More on Sorting: Quick Sort and Heap Sort

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Outline

- Another divide-and-conquer sorting algorithm
- The *heap*
- Heap sort

Sorting Algorithms Seen So Far

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Algorithm	Complexity			In place?
	worst	*average*	*best*	

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- *Basic step:* partition A in three parts based on a *chosen value* $v \in A$
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- *Can we partition A* ***in place****?*

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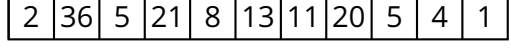

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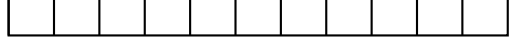

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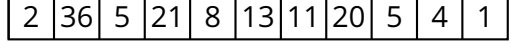

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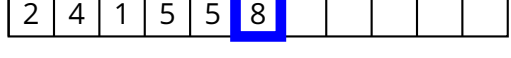

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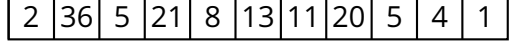

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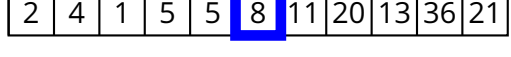

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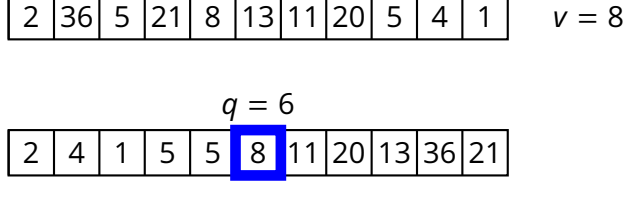

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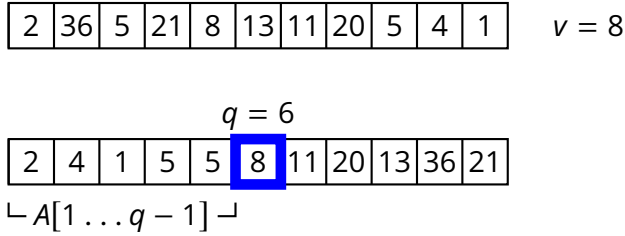

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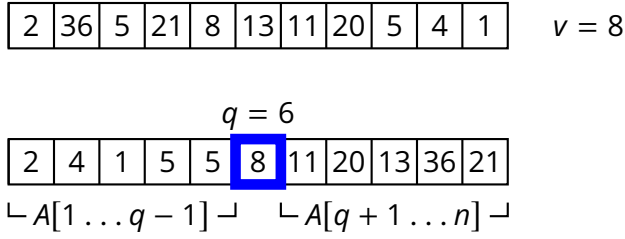

