Basics of Complexity Analysis: The RAM Model and the Growth of Functions

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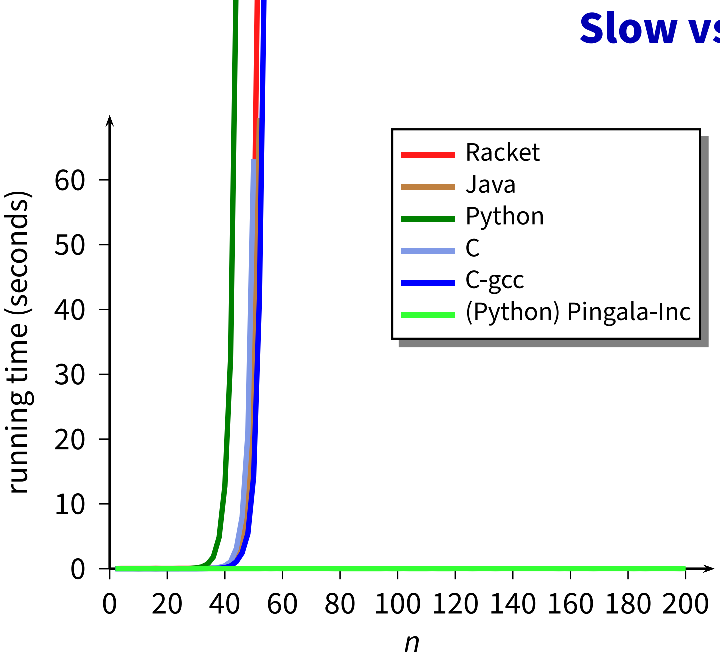
Outline

- Informal analysis of two Pingala algorithms
- The *random-access machine* model
- Measure of complexity
- Characterizing functions with their asymptotic behavior
- Big-*O*, omega, and theta notations

Slow vs. Fast Pingala



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  - in a way that is ***specific to the algorithms***
  - but ***independent of implementation details***

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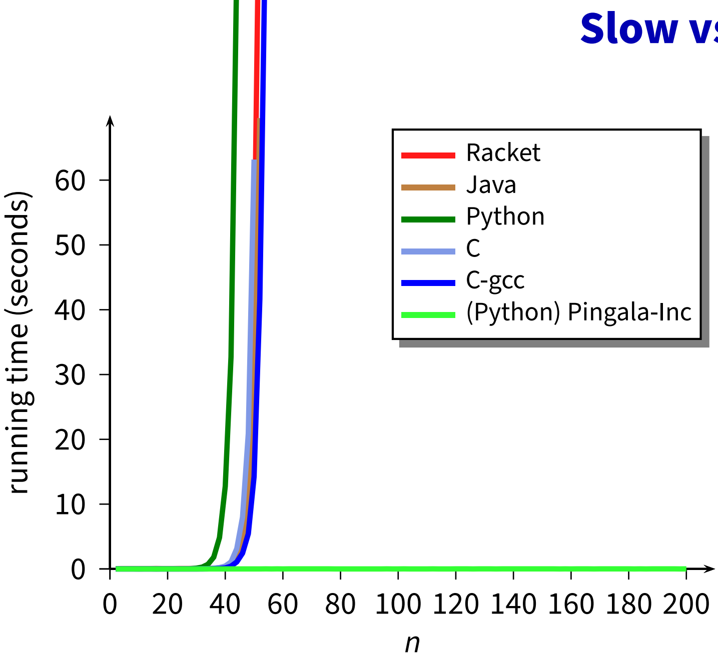

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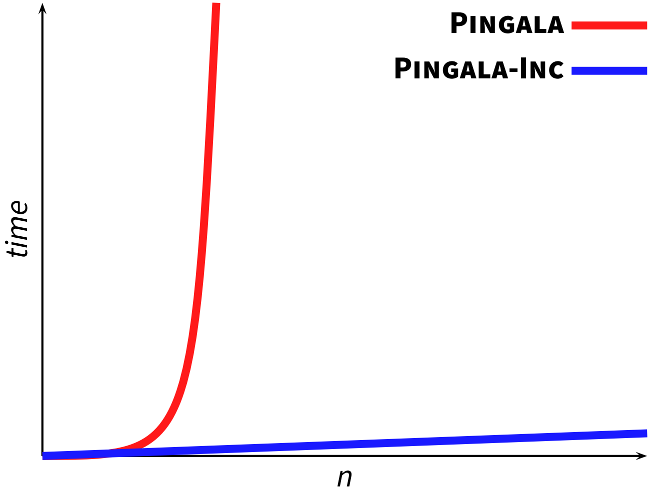

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- A ***basic step*** in the RAM model takes a ***constant time***

