Algorithms and Data Structures

Course Introduction

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Faculty of Informatics
Università della Svizzera italiana

February 22, 2016

- On-line course information
  - on iCorsi: 'INFO.B299'
    https://www2.icorsi.ch/course/view.php?id=4909
  - and on my web page:
    http://www.inf.usi.ch/carzaniga/edu/algo/
  - last edition also on-line:
    http://www.inf.usi.ch/carzaniga/edu/algo15s/

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  - ***you are responsible for reading the announcements page or the messages sent through iCorsi***

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- Office hours
  - Antonio Carzaniga: *by appointment*
  - Salvatore Ingala: *by appointment*
  - Luis Mastrangelo: *by appointment*

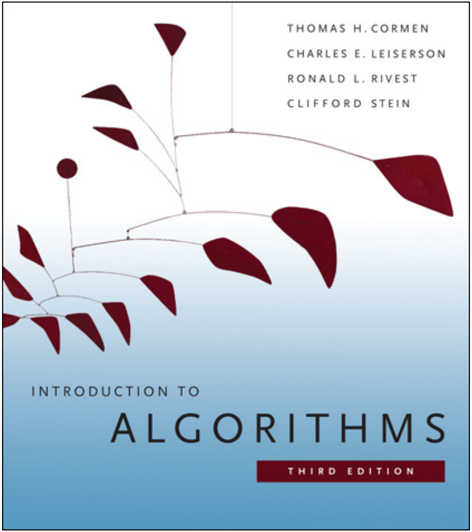
Textbook

Introduction to Algorithms

Third Edition

Thomas H. Cormen
Charles E. Leiserson
Ronald L. Rivest
Clifford Stein

The MIT Press



Evaluation

- +30% projects
  - 3–5 assignments
  - grades added together, thus resulting in a weighted average
- +30% midterm exam
- +40% final exam
- ±10% instructor’s discretionary evaluation
  - participation
  - extra credits
  - trajectory
  - …

Evaluation

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- +30% midterm exam
- +40% final exam
- ±10% instructor's discretionary evaluation
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  - ...
- −100% plagiarism penalties

Plagiarism

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A student should never take someone else's material and present it as his or her own. Doing so means committing plagiarism.

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- "material" means ideas, words, code, suggestions, corrections on one's work, etc.
- Using someone else's material may be appropriate
  - e.g., software libraries
  - ***always clearly identify the external material, and acknowledge its source. Failing to do so means committing plagiarism.***
  - the work will be evaluated based on its *added value*

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  - the work will be evaluated based on its *added value*
- Plagiarism on an assignment or an exam will result in
  - failing that assignment or that exam
  - loosing one or more points *in the final note!*
- Penalties may be escalated in accordance with the regulations of the Faculty of Informatics

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- Each late day will reduce the assignment's grade by *one third* of the total value of that assignment
  - **Corollary 1:** The grade of an assignment turned in more than two days late is 0
  - The proof of Corollary 1 is left as an exercise

Now let’s move on to the real
interesting and fun stuff. . .

Fundamental Ideas

Johannes Gutenberg invents movable type and the printing press in Mainz, circa 1450 (already known in China, circa 1200 CE)

Maybe More Fundamental Ideas

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    - these procedures were ***precise***, ***unambiguous***, ***mechanical***, ***efficient***, and ***correct***

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  - these procedures were ***precise***, ***unambiguous***, ***mechanical***, ***efficient***, and ***correct***
  - *they were **algorithms!***

Muhammad ibn Musa al-Khwārizmī

Algorithms are

the essence

`of computer programs`

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Example

- A sequence of numbers

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$$

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- The well-known Fibonacci sequence

Leonardo da Pisa (ca. 1170–ca. 1250)
son of Guglielmo “Bonaccio”
a.k.a. *Leonardo Fibonacci*

- Mathematical definition: $F_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F_{n-1} + F_{n-2} & \text{if } n > 1 \end{cases}$

The Fibonacci Sequence

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- Implementation on a computer:

Racket

```
(define (F n)
  (cond
    ((= n 0) 0)
    ((= n 1) 1)
    (else (+ (F (- n 1)) (F (- n 2))))))
```

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- Implementation on a computer:

Java

```
public class Fibonacci {
  public static int F(int n) {
    if (n == 0) {
      return 0;
    } else if (n == 1) {
      return 1;
    } else {
      return F(n-1) + F(n-2);
    } }
}
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- Implementation on a computer:

C or C++

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int F(int n) {
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- Implementation on a computer:

Ruby

```
def F(n)
  case n
    when 0
      return 0
    when 1
      return 1
    else
      return F(n-1) + F(n-2)
  end
end
```

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- Implementation on a computer:

Python

```
def F(n):
  if n == 0:
    return 0
  elif n == 1:
    return 1
  else:
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```

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- Implementation on a computer:

very concise C/C++ (or Java)

```
int F(int n) { return (n<2)?n:F(n-1)+F(n-2); }
```