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```
QUICKSORT(A, begin, end)
1  if begin < end
2        q = PARTITION(A, begin, end)
3        QUICKSORT(A, begin, q − 1)
4        QUICKSORT(A, q + 1, end)
```

Partition

Partition

- Start with $q = 1$
  - i.e., *assume all elements are greater than the pivot*
- Scan the array left-to-right, starting at position 2
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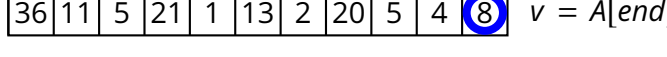

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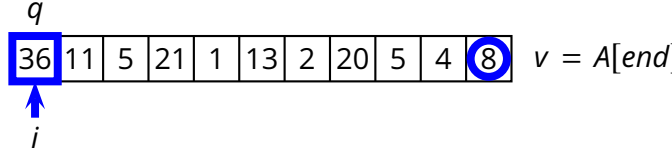

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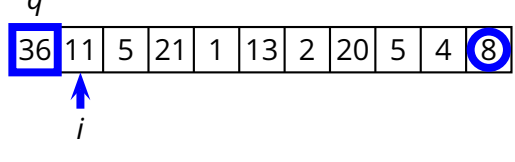

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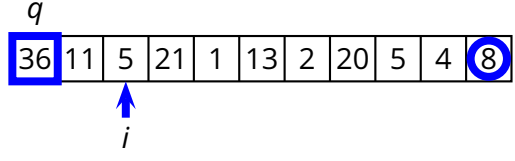

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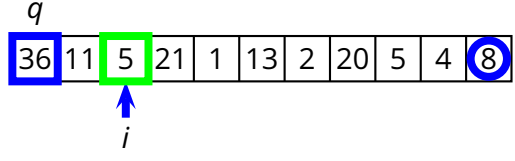

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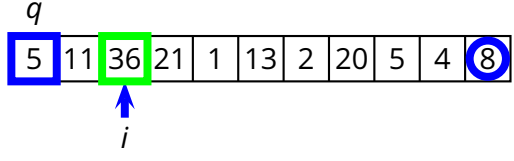

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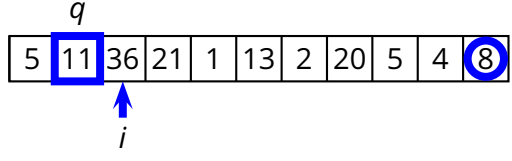

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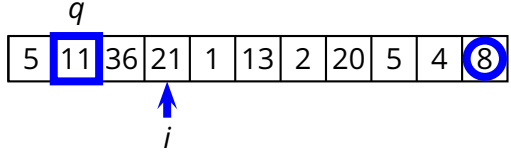

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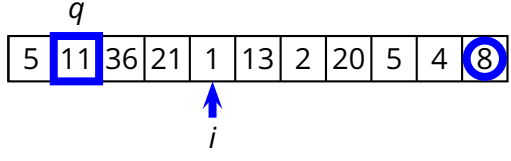

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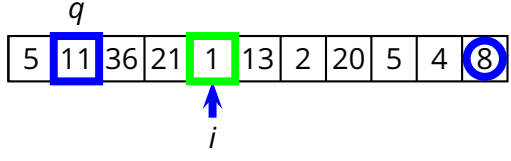

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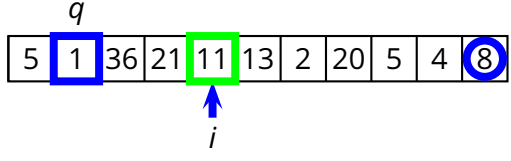

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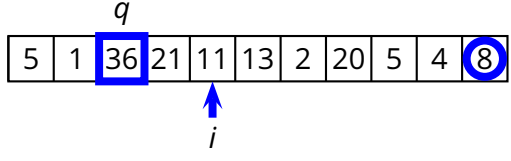

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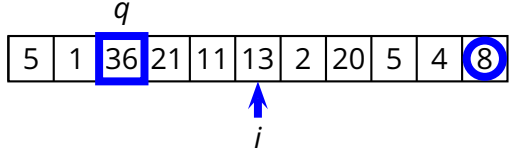

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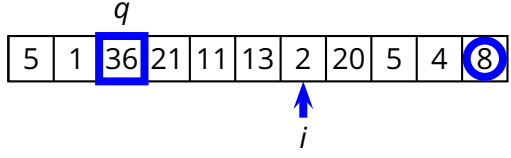

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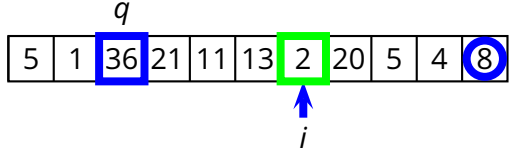

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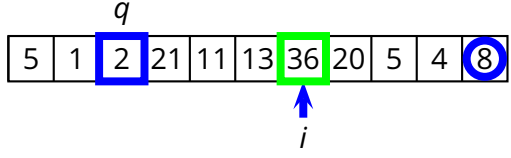

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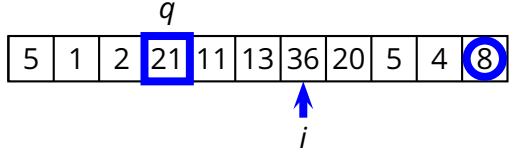

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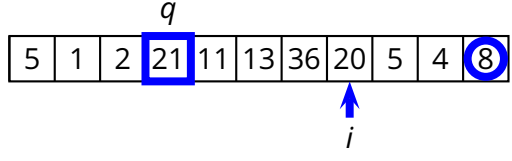

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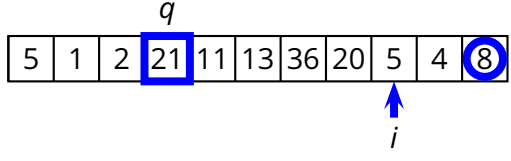

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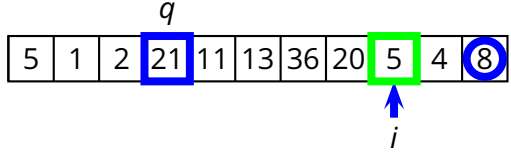

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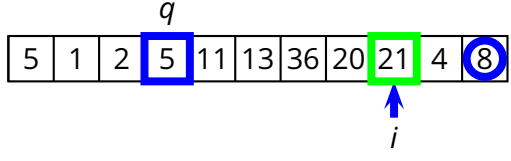

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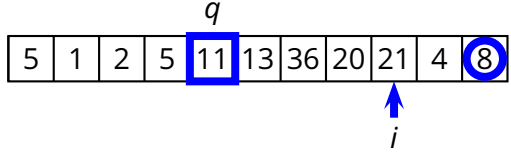

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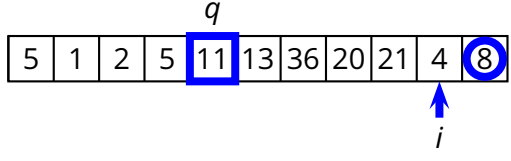

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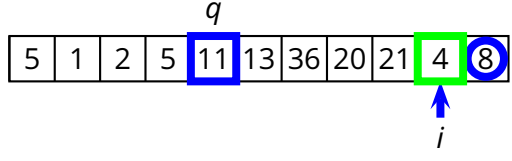

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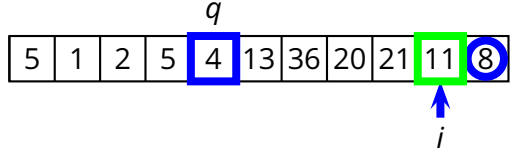

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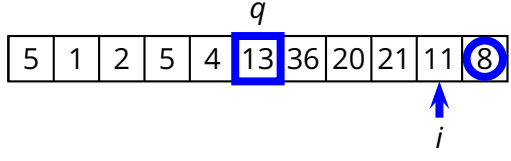

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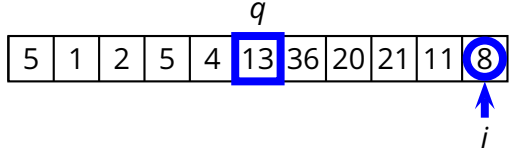

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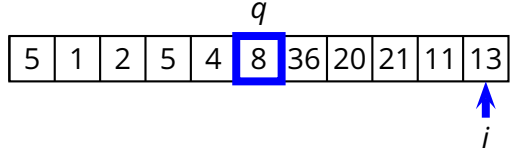

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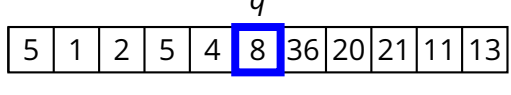

Complete QUICKSORT Algorithm