Analysis in the RAM Model

```
PINGALA-INC(n)
if n ≤ 2
        return n
pprev = 1
prev = 2
for i = 3 to n
        P = prev + pprev
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c_1	1
c_2	0
c_3	1
c_4	1
c_5	$n-1$
c_6	$n-2$
c_7	$n-2$
c_8	$n-2$
c_9	1

$$T(n) = c_1 + c_3 + c_4 + c_9 + (n-1)c_5 + (n-2)(c_6 + c_7 + c_8)$$

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$T(n) = nC_1 + C_2$ $\Rightarrow$ $T(n)$ is a *linear function* of n

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FIND(A, x)
for i = 1 to length(A)
    if A[i] == x
        return TRUE
return FALSE
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$$T(n) = Cn$$

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for i = 1 to length(A) − 1
    for j = i + 1 to length(A)
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$$T(n) = C\frac{n(n-1)}{2}$$

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  - so, we assume $c_1 = c_2 = c_3 = \cdots = c_i$
  - we also ignore the specific *value* c_i, and in fact ***we ignore every constant cost factor***

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    We write

    $$\boxed{T(n) = \Theta(n^2)}$$

    and say that "$T(n)$ is theta of n-squared"

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  - $A(n-1) = A(n^2)$ for all n

- If $f(n)$ is such that $f(n) = kA(g(n))$ for all n sufficiently large and for some constant $k > 0$, then we say that

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- If $f(n) = O(g(n))$ then we can also say that $g(n)$ asymptotically *dominates* $f(n)$, which we can also write as

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From O to Ω and Θ

