Algorithms and Data Structures

Course Introduction

Antonio Carzaniga

Faculty of Informatics
Università della Svizzera italiana

February 20, 2024

General Information

- On-line course information
  - on iCorsi: ***INF.B.SP 2024.324***
  - and on my web page: ***https://www.inf.usi.ch/carzaniga/edu/algo/***
  - previous edition also on-line: ***https://www.inf.usi.ch/carzaniga/edu/algo23s/***

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- Announcements
  - ***you are responsible for reading the announcements (posted through iCorsi)***

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- Announcements
  - ***you are responsible for reading the announcements (posted through iCorsi)***
- Personal consultations: ***by appointment***
  - Antonio Carzaniga (yours, truly)
  - Thomas Bertini
  - Michal Burgunder
  - Fabio Di Lauro
  - Koppány Encz
  - Claudio Milanesi

Assessment

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- +40% midterm exam *18 April*, 10:30-12:30, Aula Polivalente
- +60% final exam
- ±10% instructor's discretionary evaluation
  - participation
  - extra credits
  - trajectory
  - …

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- −100% plagiarism penalties

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- "material" means ideas, words, code, suggestions, corrections on one's work, etc.
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  - e.g., software libraries
  - ***always clearly identify the external material, and acknowledge its source!***
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  - the work will be evaluated based on its *added value*

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  - the work will be evaluated based on its *added value*
- Plagiarism or cheating on an assignment or an exam may result in
  - failing that assignment or that exam
  - losing one or more points *in the final note!*
- Penalties may be escalated in accordance with the regulations

A note on learning Algorithms

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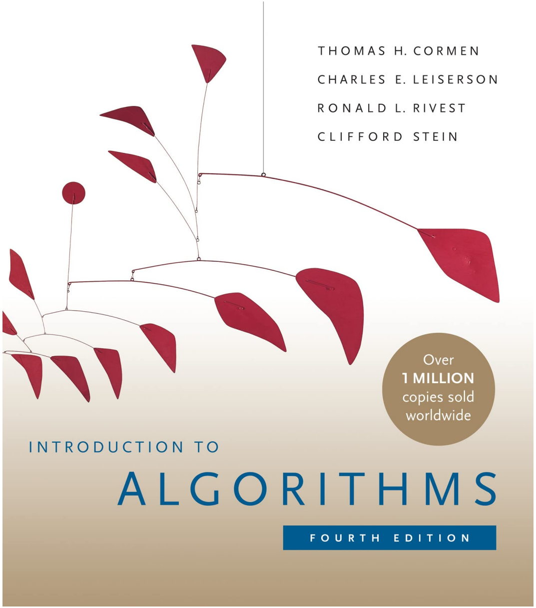
I will give you all the resources and all the help I can provide

Textbook

Introduction to Algorithms

Thomas H. Cormen
Charles E. Leiserson
Ronald L. Rivest
Clifford Stein

The MIT Press



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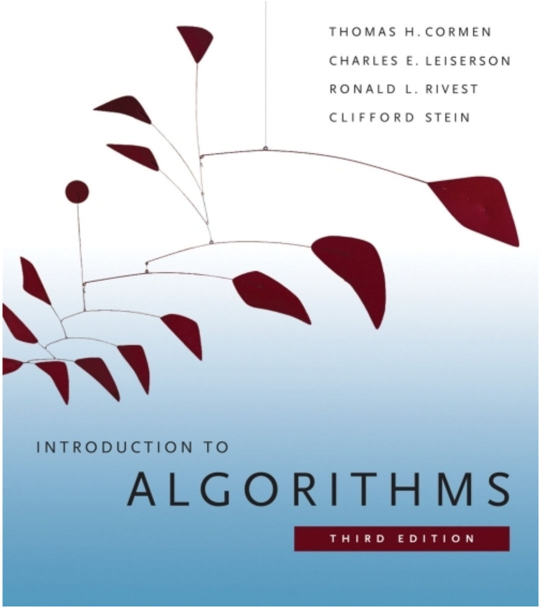

Exercises and Other Material

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- *Notes on Elementary Algorithmic Programming in Python*
  https://www.inf.usi.ch/carzaniga/edu/algo/programming.html

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- A collection of 298 exam exercises, many of them with solutions
  https://www.inf.usi.ch/carzaniga/edu/algo/exercises.pdf

Our Time and Energy

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- Personal meetings
  - extemporaneous, *any time I have time!*
  - individually or in small groups
  - questions, exercises, discussions, . . .