```
PARTITION(A, begin, end)
1  q = begin
2  v = A[end]
3  for i = begin to end
4       if A[i] ≤ v
5            swap A[i] and A[q]
6            q = q + 1
7  return q − 1
```

```
QUICKSORT(A, begin, end)
1  if begin < end
2       q = PARTITION(A, begin, end)
3       QUICKSORT(A, begin, q − 1)
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Complexity of PARTITION

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$$T(n) = \Theta(n)$$

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  - $q = begin$ or $q = end$

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  - $q = begin$ or $q = end$
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$$T(n) = \Theta(n^2)$$

Complexity of QUICKSORT (2)

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QUICKSORT(A, begin, end)
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- Best case
  - $q = \lceil n/2 \rceil$

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  - $q = \lceil n/2 \rceil$
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$$T(n) = \Theta(n \log n)$$

Sorting Algorithms Seen So Far

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Algorithm	Complexity			In place?
	worst	*average*	*best*	
INSERTION-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n)$	yes
SELECTION-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	yes
BUBBLE-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	yes
MERGE-SORT	$\Theta(n \log n)$	$\Theta(n \log n)$	$\Theta(n \log n)$	no

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QUICKSORT				

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MERGE-SORT	$\Theta(n \log n)$	$\Theta(n \log n)$	$\Theta(n \log n)$	no
QUICKSORT	$\Theta(n^2)$	$\Theta(n \log n)$	$\Theta(n \log n)$	yes

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QUICKSORT	$\Theta(n^2)$	$\Theta(n \log n)$	$\Theta(n \log n)$	yes
??	$\Theta(n \log n)$			***yes***

Binary Heap

Binary Heap

- Our first real ***data structure***

Binary Heap

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- Interface

Binary Heap

- Our first real ***data structure***
- Interface
  - **BUILD-MAX-HEAP**(A) rearranges A into a max-heap
  - **HEAP-INSERT**(H, *key*) inserts *key* in the heap
  - **HEAP-EXTRACT-MAX**(H) extracts the maximum key
  - H.*heap-size* is the number of keys in H

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- Two kinds of binary heaps

Binary Heap

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Binary Heap: Structure

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- Conceptually a full binary tree

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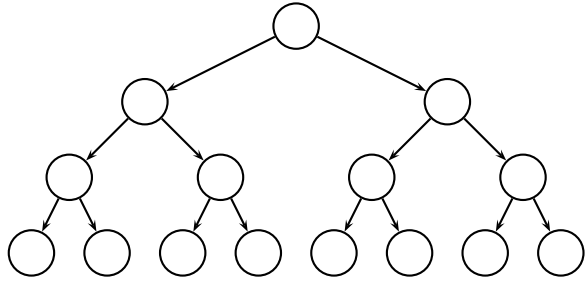

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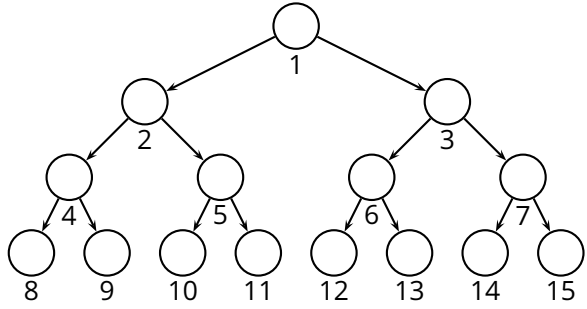