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- When $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$ we also write

$$f(n) = \Theta(g(n))$$

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  - $\pi(n) = O(n)$ trivial ***upper bound***
  - $\pi(n) = \Omega(1)$ trivial ***lower bound***
  - $\pi(n) = \Theta(n/\log n)$ non-trivial ***tight bound***

Characterizing *Unknown* Functions

- The idea of the O, Ω, and Θ notations is very often to characterize a function that is *not completely known*

  Example:

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  In fact, the fundamental *prime number theorem* says that

$$\lim_{n\to\infty} \frac{\pi(n)\ln n}{n} = 1$$

Θ-Notation

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- Given a function $g(n)$, we define the *family of functions* $\Theta(g(n))$

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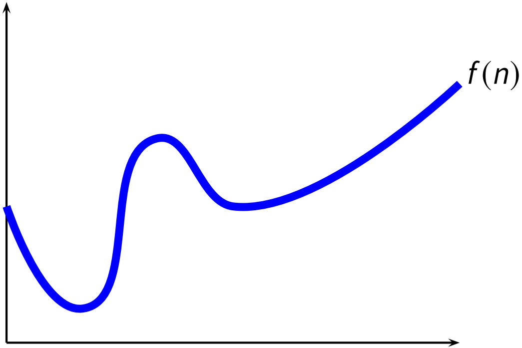

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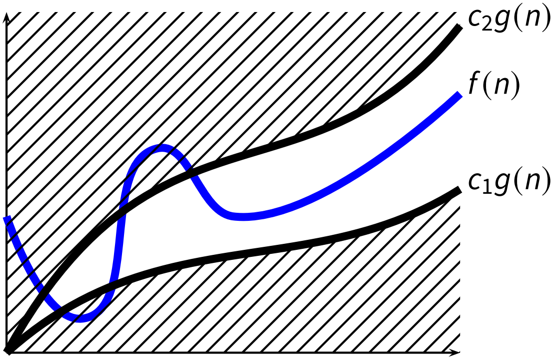

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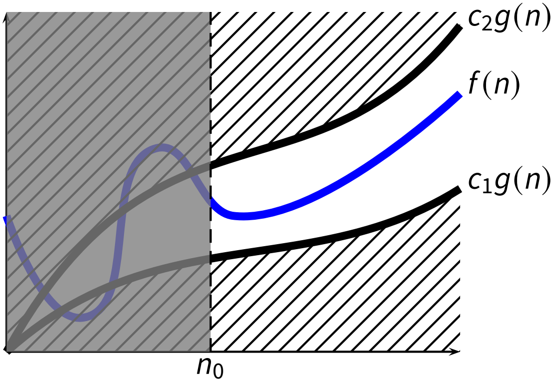

$$\Theta(g(n)) = \{f(n) : \exists c_1 > 0, \exists c_2 > 0, \exists n_0 > 0$$
$$: 0 \le c_1g(n) \le f(n) \le c_2g(n) \text{ for all } n \ge n_0\}$$

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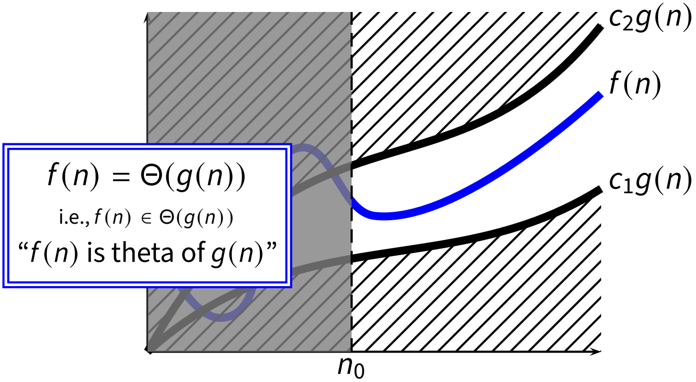

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Examples

- $T(n) = n^2 + 10n + 100$

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- $T(n) =$ complexity of **PINGALA-INC** $\quad \Rightarrow T(n) = \Theta(n)$
- We characterize the behavior of $T(n)$ *in the limit*
- The Θ-notation is an ***asymptotic notation***

