# Algorithms and Data Structures

## Course Introduction

Antonio Carzaniga

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Università della Svizzera italiana

February 21, 2017

# General Information

- On-line course information
  - on iCorsi: 'INFO.ALGO17' ***https://www2.icorsi.ch/course/view.php?id=5726***
  - and on my web page: ***http://www.inf.usi.ch/carzaniga/edu/algo/***
  - last edition also on-line: ***http://www.inf.usi.ch/carzaniga/edu/algo16s/***

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  - ***you are responsible for reading the announcements page or the messages sent through iCorsi***

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- Office hours
  - Antonio Carzaniga: *by appointment*
  - Enrique Fynn: *by appointment*
  - Seyed Ali Bahreinian: *by appointment*

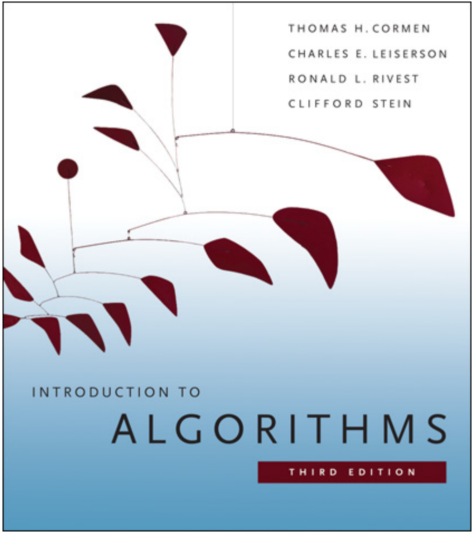
# Textbook

*Introduction to Algorithms*

Third Edition

Thomas H. Cormen
Charles E. Leiserson
Ronald L. Rivest
Clifford Stein

The MIT Press



# Evaluation

- +30% projects
  - 3–5 assignments
  - grades added together, thus resulting in a weighted average
- +30% midterm exam
- +40% final exam
- ±10% instructor's discretionary evaluation
  - participation
  - extra credits
  - trajectory
  - ...

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- −100% plagiarism penalties

# Plagiarism

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- "material" means ideas, words, code, suggestions, corrections on one's work, etc.
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  - e.g., software libraries
  - ***always clearly identify the external material, and acknowledge its source. Failing to do so means committing plagiarism.***
  - the work will be evaluated based on its *added value*

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  - the work will be evaluated based on its *added value*
- Plagiarism on an assignment or an exam will result in
  - failing that assignment or that exam
  - loosing one or more points *in the final note!*
- Penalties may be escalated in accordance with the regulations of the Faculty of Informatics

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  - at the instructor's discretion
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- Each late day will reduce the assignment's grade by *one third* of the total value of that assignment
  - **Corollary 1:** The grade of an assignment turned in more than two days late is 0
  - The proof of Corollary 1 is left as an exercise

Now let's move on to the real
interesting and fun stuff. . .

# Fundamental Ideas

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Johannes Gutenberg invents movable type and the printing press in Mainz, circa 1450 (already known in China, circa 1200 CE)

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    - methods for adding, multiplying, and dividing numbers (and more)
    - these procedures were ***precise***, ***unambiguous***, ***mechanical***, ***efficient***, and ***correct***
    - *they were **algorithms!***

Muhammad ibn Musa al-Khwārizmī

**Algorithms** are

***the essence***

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# Example

- A sequence of numbers

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$$

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$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$$

- The well-known Fibonacci sequence

Leonardo da Pisa (ca. 1170–ca. 1250)
son of Guglielmo “Bonaccio”
a.k.a. *Leonardo Fibonacci*

- Mathematical definition: $F_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F_{n-1} + F_{n-2} & \text{if } n > 1 \end{cases}$

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- Implementation on a computer:

**Racket**

```
(define (F n)
  (cond
    ((= n 0) 0)
    ((= n 1) 1)
    (else (+ (F (- n 1)) (F (- n 2))))))
```

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- Implementation on a computer:

Java

```
public class Fibonacci {
  public static int F(int n) {
    if (n == 0) {
      return 0;
    } else if (n == 1) {
      return 1;
    } else {
      return F(n-1) + F(n-2);
    } }
}
```

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- Implementation on a computer:

**C or C++**

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int F(int n) {
  if (n == 0) {
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- Implementation on a computer:

Ruby

```
def F(n)
  case n
    when 0
      return 0
    when 1
      return 1
    else
      return F(n-1) + F(n-2)
  end
end
```

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- Implementation on a computer:

Python

```
def F(n):
  if n == 0:
    return 0
  elif n == 1:
    return 1
  else:
    return F(n-1) + F(n-2)
```

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- Implementation on a computer:

very concise C/C++ (or Java)

```
int F(int n) { return (n<2)?n:F(n-1)+F(n-2); }
```

# The Fibonacci Sequence

- Mathematical definition: $F_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F_{n-1} + F_{n-2} & \text{if } n > 1 \end{cases}$
- Implementation on a computer:

“pseudo-code”

```
FIBONACCI(n)
if n == 0
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elseif n == 1
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else return FIBONACCI(n − 1) + FIBONACCI(n − 2)
```

# Questions on Our First Algorithm

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   - for every valid input, does it terminate?
   - if so, does it do the right thing?