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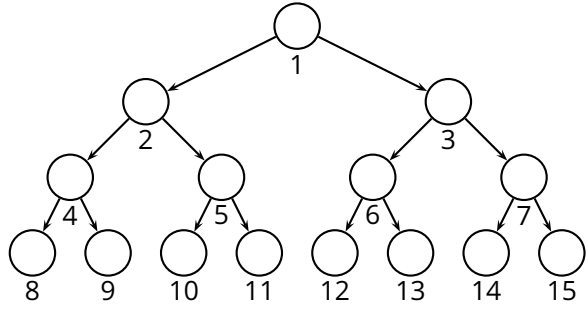

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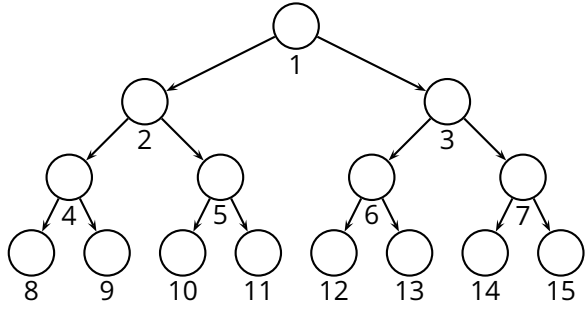

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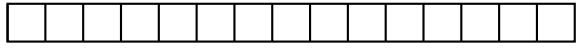

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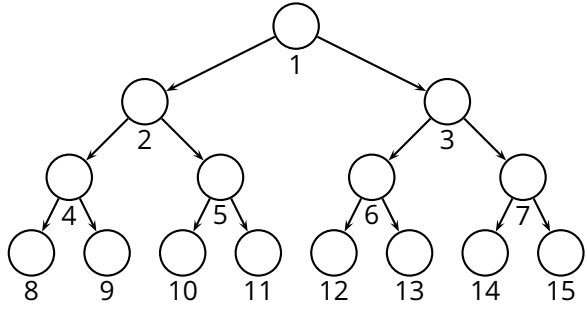

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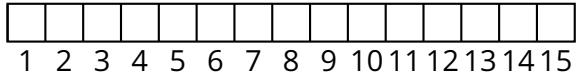

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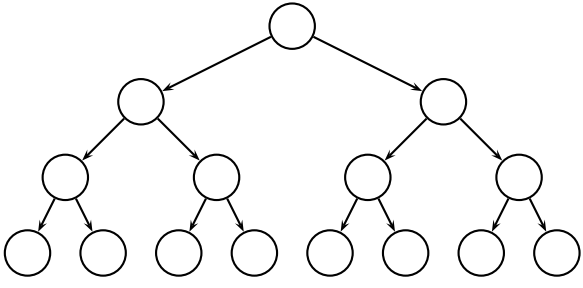

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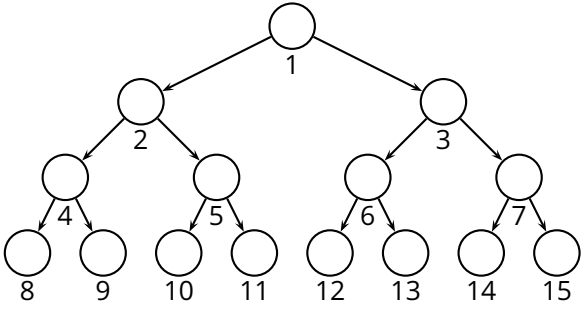

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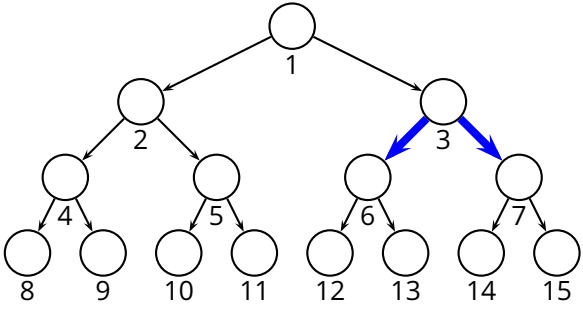

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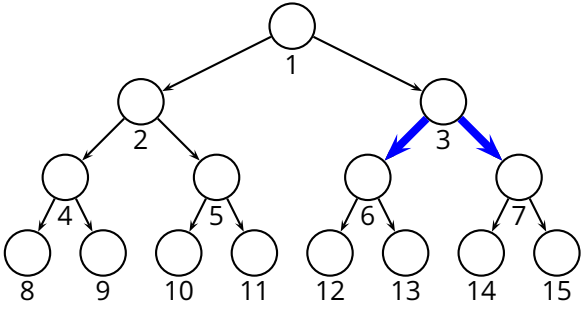