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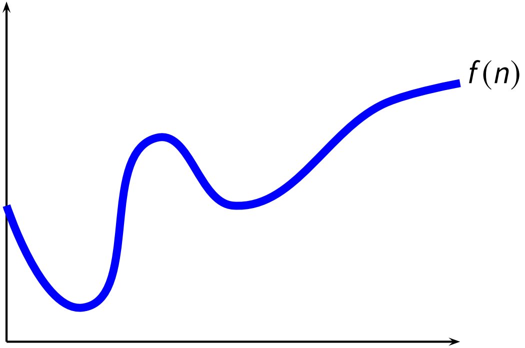

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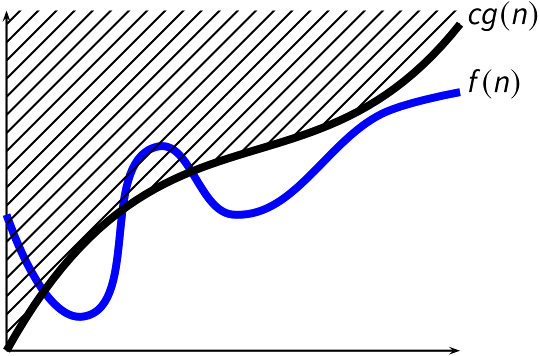

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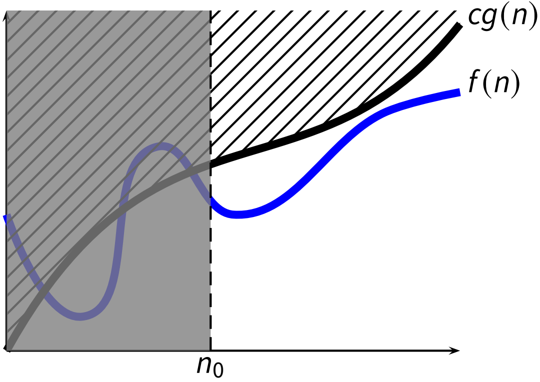

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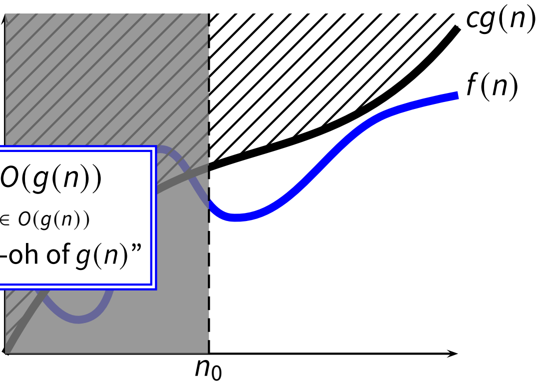

$$f(n) = O(g(n))$$

i.e., $f(n) \in O(g(n))$

"$f(n)$ is big-oh of $g(n)$"

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FINDEQUALS(A)
for i = 1 to length(A) − 1
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$$T(n) = \Theta(n^2)$$

- $n = length(A)$ is the ***size of the input***
- we measure the ***worst-case complexity***

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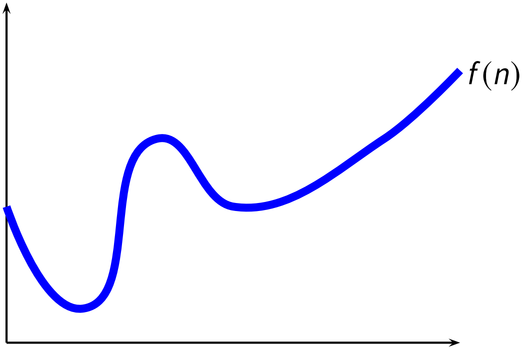