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3. Can we do better?

# Correctness

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- The algorithm is clearly correct
    - assuming $n \geq 0$

# Performance

- How long does it take?

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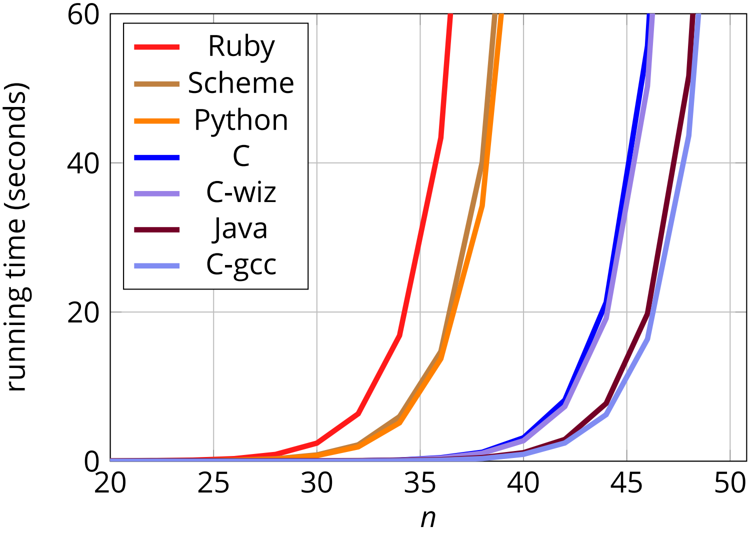
  Let's try it out. . .


60
40
20
0
20
25
30
35
40
45
50
n
running time (seconds)
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- Different implementations perform differently
  - it is better to let the compiler do the optimization
  - simple language tricks don’t seem to pay off
- However, the differences are not substantial
  - *all* implementations sooner or later seem to hit a wall…
- Conclusion: ***the problem is with the algorithm***

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$T(0) = 2; T(1) = 3$

$T(n) = T(n-1) + T(n-2) + 3 \quad \Rightarrow T(n) \geq F_n$

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- $T(n)$ ***grows exponentially*** with $n$
- Can we do better?

# A Better Algorithm

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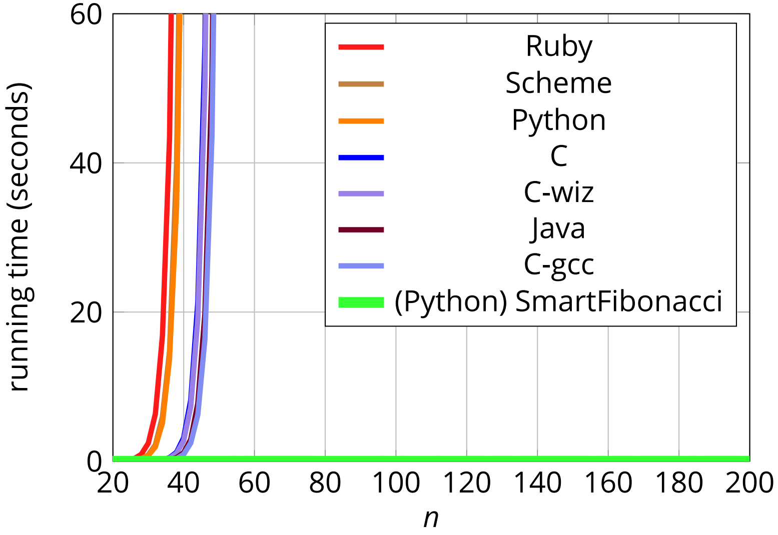
```
SMARTFIBONACCI(n)
if n == 0
        return 0
elseif n == 1
        return 1
else pprev = 0
        prev = 1
        for i = 2 to n
                f = prev + pprev
                pprev = prev
                prev = f
return f
```


60
40
20
0
running time (seconds)
20
40
60
80
100
120
140
160
180
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n
Ruby
Scheme
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(Python) SmartFibonacci

# Complexity of SmartFibonacci

```
SmartFibonacci(n)
if n == 0
        return 0
elseif n == 1
        return 1
else prev = 0
        pprev = 1
        for i = 2 to n
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```

$T(n) =$

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return f
```

$T(n) = 6 + 6(n-1)$

# Complexity of SMARTFIBONACCI

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                prev = f
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```

$T(n) = 6 + 6(n - 1) = 6n$

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$T(n) = 6 + 6(n-1) = 6n$

The *complexity* of **SmartFibonacci**($n$) is ***linear*** in $n$