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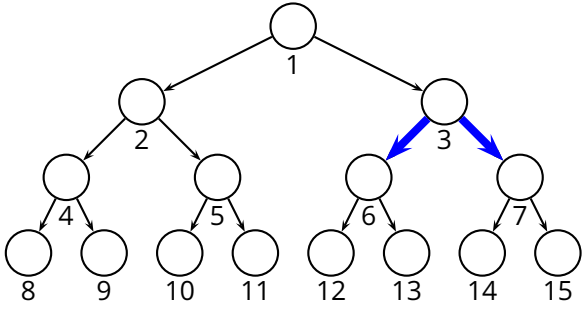

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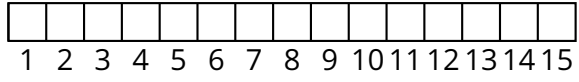

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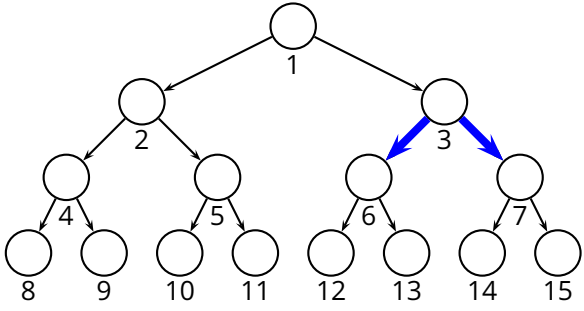

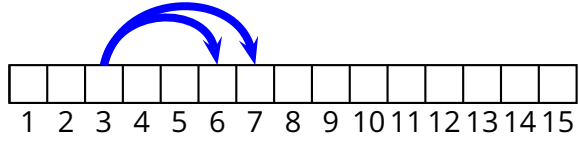

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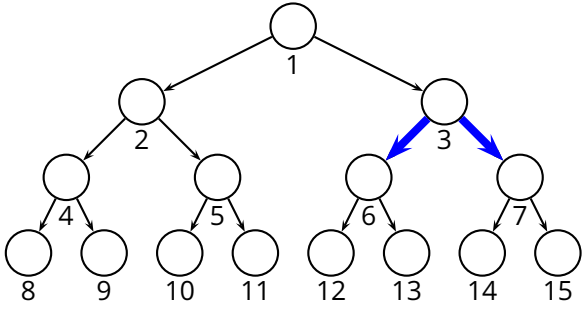

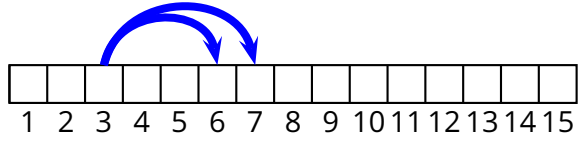

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- ***Max-heap property:*** for all $i > 1$ $\boxed{A[\textbf{PARENT}(i)] \geq A[i]}$

Example

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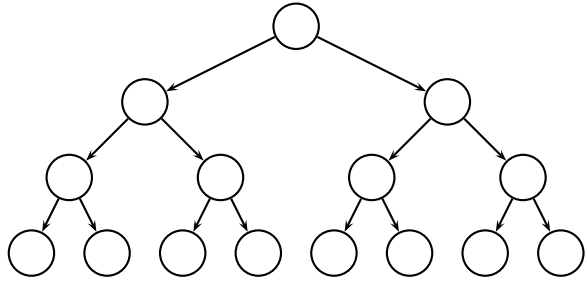

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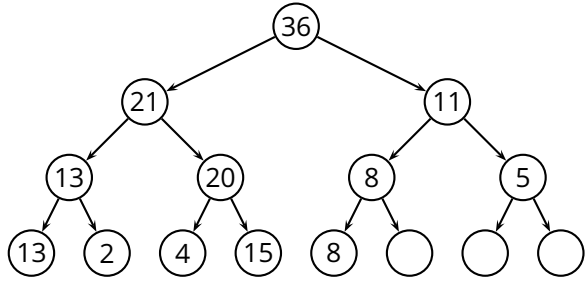

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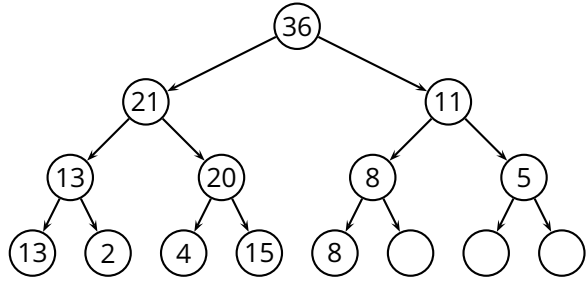

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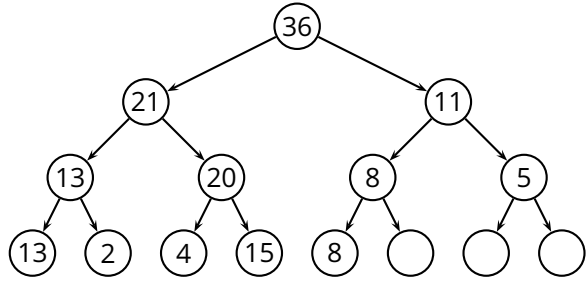

- Where is the max element?
- How can we implement **HEAP-EXTRACT-MAX**?