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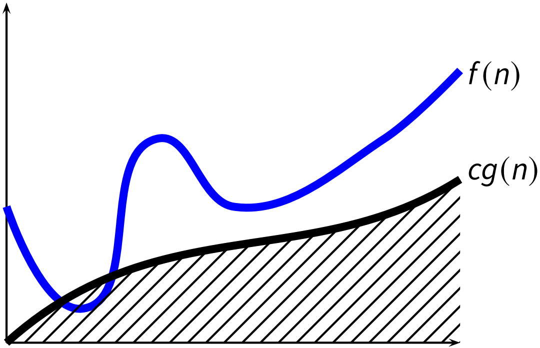

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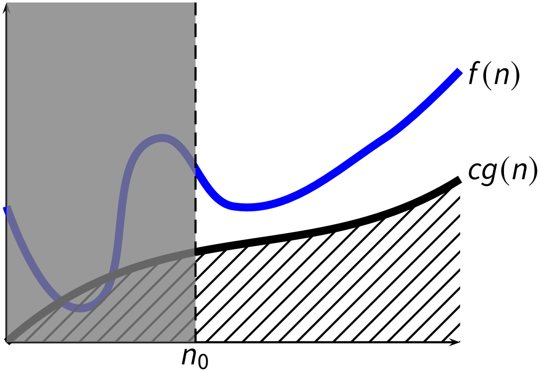

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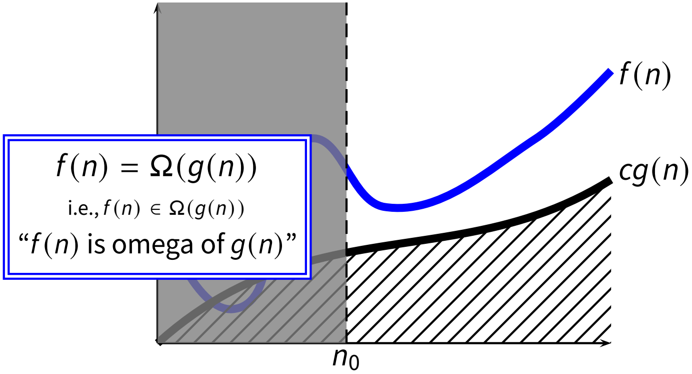

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Θ, O, and Ω as Relations

- *Theorem:* for any two functions $f(n)$ and $g(n)$,
  $f(n) = \Omega(g(n)) \wedge f(n) = O(g(n)) \Leftrightarrow f(n) = \Theta(g(n))$

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- The above theorem can be interpreted as saying

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- We use the o-notation to denote upper bounds that are *not* asymtotically tight. So, given a function $g(n)$, we define the family of functions $o(g(n))$

