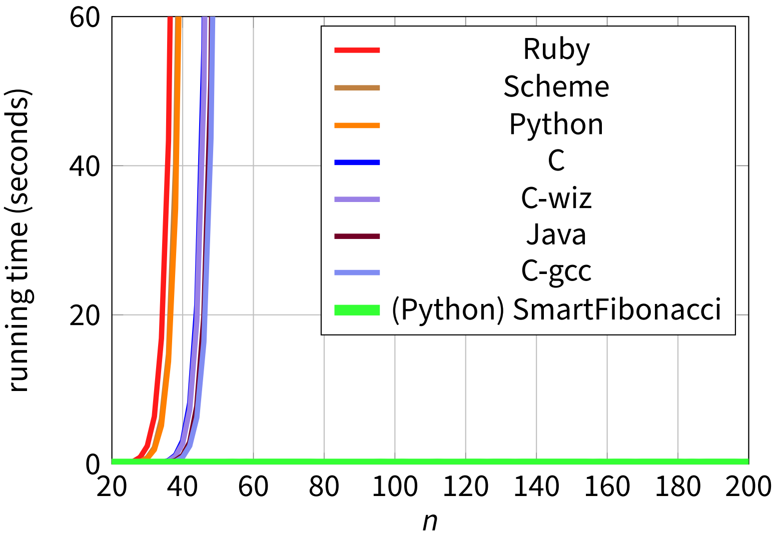
```
SmartFibonacci(n)
if n == 0
    return 0
elseif n == 1
    return 1
else pprev = 0
    prev = 1
    for i = 2 to n
        f = prev + pprev
        pprev = prev
        prev = f
return f
```

Results



running time (seconds)
60
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n
Ruby
Scheme
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Java
C-gcc
(Python) SmartFibonacci

Complexity of SmartFibonacci

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$T(n) =$

Complexity of SmartFibonacci

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```

$T(n) = 6 + 6(n - 1)$

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```

$T(n) = 6 + 6(n-1) = 6n$

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$T(n) = 6 + 6(n-1) = 6n$

The *complexity* of **SmartFibonacci**(n) is ***linear*** in n