Heap-Extract-Max

- **HEAP-EXTRACT-MAX** procedure
  - extract the max key
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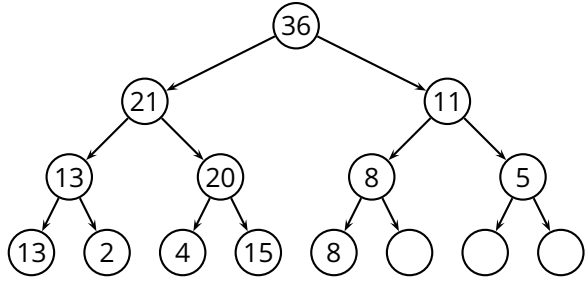

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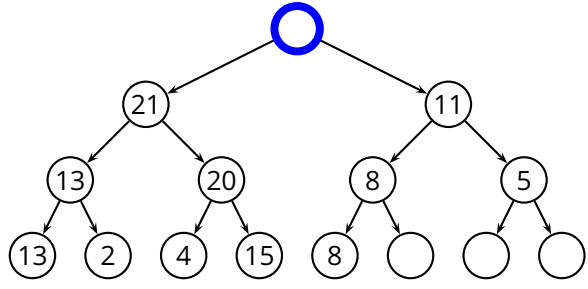

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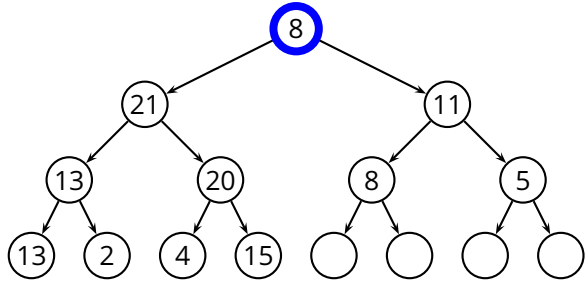

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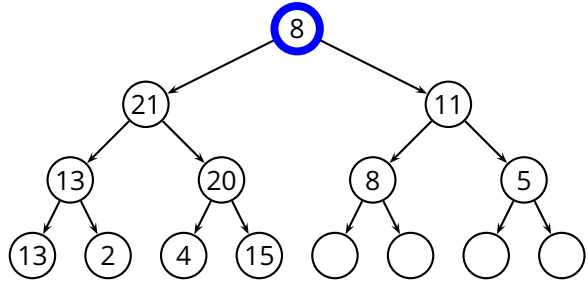

- Now we have two subtrees where the *max-heap property* holds

- **MAX-HEAPIFY**(A, i) procedure
  - *assume:* the *max-heap property* holds in the subtrees of node i
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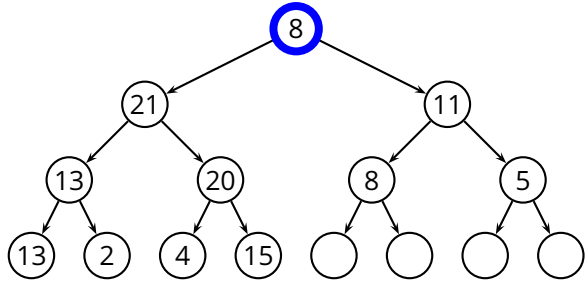

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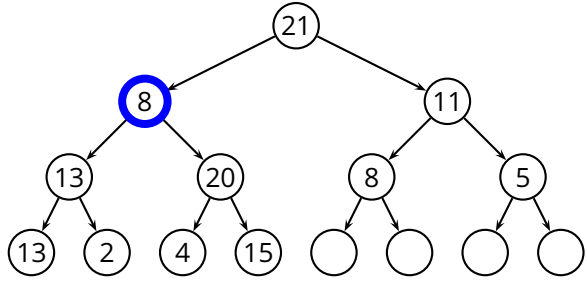

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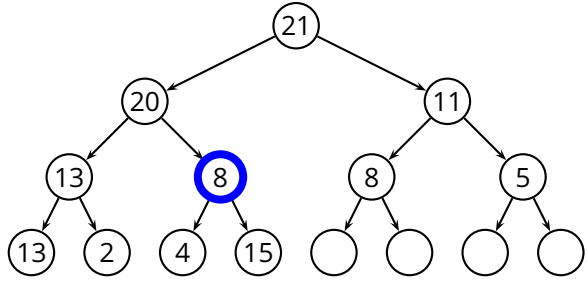

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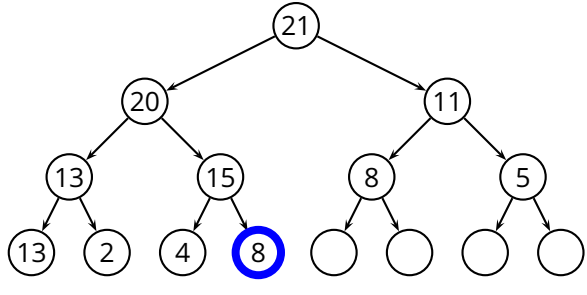