$$o(g(n)) = \{f(n) : \forall c > 0, \exists n_0 > 0 \\ : 0 \leq f(n) < cg(n) \text{ for all } n \geq n_0\}$$

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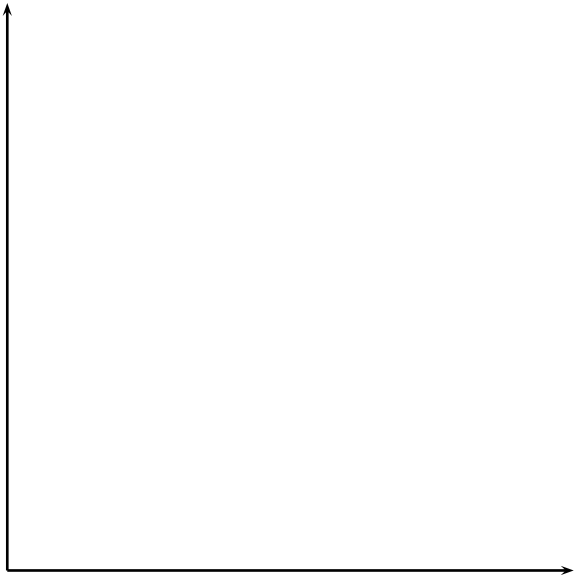

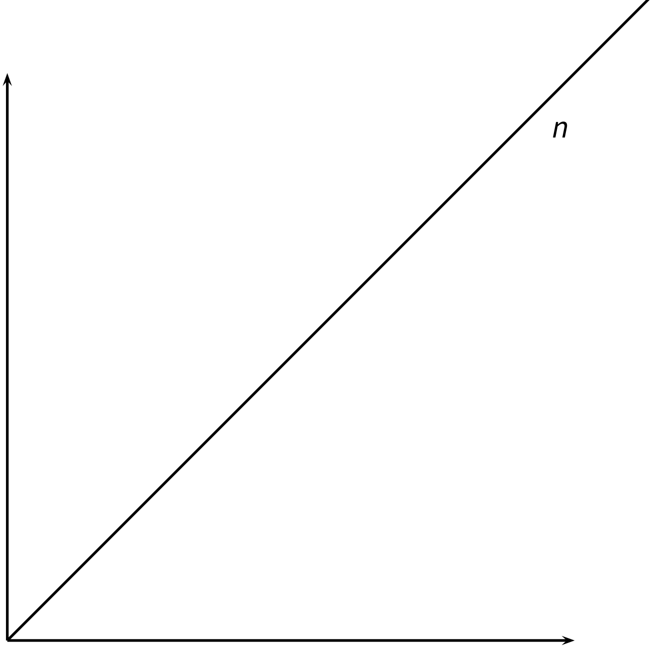
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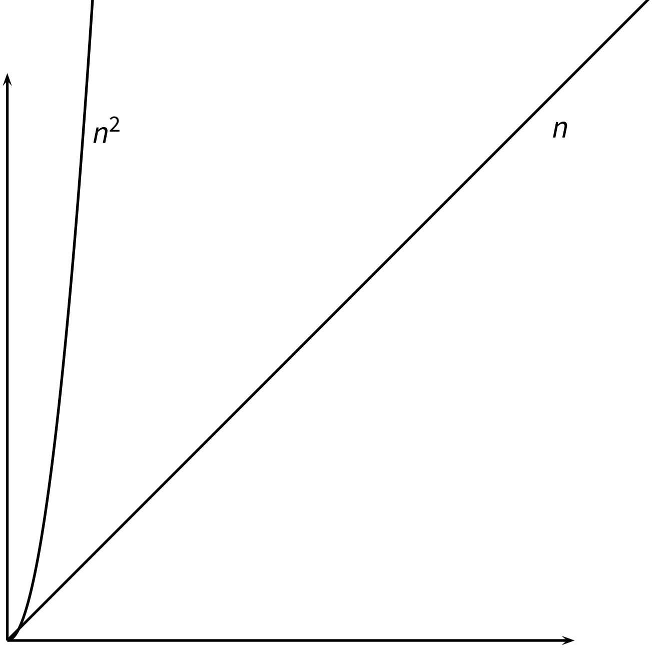
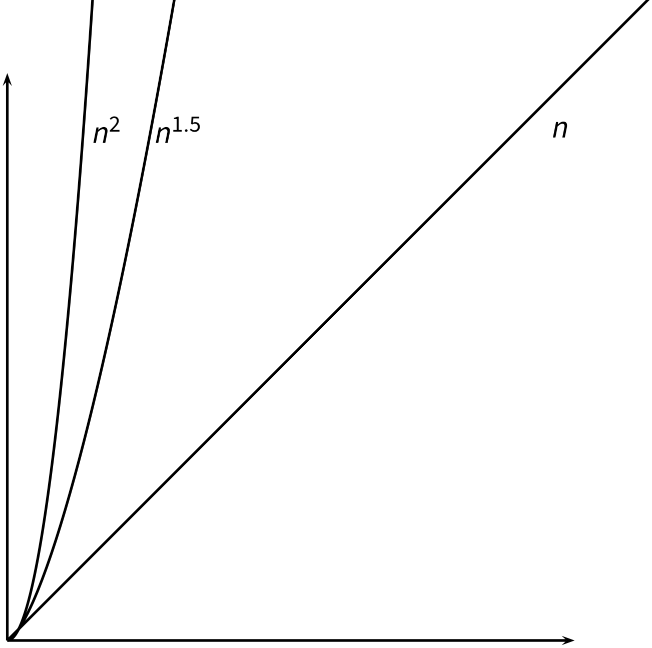
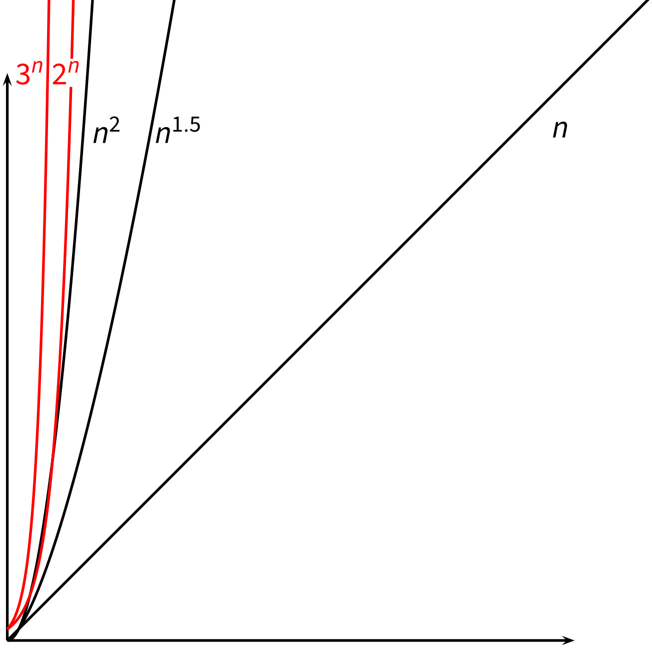


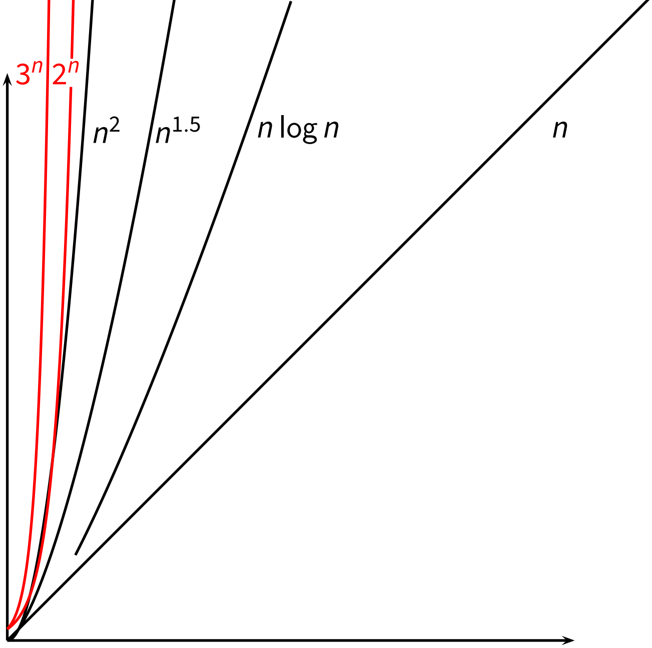
n^2
n



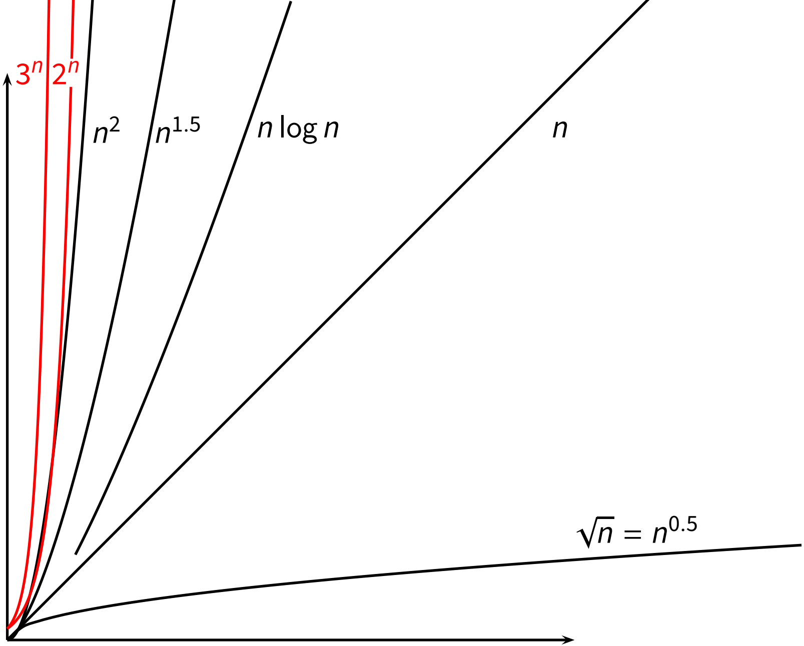
n^2
$n^{1.5}$
n



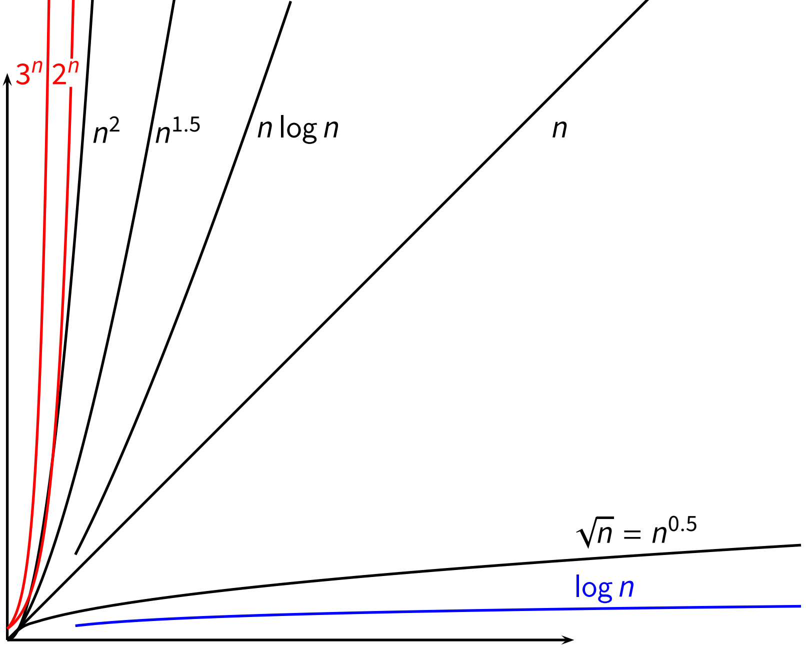
3^n
2^n
n^2
$n^{1.5}$
n



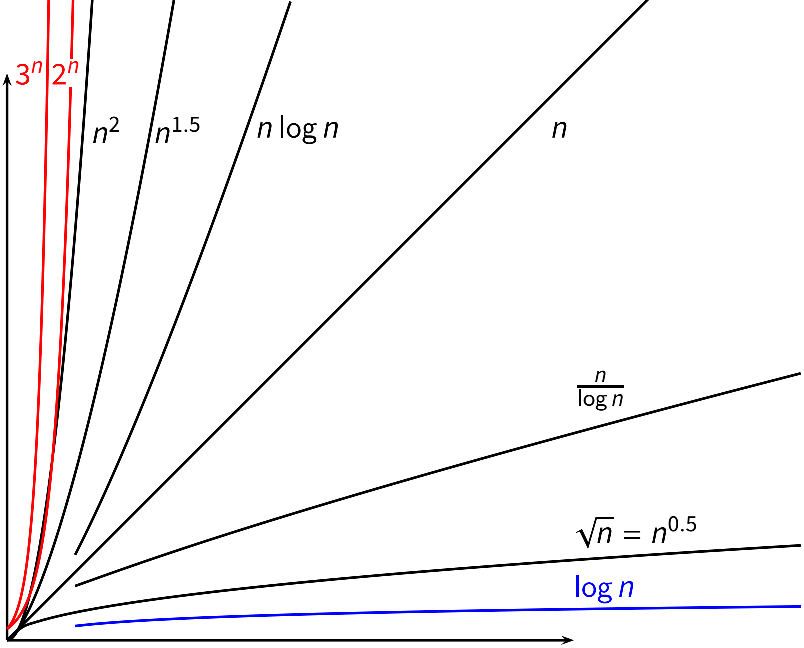
3^n
2^n
n^2
$n^{1.5}$
$n \log n$
n



3^n
2^n
n^2
$n^{1.5}$
$n \log n$
n
$\sqrt{n} = n^{0.5}$



3^n
2^n
n^2
n^1.5
n log n
n
√n = n^0.5
log n



3^n
2^n
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n
$\frac{n}{\log n}$
$\sqrt{n} = n^{0.5}$
$\log n$