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  - individually or in small groups
  - questions, exercises, discussions, …
- ***Exercise sessions***
  - every ***Wednesday 15:30–17:00 in C1.04***
  - supervised exercises, analysis of solutions, discussions

an introductory example…

Fundamental Ideas

Fundamental Ideas

Johannes Gutenberg invents movable type and the printing press in Mainz, circa 1450
(already known in China and Korea, circa 1200 CE)

Maybe More Fundamental Ideas

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  - these procedures were ***precise***, ***unambiguous***, ***mechanical***, ***efficient***, and ***correct***
  - *they were **algorithms!***

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Algorithms are

the essence

`of computer programs`

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Example: Poetic Rhythms

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How many 1,2-rhythms can you compose over a total of* n *beats?

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We have $n = 4$ total beats. How many different rhythms can we have?

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1–1–1–1	*Ta–Ta–Ta–Ta–*
1–1–2	*Ta–Ta–Ta-a–*
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$$P(3) = ?$$

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How many rhythms can you compose over a total of n beats?

Example:

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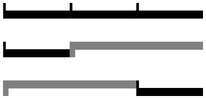

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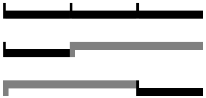

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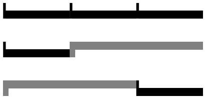

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We want a general *algorithm* to compute $P(n)$

$n = 5$:

A First Algorithm

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$\textsc{Pingala}(5) = 8$

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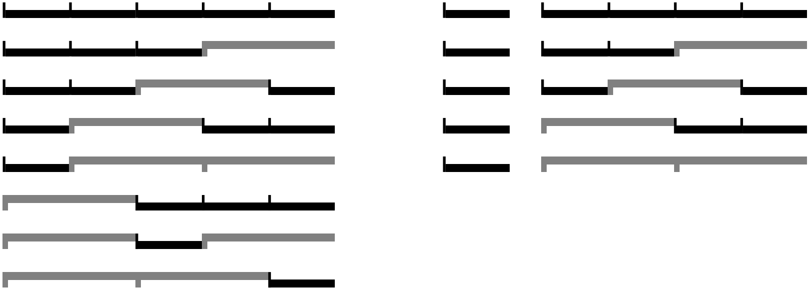

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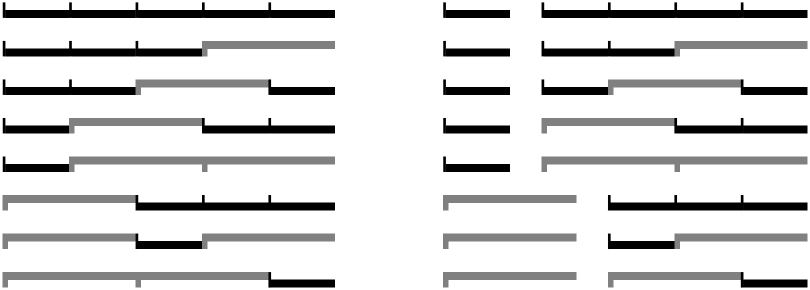

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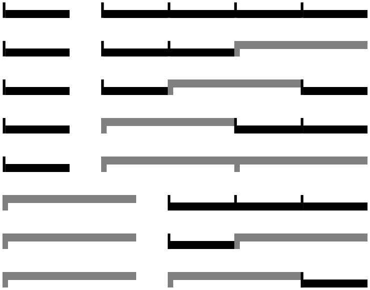

PINGALA(5) = 8

PINGALA(5) = PINGALA(4) + PINGALA(3)

A First Algorithm

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PINGALA(n)
if n ≤ 2
        return n
return PINGALA(n − 1) + PINGALA(n − 2)
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Questions on Our First Algorithm

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2. Is the algorithm ***efficient?***
   - How much ***time*** does it take to complete?

3. Can we do better?

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- For now we wave our hands…
  - “the algorithm is clearly correct!”
  - assuming $n > 0$