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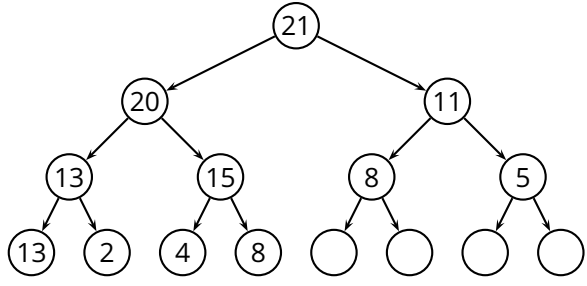

Max-Heapify

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MAX-HEAPIFY(A, i)
 1  l = LEFT(i)
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$$\boxed{T(n) = \Theta(\log n)}$$

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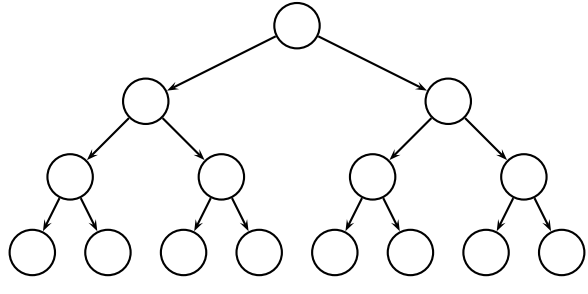

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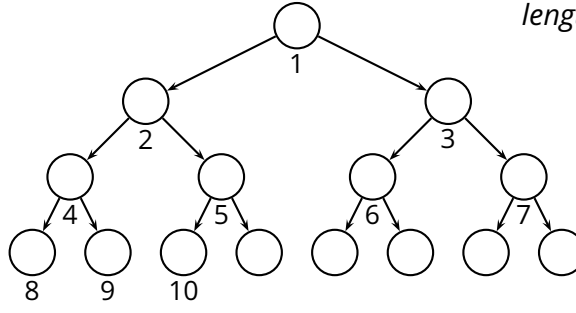

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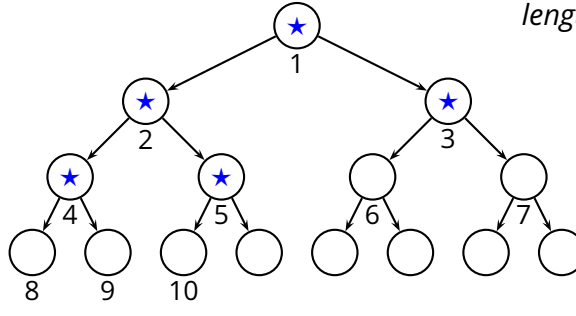

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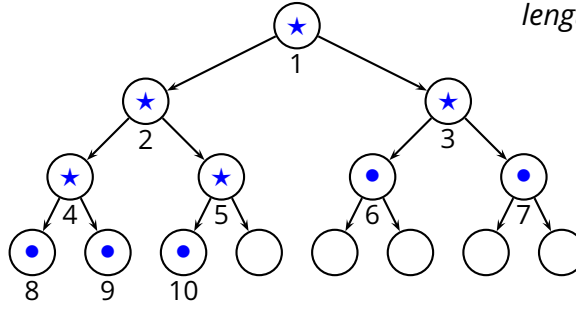

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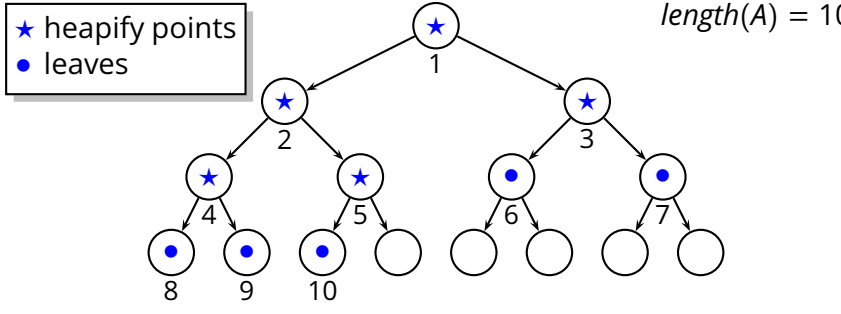

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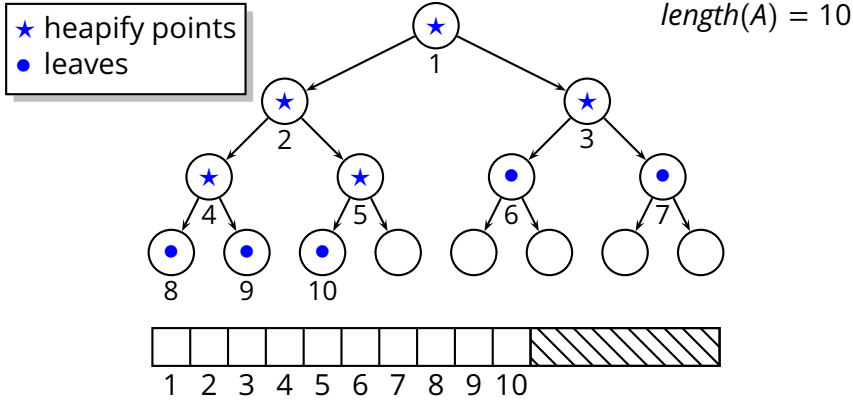