The Fibonacci Sequence

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- Implementation on a computer:

“pseudo-code”

```
FIBONACCI(n)
if n == 0
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Questions on Our First Algorithm

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    - for every valid input, does it terminate?
    - if so, does it do the right thing?

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3. Can we do better?

Correctness

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- The algorithm is clearly correct
  - assuming $n \geq 0$

Performance

- How long does it take?

Performance

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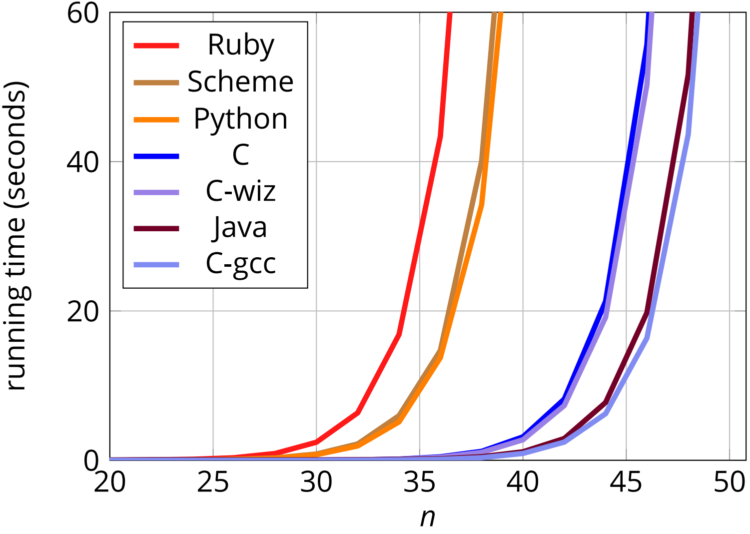
  Let's try it out. . .



60
40
20
0
20
25
30
35
40
45
50
running time (seconds)
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Ruby
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Comments

- Different implementations perform differently
  - it is better to let the compiler do the optimization
  - simple language tricks don't seem to pay off
- However, the differences are not substantial
  - *all* implementations sooner or later seem to hit a wall…
- Conclusion: ***the problem is with the algorithm***

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$T(n) = T(n-1) + T(n-2) + 3$

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$T(0) = 2$; $T(1) = 3$

$T(n) = T(n-1) + T(n-2) + 3 \quad \Rightarrow T(n) \geq F_n$

Complexity of Our First Algorithm (2)

- So, let's try to understand how F_n grows with n

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$$T(n) \geq (\sqrt{2})^n \approx (1.4)^n$$

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- Can we do better?

A Better Algorithm

- Again, the sequence is 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, . . .

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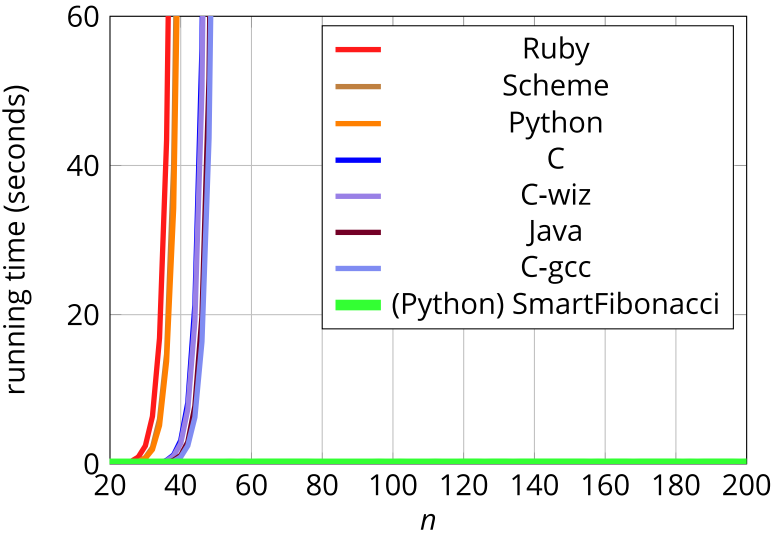
```
SMARTFIBONACCI(n)
if n == 0
    return 0
elseif n == 1
    return 1
else pprev = 0
    prev = 1
    for i = 2 to n
        f = prev + pprev
        pprev = prev
        prev = f
return f
```

Results



running time (seconds)
60
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120
140
160
180
200
n
Ruby
Scheme
Python
C
C-wiz
Java
C-gcc
(Python) SmartFibonacci

Complexity of SmartFibonacci

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else prev = 0
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Complexity of SMARTFIBONACCI

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$T(n) =$

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$T(n) = 6 + 6(n-1)$

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$T(n) = 6 + 6(n - 1) = 6n$

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$T(n) = 6 + 6(n - 1) = 6n$

The *complexity* of **SmartFibonacci**(n) is ***linear*** in n