Performance

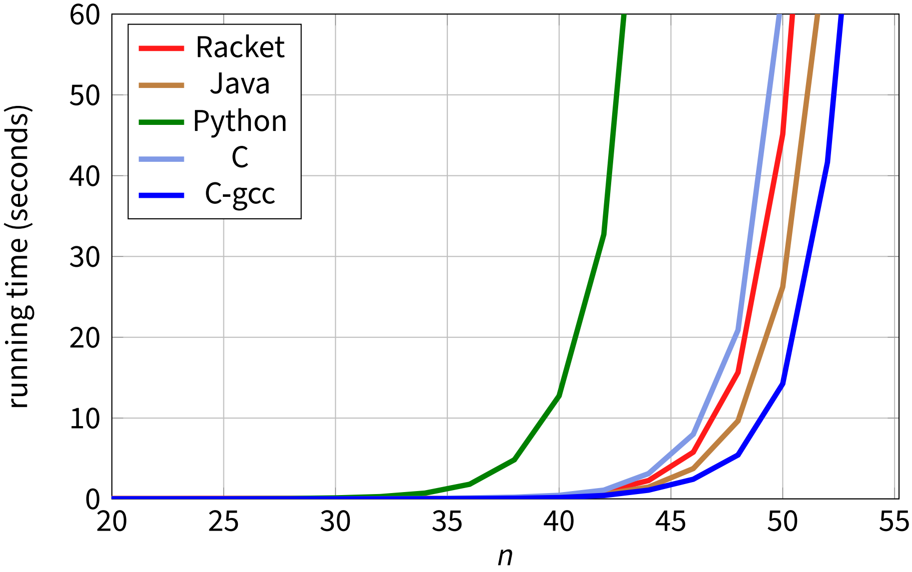
- How long does it take?

Performance

- How long does it take?

  Let's try it out…

Results



Racket
Java
Python
C
C-gcc
running time (seconds)
0
10
20
30
40
50
60
20
25
30
35
40
45
50
55
n

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  - compiler optimizations can make a difference

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  - with different languages you get different performances
  - compiler optimizations can make a difference
- However, the differences are not substantial
  - *all* implementations sooner or later seem to hit a wall…
- Conclusion: ***the problem is with the algorithm***

Complexity of Our First Algorithm

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1  if n ≤ 2
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$T(1) = T(2) = 2$

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$$T(1) = T(2) = 2$$
$$T(n) = T(n-1) + T(n-2) + 2 \quad \Rightarrow T(n) \geq P(n)$$

Complexity of Our First Algorithm (2)

- So, let's try to understand how $T(n) =$ grows with n

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- Can we do better?

A Better Algorithm

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```
PINGALA-MEM(n, M)
if n ≤ 2
    return n
if M == ∅
    M = array of n NIL elements
if M[n] == NIL
    M[n] = PINGALA-MEM(n − 1, M) + PINGALA-MEM(n − 2, M)
return M[n]
```

An Even Better Algorithm

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Idea: we can build $P(n)$ from the ground up, with just a couple of extra variables!

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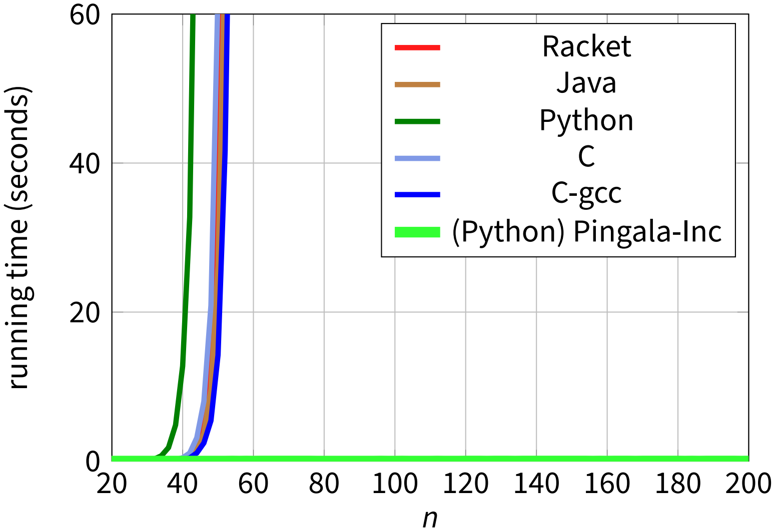
```
PINGALA-INC(n)
if n ≤ 2
    return n
pprev = 1
prev = 2
for i = 3 to n
    P = prev + pprev
    pprev = prev
    prev = P
return P
```

Results



Racket
Java
Python
C
C-gcc
(Python) Pingala-Inc
running time (seconds)
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0
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40
60
80
100
120
140
160
180
200
n

Complexity of Pingala-Inc

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        P = prev + pprev
        pprev = prev
        prev = P
return P
```

$T(n) =$

Complexity of PINGALA-INC

```
PINGALA-INC(n)
if n ≤ 2
        return n
pprev = 1
prev = 2
for i = 3 to n
        P = prev + pprev
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$T(n) = 4 + 5(n - 2)$

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The *complexity* of **PINGALA-INC**(n) is ***linear*** in n