Building a Heap

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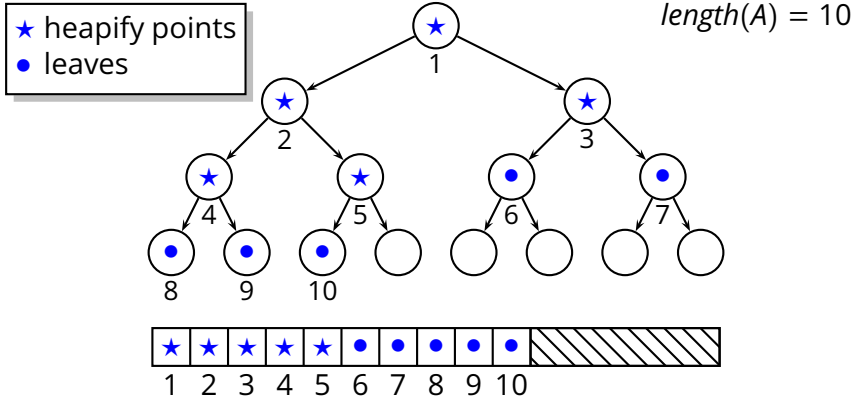
2 **for** $i = \lfloor length(A)/2 \rfloor$ **downto** 1
3 **MAX-HEAPIFY**(A, i)



Heap Sort

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HEAP-SORT(A)
1  BUILD-MAX-HEAP(A)
2  for i = length(A) downto 1
3      swap A[i] and A[1]
4      A.heap-size = A.heap-size − 1
5      MAX-HEAPIFY(A, 1)
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- What is the complexity of **HEAP-SORT**?

- Idea: we can use a heap to sort an array

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HEAP-SORT(A)
1  BUILD-MAX-HEAP(A)
2  for i = length(A) downto 1
3      swap A[i] and A[1]
4      A.heap-size = A.heap-size − 1
5      MAX-HEAPIFY(A, 1)
```

- What is the complexity of **HEAP-SORT**?

$$\boxed{T(n) = \Theta(n \log n)}$$

- Idea: we can use a heap to sort an array

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```

- What is the complexity of **HEAP-SORT**?

$$\boxed{T(n) = \Theta(n \log n)}$$

- Benefits
  - in-place sorting; worst-case is $\Theta(n \log n)$

Summary of Sorting Algorithms

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Algorithm	Complexity			In place?
	worst	*average*	*best*	
INSERTION-SORT				

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INSERTION-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n)$	yes
SELECTION-SORT				

Summary of Sorting Algorithms

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	worst	*average*	*best*	
INSERTION-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n)$	yes
SELECTION-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	yes
BUBBLE-SORT				

Summary of Sorting Algorithms

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	worst	*average*	*best*	
INSERTION-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n)$	yes
SELECTION-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	yes
BUBBLE-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	yes
MERGE-SORT				

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INSERTION-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n)$	yes
SELECTION-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	yes
BUBBLE-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	yes
MERGE-SORT	$\Theta(n \log n)$	$\Theta(n \log n)$	$\Theta(n \log n)$	no

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BUBBLE-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	yes
MERGE-SORT	$\Theta(n \log n)$	$\Theta(n \log n)$	$\Theta(n \log n)$	no
QUICK-SORT				

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SELECTION-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	yes
BUBBLE-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	yes
MERGE-SORT	$\Theta(n \log n)$	$\Theta(n \log n)$	$\Theta(n \log n)$	no
QUICK-SORT	$\Theta(n^2)$	$\Theta(n \log n)$	$\Theta(n \log n)$	yes

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INSERTION-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n)$	yes
SELECTION-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	yes
BUBBLE-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	yes
MERGE-SORT	$\Theta(n \log n)$	$\Theta(n \log n)$	$\Theta(n \log n)$	no
QUICK-SORT	$\Theta(n^2)$	$\Theta(n \log n)$	$\Theta(n \log n)$	yes
HEAP-SORT				

Summary of Sorting Algorithms

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	worst	*average*	*best*	
INSERTION-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n)$	yes
SELECTION-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	yes
BUBBLE-SORT	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	yes
MERGE-SORT	$\Theta(n \log n)$	$\Theta(n \log n)$	$\Theta(n \log n)$	no
QUICK-SORT	$\Theta(n^2)$	$\Theta(n \log n)$	$\Theta(n \log n)$	yes
HEAP-SORT	$\Theta(n \log n)$	$\Theta(n \log n)$	$\Theta(n \log n)$	yes